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Geometrical interpretation of negative radiation forces of acoustical Bessel beams on spheres

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Various researchers have predicted situations where the acoustical or optical radiation force on a sphere centered on a Bessel beam is opposite the direction of beam propagation. We develop the analogy between acoustical and optical radiation forces of arbitrary-order helicoidal and ordinary Bessel beams to gain insight into negative radiation forces. The radiation force is expressed in terms of the asymmetry of the scattered field, the scattered power, the absorbed power, and the conic angle of the Bessel beam and is related to the partial-wave coefficients for the scattering. Negative forces only occur when the scattering into the backward hemisphere is suppressed relative to the scattering into the forward hemisphere. Absorbed power degrades negative radiation forces.

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I. INTRODUCTION

Recent predictions that spherical particles placed on the axis of an electromagnetic Bessel beam (EMBB) may be attracted to the source of the beam for appropriately selected particle properties and beam parameters [1,2] make it appropriate to consider prior results predicting negative radiation forces on spherical objects placed on the axis of acoustical Bessel beams [3–6]. Since Bessel beams are translationally invariant, the force in the EMBB case is not associated with the gradient force commonly used to trap small objects at the focus of converging light beams. For situations producing negative acoustical radiation forces for which the associated scattering pattern is also evaluated, the scattering into the backward hemisphere is suppressed relative to the scattering into the forward hemisphere [3–5]. This result is suggestive of the correlation between the shape of the scattering pattern, as characterized by the asymmetry factor, and negative radiation forces predicted in one of the EMBB investigations [1]. In the present research we identify the analogy between optical and acoustical radiation forces by showing the similarity of the dependence on the asymmetry in the scattering by spheres illuminated by Bessel beams. The research illustrates geometrical aspects of the momentum transport associated with negative forces and the importance of the conic angle of the beam.

This Rapid Communication shows that the acoustical radiation force can be expressed directly in terms of the asymmetry of the acoustical scattering pattern and the computed acoustical extinction. Throughout this analysis the propagation of waves in the external media is assumed to be free of absorption, as was also the case in the calculation of radiation forces in EMBBs. The analysis includes the case of helicoidal acoustical Bessel beams with an $\exp(im\phi)$ azimuthal phase dependence where the beam order m is an arbitrary integer [5–7]. The nonhelicoidal case is recovered by setting $m = 0$ [3,4,8]. The notation follows a generalization of the notation used in the closely related prior work on acoustical beams [3–5,7,8] and light scattering [9,10]. In addition to considering the asymmetry in the acoustical scattering it is also necessary to consider the scattered power and, when the sphere is absorptive, absorbed power. The results clarify the degradation of negative radiation forces introduced by energy absorption by the sphere.

While the emphasis is on gaining insight into the reason why negative forces are predicted to be produced in idealized situations, if such forces can be produced with acoustical or optical beams there may be diverse applications for the manipulation of spherical and nearly spherical objects with beams of traveling waves [1,2,11,12].

II. ACOUSTICAL BESSEL BEAMS AND SCATTERING BY A SPHERE

The spatial part of the complex velocity potential for a helicoidal Bessel beam in an ideal fluid may be expressed as follows in cylindrical coordinates: $\psi_i = \psi_0 i^m J_m(\mu\rho)\exp(i\kappa z + im\phi)$, where ψ_0 is a real-valued amplitude constant, z is the axial coordinate, ρ is the radial coordinate, J_m is a Bessel function of order m, κ and μ denote the axial and radial wave numbers, and the wave number $k = \sqrt{\mu^2 + \kappa^2} = \omega/c_0$, with c_0 the speed of sound in the surrounding fluid and ω the beam's frequency for the time dependence $\exp(-i\omega t)$. [The dependence of the pressure and the velocity on the velocity potential is given in the paragraph following Eq. (4).] The i^m is included for convenience and for making the distinction between positive and negative m primarily the dependence on ϕ since $i^m J_m = i^{-m} J_{-m}$. The Bessel beam has the geometric parameter $\beta = \arctan(\mu/\kappa)$, which is the conic angle of the beam's wave-vector components relative to the z axis [3,13].

The property that the Bessel beam is equivalent to the superposition of plane-wave components allows the beam to be expressed as a partial-wave expansion. Following the approach of Durnin *et al.* [13], the Bessel beam is expressed in spherical coordinates as the superposition (see the Appendix of Ref. [7]) $\psi_i = (2\pi)^{-1}\psi_0 \int_0^{2\pi} \exp(i\mathbf{k}'\cdot\mathbf{r} + im\phi')d\phi'$, where $\mathbf{k}'(k,\beta,\phi') = k\mathbf{n}'$ is the wave-vector component with $\mathbf{n}'\cdot\mathbf{n}_z = \cos\beta$, $\mathbf{n}_z$ is a unit vector along the z axis, and $\mathbf{r}(r,\theta,\phi) = r\mathbf{n}$ is the field point having a polar angle θ relative to the z axis. The partial-wave expansion of ψ_i is given by inserting the plane-wave expansion $\exp(ikr\mathbf{n}'\cdot\mathbf{n}) = \sum_{n=0}^{\infty}(2n + 1)i^n j_n(kr)P_n(\mathbf{n}'\cdot\mathbf{n})$ (see p. 471 of Ref. [14]) into the

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superposition representation of ψ_i. This yields a summation of terms containing the integration $A_{nm} = (2\pi)^{-1}\int_0^{2\pi} P_n(\mathbf{n}\cdot\mathbf{n}')e^{im\phi'}d\phi'$, which may be evaluated by using one form of the spherical harmonic addition theorem [Eq. (3.68) in Ref. [14]] and by noting that only certain terms survive the integration. This gives

$$\psi_i = \psi_0 e^{im\phi}\sum_{n=|m|}^{\infty}(2n+1)\frac{(n-m)!}{(n+m)!}i^n j_n(kr)P_n^m(b)P_n^m(w), \tag{1}$$

where $w = \cos\theta$, $b = \cos\beta$, j_n is a spherical Bessel function, and P_n^m is an associated Legendre function. The series converges with a finite number of terms. With $j_n(kr) = h_n^{(1)}(kr)/2 + h_n^{(2)}(kr)/2$, where $h^{(1)}$ and $h^{(2)}$ are a spherical Hankel function of the first and second kinds, the incident beam is a superposition of outgoing and incoming spherical-wave components ψ_{i1} and ψ_{i2}. Only the far-field form is needed here; $j_n(kr)$ is replaced by the asymptotic form $j_n(kr) \sim i^{-(n+1)}e^{ikr}/2kr + i^{n+1}e^{-ikr}/2kr$ for $kr \gg n$.

The analytical solution of the scattering by an isotropic sphere with a radius a in an inviscid fluid centered on the beam's axis can be expressed in terms of a superposition of the scattering for each plane-wave component [7,8]. The superposition of the scattering from the plane-wave components of the beam is $\psi_s = (2\pi)^{-1}\psi_0\int_0^{2\pi}\psi_s'\exp(im\phi')d\phi'$, where ψ_s' is the scattering for the plane-wave component $\exp(ikr\mathbf{n}'\cdot\mathbf{n})$, given in the far field as $\psi_s' = (a/2r)f(ka,\mathbf{n}'\cdot\mathbf{n})\exp(ikr)$, with $f(ka,\mathbf{n}'\cdot\mathbf{n}) = -(i/ka)\sum_{n=0}^{\infty}(2n+1)(s_n-1)P_n(\mathbf{n}'\cdot\mathbf{n})$. The notation is similar to the notation used for quantum mechanical scattering. The scattering functions s_n are functions of ka determined by the composition of the sphere and the acoustical properties of the surrounding fluid. These functions are known from the analysis of plane-wave scattering for a wide variety of spheres [3–8,15]. Hence the scattering of the beam in the far field is expressed in the terms of a partial-wave series as

$$\psi_s(r,\theta,\phi) = \psi_0(a/2r)\exp(ikr)F(w,\phi), \tag{2}$$

$$\begin{aligned} F(w,\phi) &= (2\pi)^{-1}\int_0^{2\pi} f(ka,\mathbf{n}'\cdot\mathbf{n})\exp(im\phi')d\phi' \\ &= \frac{\exp(im\phi)}{ika}\sum_{n=|m|}^{\infty}(s_n-1)(2n+1) \\ &\quad\times\frac{(n-m)!}{(n+m)!}P_n^m(b)P_n^m(w), \end{aligned} \tag{3}$$

where the integral for $F(w,\phi)$ is evaluated like that for the incident beam with a summation containing terms proportional to A_{nm}. The scattered field has the same azimuthal phase dependence as the incident beam due to the axial symmetry of the scatterer. The scattering reduces to that for the $m = 0$ and 1 cases [7,8]. Note that in the case of an ideal sphere causing no dissipation of energy, the complex functions s_n are unimodular: $|s_n| = 1$; otherwise, $|s_n| < 1$. It is convenient to write the normalized partial-wave amplitude as $(s_n-1)/2 = \alpha_n + i\beta_n$, with α_n and β_n the real and imaginary parts.

III. AXIAL RADIATION FORCE ON THE SPHERE

Using the far-field scattering, the static radiation force is evaluated by a surface integration of the time-averaged radiation stress tensor for the total sound field over a fixed spherical surface of radius r with $kr \gg 1$ [3,5,16], $\mathbf{F} = \int_S \langle\mathbf{S}_T\rangle\cdot d\mathbf{A}$, giving

$$\mathbf{F} = \int_S\left(\frac{\rho_0}{2}\langle|\mathbf{u}|^2\rangle - \frac{1}{2\rho_0 c_0^2}\langle p^2\rangle\right)d\mathbf{A} - \int_S \rho_0\langle\mathbf{uu}\rangle\cdot d\mathbf{A}, \tag{4}$$

where the area differential $d\mathbf{A}$ is directed radially outward, ρ_0 is the density of the surrounding fluid, and $\langle\,\rangle$ denotes a time average of the quantity in the angular brackets. The real-valued time-varying first-order pressure and velocity are expressed in terms of velocity potential as $p = \mathrm{Re}(i\omega\rho_0\psi e^{-i\omega t})$, and $\mathbf{u} = \mathrm{Re}(\nabla\psi e^{-i\omega t})$. By writing the fields as the superposition of the incident beam and the scattering field using Eq. (2), the axial radiation force on the sphere becomes [3,5,16] $F_z = \pi a^2 I_0 c_0^{-1} Y_p$, where $Y_p = Y_1 + Y_2 - Y_3$ is the dimensionless radiation force and

$$Y_1 = -(4\pi)^{-1}\int_0^{2\pi}\int_{-1}^{1}|F(w,\phi)|^2 w\,dw\,d\phi, \tag{5}$$

$$Y_2 = -r(2\pi\psi_0 a)^{-1}\mathrm{Re}\int_0^{2\pi}\int_{-1}^{1}\psi_i^* F(w,\phi)e^{ikr}w\,dw\,d\phi, \tag{6}$$

$$Y_3 = -r(2\pi\psi_0 ka)^{-1}\mathrm{Im}\int_0^{2\pi}\int_{-1}^{1}\left(\frac{\partial\psi_i}{\partial z}\right)^* F(w,\phi)e^{ikr}dw\,d\phi, \tag{7}$$

where $I_0 = (\rho_0 c_0/2)(k\psi_0)^2$ characterizes the beam's intensity, Re and Im designate real and imaginary parts of a complex quantity, and $*$ denotes complex conjugation. The integral in Eq. (4) containing only the incident wave vanishes since the radiation force vanishes in the absence of a sphere. The integrals are then evaluated using the far-field partial-wave representations of ψ_i in Eq. (1) and of the scattering F in Eq. (3). Both Y_2 and Y_3 have two summations, letting $Y_2 = Y_{21} + Y_{22}$ and $Y_3 = Y_{31} + Y_{32}$, which are associated with the outgoing and incoming components ψ_{i1} and ψ_{i2} of the incident beam. For Y_3 the first step is to set $\partial\psi_{i1,i2}/\partial z = \hat{z}\cdot\nabla\psi_{i1,i2} \simeq w\partial\psi_{i1,i2}/\partial r$ and $\partial\psi_{i1,i2}/\partial r \simeq (+,-)ik\psi_{i1,i2}$ for $kr \gg 1$. Then it is trivial to identify $Y_{31} = -Y_{21}$ and $Y_{32} = Y_{22}$, and hence $Y_2 - Y_3 = 2Y_{21}$. The task then reduces to evaluating Y_1 and Y_{21} only. Their integration over ϕ yields a factor of 2π by noticing that the integrands are independent of ϕ. The remaining double summation contains an integration over w: $I_{nq}^m = \int_{-1}^{1} wP_n^m(w)P_q^m(w)dw$, which is evaluated by using the relations

$$(2n+1)wP_n^m(w) = (n-m+1)P_{n+1}^m(w) + (n+m)P_{n-1}^m(w), \tag{8}$$

$$\int_{-1}^{1} P_n^m(w)P_q^m(w)dw = \frac{2}{2n+1}\frac{(n+m)!}{(n-m)!}\delta_{n,q}, \tag{9}$$

where $\delta_{n,q} = 1$ if $n = q$ and $\delta_{n,q} = 0$ if $n \neq q$. The results for Y_1 and $Y_{23} = Y_2 - Y_3$ are

$$Y_1 = -\left(\frac{2}{ka}\right)^2 \sum_{n=|m|}^{\infty} [2(\alpha_n\alpha_{n+1} + \beta_n\beta_{n+1})] \times \frac{(n-m+1)!}{(n+m)!} P_n^m(b) P_{n+1}^m(b), \tag{10}$$

$$Y_{23} = -\left(\frac{2}{ka}\right)^2 \sum_{n=|m|}^{\infty} \alpha_n \frac{(n-m)!}{(n+m)!} P_n^m(b)\big[(n-m+1)P_{n+1}^m(b)\big] - \left(\frac{2}{ka}\right)^2 \sum_{n=|m|}^{\infty} \alpha_n \frac{(n-m)!}{(n+m)!} P_n^m(b)\big[(n+m)P_{n-1}^m(b)\big]. \tag{11}$$

Simplifying Eq. (11) by using the recurrence relation given in Eq. (8) with w being read as b gives

$$Y_{23} = \frac{b}{(ka)^2} \sum_{n=|m|}^{\infty} (-4\alpha_n)(2n+1)\frac{(n-m)!}{(n+m)!}\big[P_n^m(b)\big]^2, \tag{12}$$

which is used below in the geometrical interpretation of the radiation force.

One may rewrite the second summation in Eq. (11) by discarding the vanishing term $n = |m|$, reindexing with $n = n' + 1$, and replacing n' by n such that

$$Y_{23} = -\left(\frac{2}{ka}\right)^2 \sum_{n=|m|}^{\infty} (\alpha_n + \alpha_{n+1})\frac{(n-m+1)!}{(n+m)!} P_n^m(b) P_{n+1}^m(b). \tag{13}$$

Then the result for Y_p with Y_1 in Eq. (10) and Y_{23} in Eq. (13) agrees with the results for the cases $m = 0$ and 1 in Refs. [3,5] and with the result of a different approach given in Ref. [6] ($Y_{J_m,p}$ therein).

IV. GEOMETRICAL INTERPRETATION OF THE RADIATION FORCE

We can connect the radiation force to the asymmetry of the scattering and the scattered and absorbed power. Using the far field scattering, the absorbed power is evaluated by the surface integral of the time-averaged energy flux of the total field over a fixed spherical surface of radius r with $kr \gg 1$: $P_{\text{abs}} = -\int_S \langle p\mathbf{u}\rangle \cdot d\mathbf{A}$. We could write the total field as a superposition of the incident wave and the scattering field. The term containing only the incident wave may be omitted since the absorption vanishes in the absence of the sphere. The two terms crossing the beam and the scattering are the extinction power $P_{\text{ext}} = -\int_S \langle p_i\mathbf{u}_s + p_s\mathbf{u}_i\rangle \cdot d\mathbf{A}$, that is, the power extracted from the beam. The term containing only the scattering is associated with the scattered power $P_{\text{sca}} = \int_S \langle p_s\mathbf{u}_s\rangle \cdot d\mathbf{A}$. It has $P_{\text{abs}} = P_{\text{ext}} - P_{\text{sca}}$. Then it follows from a procedure analogous to the evaluation of Eq. (4) that $P_{\text{sca,ext,abs}} = \pi a^2 I_0 Q_{\text{sca,ext,abs}}$, where the Q's denote acoustical counterparts of optical efficiency factors [9,10]:

$$Q_{\text{sca}} = (4\pi)^{-1} \int_0^{2\pi}\int_{-1}^{1} |F(w,\phi)|^2 dw\, d\phi, \tag{14}$$

$$Q_{\text{ext}} = Q_{is} + Q_{si} = -r^2(\pi a^2 \psi_0^2 k)^{-1} \text{Im} \int_0^{2\pi}\int_{-1}^{1} \left(\frac{\partial\psi_i}{\partial r}\psi_s^* + \frac{\partial\psi_s}{\partial r}\psi_i^*\right) \times dw\, d\phi, \tag{15}$$

and $Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}$. Again, the integrals are evaluated using the far-field partial-wave representations of ψ_i in Eq. (1) and of the scattering F in Eq. (3). With $\psi_i = \psi_{i1} + \psi_{i2}$, each term in Q_{ext} involves two summations, letting $Q_{is} = Q_{is1} + Q_{is2}$ and $Q_{si} = Q_{si1} + Q_{si2}$. By recognizing $\partial\psi_{i1}/\partial r \simeq ik\psi_{i1}$, $\partial\psi_{i2}/\partial r \simeq -ik\psi_{i2}$, and $\partial\psi_s/\partial r \simeq ik\psi_s$ for $kr \gg 1$, it is straightforward to identify $Q_{is1} = Q_{si1}$ and $Q_{is2} = -Q_{si2}$, and hence $Q_{\text{ext}} = 2Q_{is1}$. Now the task reduces to evaluating Q_{sca} and Q_{is1} only. Again, their integration over ϕ yields a factor of 2π by noticing the independence of the integrands on ϕ. The remaining double summation contains an integration over w given in Eq. (9). The results in terms of partial-wave coefficients are

$$Q_{\text{sca}} = \frac{1}{(ka)^2} \sum_{n=|m|}^{\infty} (|s_n - 1|^2)(2n+1)\frac{(n-m)!}{(n+m)!}\big[P_n^m(b)\big]^2, \tag{16}$$

$$Q_{\text{ext}} = \frac{1}{(ka)^2} \sum_{n=|m|}^{\infty} (2 - s_n - s_n^*)(2n+1)\frac{(n-m)!}{(n+m)!}\big[P_n^m(b)\big]^2, \tag{17}$$

$$Q_{\text{abs}} = \frac{1}{(ka)^2} \sum_{n=|m|}^{\infty} (1 - |s_n|^2)(2n+1)\frac{(n-m)!}{(n+m)!}\big[P_n^m(b)\big]^2. \tag{18}$$

The sign of m does not affect Q_{sca}, Q_{ext}, Q_{abs}, Y_1, and Y_{23}. Notice that each term in Q_{sca} and Q_{ext} is positive, which is consistent with $P_{\text{sca}} > 0$ and $P_{\text{ext}} > 0$. The expression Q_{abs} indicates that if $|s_n| = 1$ for the ideal case of no energy dissipation, all terms vanish so that $P_{\text{abs}} = 0$. To have energy absorption, there must be terms with $|s_n| < 1$.

The first part of the force Y_1 is associated with only the scattering. From Y_1 in Eq. (5) and Q_{sca} in Eq. (14) we have

$$\frac{Y_1}{Q_{\text{sca}}} = -\frac{\int_0^{2\pi}\int_{-1}^{1} |F(w,\phi)|^2 w\, dw\, d\phi}{\int_0^{2\pi}\int_{-1}^{1} |F(w,\phi)|^2 dw\, d\phi} = -\langle w\rangle, \tag{19}$$

that is, $Y_1 = -Q_{\text{sca}}\langle w\rangle$, where the asymmetry $\langle w\rangle$ means the average of $w = \cos\theta$ over the angle distribution of scattered power. The rest of the force Y_{23} is associated with the interference of the incident beam and the scattered field. The extinction power Q_{ext} is also associated with such interference.

By recognizing $-4\alpha_n = 2 - s_n - s_n^*$, it follows immediately from Y_{23} in Eq. (12) and $Q_{\rm ext}$ in Eq. (17) that $Y_{23} = Q_{\rm ext} b = (Q_{\rm sca} + Q_{\rm abs})b$. [This relation is also obtained by recognizing $-4\alpha_n = |s_n - 1|^2 + (1 - |s_n|^2)$ and using Eqs. (12), (16), and (18).] Together we have

$$\begin{aligned} Y_p &= Q_{\rm ext} b - Q_{\rm sca}\langle w\rangle \\ &= Q_{\rm sca}(b - \langle w\rangle) + Q_{\rm abs} b, \end{aligned} \tag{20}$$

$$\begin{aligned} F_z &= P_{\rm ext} c_0^{-1} b - P_{\rm sca} c_0^{-1}\langle w\rangle \\ &= P_{\rm sca} c_0^{-1}(b - \langle w\rangle) + P_{\rm abs} c_0^{-1} b, \end{aligned} \tag{21}$$

which are expressions for the radiation force in terms of the scattered power, absorption, and asymmetry of the scattering; Eq. (21) may be interpreted using the conservation of momentum. The factor $\langle w\rangle$ gives the axial projection of momentum transport associated with the scattering field. This is analogous to the dependence of the optical radiation force on the asymmetry of the scattering [1,9]. The rate at which momentum is removed from the incident Bessel beam as a consequence of scattering and absorption is explicitly proportional to the projection factor $b = \cos\beta$, where β is the beam's conic angle. The part of the force associated with the interference of the incident beam and scattered field is proportional to the extinction power with a projection factor b. There is also an implicit dependence on β in Eqs. (16)–(18). The angle β also affects the asymmetry factor $\langle w\rangle$ through F in Eq. (3) and the expression for Y_1 in Eq. (10). In the limit $\beta = 0$ Eq. (21) agrees with standard acoustical results [17] (see also references cited in Ref. [3]). In an analysis of the radiation force from an EMBB, the term proportional to $P_{\rm abs}$ was not included [1].

V. NEGATIVE RADIATION FORCES ON SPHERES

In the absence of absorption F_z is negative when $\langle w\rangle > \cos\beta$ for both acoustical and optical beams. The recasting of the acoustical radiation force into Eqs. (20) and (21) explains the correlation between the shape of the scattering pattern and the existence of negative forces [3–5]: When the scattering into the backward hemisphere is suppressed relative to scattering into the forward hemisphere, the asymmetry factor $\langle w\rangle$ is positive and relatively large in magnitude. Figure 1 shows the example of an empty aluminum shell in water where the ratio of the inner to the outer radius of the shell is 0.96. Absorption is negligible for aluminum and the shear and longitudinal wave velocities used were 3160 and 6370 m/s, respectively. The other parameters used in the evaluation of the s_n [15] were the density ratio 2.712 and the velocity of sound in water, 1479 m/s. Other situations computed to give negative forces include various liquid drops and solid spheres in water [3–6]. The results suggest that, even in the favorable case of no absorption, it appears to be necessary for β to exceed approximately 40°: increasing β decreases the factor b in the positive force term.

In the absence of absorption the parameter space to be explored associated with negative forces corresponds to the selection of the unimodular $s_n = \exp(2i\delta_n)$. The δ_n are partial-wave phase shifts having the property that the partial-wave amplitude vanishes if δ_n vanishes. (It becomes a separate question as to the kind of sphere associated with a given set of δ_n. In the EMBB case there are separate δ_n for the electric and magnetic multipoles [10].) The number of significant partial waves grows with increasing ka. If an acoustical beam has $m = 0$ and ka is sufficiently small that only the monopole and dipole terms are significant, Y_1 is minimized by taking $\delta_0 = \delta_1 = \pi/2$, which follows analytically from the form of Y_1 in Eq. (10). The case of negative forces associated with low ka scattering by small fluid spheres previously discussed [$G = 0$ in Eq. (15) of Ref. [3]] corresponds to going part of the way down a valley in Y_1 having $\delta_0 = \delta_1$. Near the origin in the (δ_0,δ_1) domain a valley in Y_p is present when $\beta > 55°$ having $Y_p < 0$. The valley lies close to the valley in Y_1 since Y_{23} given in Eq. (13) or (12) does not depend on $\mathrm{Im}(s_n)$.

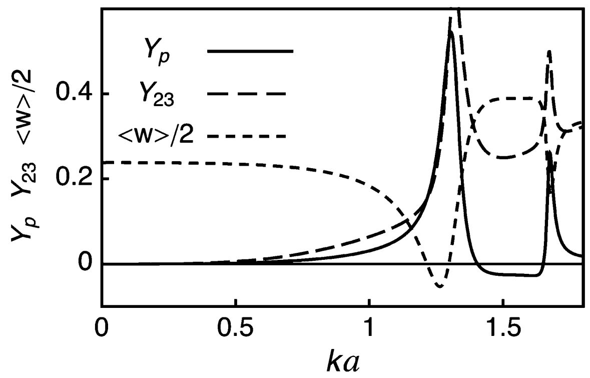

FIG. 1. Radiation force function Y_p, $Y_{23} = Q_{\rm sca}\cos\beta$, and normalized asymmetry function $\langle w\rangle/2$ evaluated for an empty aluminum shell in water in an acoustical Bessel beam with $\beta = 45°$ and $m = 0$. The radiation force F_z is negative in the region near $ka = 1.53$, where $Y_p = -0.025\,45$. In that region $\langle w\rangle$ is maximized and is greater than $\cos\beta$ as required by Eq. (20).

The practicality of using acoustical Bessel beams to attract spheres will also depend on the transverse stability of the sphere and complications associated with thermal viscous dissipation. The finite-element method has been used to compute the scattering with the sphere slightly displaced from the axis of the beam and to compute the associated radiation force by evaluating Eq. (4) [18]. It is not difficult to find stable situations where F_z is predicted by Eq. (21) to be negative. The influence of the thermal viscous response of fluids has been analyzed in the case of acoustical plane waves with the result that the influence on the radiation force may be significant if the radius a of the sphere is not much greater than the thickness of the oscillating thermal viscous boundary layer [19]. It is usually possible to measure the radiation force on a sphere in water in such a way that forces associated with acoustical streaming induced by the incident wave and other effects of viscosity and the nonlinearity of the acoustical medium may be neglected [20,21]. Acoustic radiation pressure and streaming forces are expressed using phonon concepts in Ref. [22].

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