Shearing self-propelled suspensions: Arrest of coarsening and suppression of giant density fluctuations

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(Received 10 January 2011; published 29 September 2011)

We study the effect of a linear shear flow on a collection of interacting active, self-propelled particles modeled via the Vicsek model. The imposed flow has a dramatic effect on the behavior of the model. We find that in the presence of shear there is no order-disorder transition, and that coarsening of the domains is arrested. Shear also suppresses the so-called giant density fluctuations that are observed in the quiescent limit.

DOI: 10.1103/PhysRevE.84.031930 PACS number(s): 87.10.−e, 83.10.Mj, 05.65.+b, 05.70.Ln

Active matter has been a growing field of research in physics over the last decade or so. A suspension of self-propelled (SP) particles, modeling, for instance, microbial or bacterial fluids, fish schools, or synthetic swimming microrobots, is a primary example of an active material [1–3]. Active and SP particles continuously burn energy from their surroundings or from internal sources, typically to move; this drives them out of equilibrium and renders active suspensions sharply different from their passive counterparts [1,4–7].

Elucidating the physical, and possibly universal, properties of such active and self-propelled matter is an important goal, which has prompted physicists to develop and characterize simplified models, such as the Vicsek model [8–11], which was introduced in 1995 and has by now risen to be a paradigm for the physics of SP particles. In the Vicsek model SP particles tend to preferentially align their velocity locally with their neighbors; this alignment is not deterministic but stochastic, as noise is fed into the system at all times. By decreasing the strength of noise, one observes a transition from a disordered phase to an ordered one in which flocks of SP particles form and move coherently, with long-range order even in two dimensions. This behavior is markedly at odds with the thermodynamics of passive systems, where according to the Mermin-Wagner theorem a continuous symmetry cannot be spontaneously broken in less than three dimensions [12]. Similarly, the central limit theorem constraints density fluctuations in passive equilibrium systems to scale as the square root of the number of particles, whereas suspensions of SP particles modeled like by Vicsek show giant density fluctuations, which are also found in experiments with vibrated granular rods [13], although the extent of the universality of the scaling exponents characterizing these fluctuations as well as their ultimate origin remain to be determined [14,15]. Besides being the model of choice to study universal effects in active matter, the Vicsek model is a useful starting point to describe even quantitatively a number of specific self-propelled systems ranging from bacterial fluids to starling flocks [2,16].

Our program in this work is to extend the comparison between passive and active materials to the case of externally driven, sheared, suspensions. This work then provides a theoretical framework to understand the generic properties of sheared active microbial suspensions [17]. In passive materials, an imposed flow may lead to nonequilibrium steady states that are spectacularly different from the thermodynamic ones. For instance, wormlike micelles and liquid crystalline systems can form bands when sheared [18]. In a binary fluid undergoing spinodal decomposition a linear externally imposed shear is instead, to some extent, irrelevant when hydrodynamic couplings between the order parameter and velocity fields are disregarded [19–21]. Retaining them changes the picture and effectively arrests spinodal decomposition, leaving domains of a well-defined size [22]. Here we consider, for the first time, the response of a suspension of Vicsek SP particles to an imposed linear shear; hydrodynamic couplings between the particles are therefore neglected. As in the Vicsek model flocks coalesce at low noise into a single aggregate; one may think of a coarsening binary fluid subject to shear (and without hydrodynamic coupling) as the passive analog of our system. We show that shear arrests the coarsening of flocking domains, thereby providing another example in which active and passive systems behave in a qualitatively different way, this time in an externally driven, thus nonequilibrium, context. Interestingly, shear also drastically changes the nature of the density fluctuations, bringing them back to "normal," i.e., proportional to the square root of the number of particles. Therefore a sheared suspension of active SP particles is a physical system that is qualitatively different from both an unsheared active fluid and a sheared passive fluid. Our results lead to a series of predictions for externally driven active systems, such as bacterial fluids, fish schools, or even inanimate active matter such as vibrated granular rods, whose collective dynamics in the absence of shear are well captured by the Vicsek model as discussed in Refs. [2,3].

Our starting point is the original Vicsek model. This is defined in terms of N point (off-lattice) particles with positions $\vec{x}_i$, and velocities $\vec{v}_i$, of fixed magnitude v_0 ($i = 1, \ldots, N$). In two dimensions, to which we restrict this paper, the direction of motion of the ith particle can be described via a single angle θ_i. To explore the effect of shear, we have generalized the Vicsek update rules to include an imposed linear velocity profile along the x direction, $\vec{v}_s = \dot{\gamma} y \hat{e}_x$, where y labels the velocity gradient direction and $\dot{\gamma}$ is the shear rate. The dynamics of the particle positions and directions are thus given explicitly by the formulas

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t) + (\vec{v}_i(t) + \dot{\gamma} y \hat{e}_x)\Delta t, \tag{1}$$

$$\theta_i(t+\Delta t) = \mathrm{Arg}\left\{\sum_{j\sim i} \exp[i\theta_j(t)]\right\} + \eta\psi_i(t), \tag{2}$$

where t and $t+\Delta t$ denote two successive time steps. The function Arg returns the argument of a complex variable, the parameter $\eta > 0$ measures the noise strength, and $\psi_i(t)$ is a uniform random variable between 0 and 2π. The sum in Eq. (2) is performed over j particles that are within a distance up to r_0 from the ith SP particle. Note that in the quiescent limit, $\dot{\gamma} = 0$, we recover the *backward update* rule, originally employed by Vicsek *et al.* and that, after some debate, has been clarified to lead to a continuous phase transition [10,11]. In order to account for the imposed laminar flow, we implement Lees-Edwards boundary conditions at the top and bottom surfaces of the system (periodic boundary conditions are employed along the other direction) [23].

A natural order parameter for this system is the modulus of macroscopic mean velocity normalized to v_0, φ, given by

$$\varphi(t) \equiv \frac{1}{N v_0}\left|\sum_i \vec{v}_i'(t)\right|, \tag{3}$$

where the sum extends to all particles, and $\vec{v}_i' \equiv \vec{v}_i - \vec{v}_s$ is the average of the velocities after the imposed shear has been subtracted away. In our simulations, we set, without loss of generality, $\Delta t = 1$ and $r_0 = 1$ [8], whereas the velocity was set to $v_0 = 0.2$ (similar results are obtained for different values of the velocity in the range considered, e.g., in Ref. [8]).

A useful quantity in a finite-size scaling analysis of a nonequilibrium phase transition is the variance $\chi = L^2[\langle\varphi^2\rangle - \langle\varphi\rangle^2]$ where our system is a square box of size L, and $\langle\cdots\rangle$ denotes averages over configurations [8,9]. Note that for an equilibrium system, χ would be akin to a magnetic susceptibility. In the absence of shear, χ sharply peaks at the transition, which should be attained for $\eta = \eta_c \simeq 0.09$ [8–10] at a density $\rho \equiv N/L^2 = 1/8$. The peak diverges as $L \to \infty$ [8]; this is confirmed by our data in Fig. 1(a), where the $\dot{\gamma} = 0$ case is also shown.

What happens when the suspension is instead subject to a linear shear? Our simulations in this case lead to a completely different picture [Fig. 1(b)]: The peak of the variance of χ drifts to larger values of noise strength η, but, most importantly, the height of the peak does *not* increase any more with L. Indeed the variance χ does not display, in η, any sign of singularity; rather, as L increases, it converges to a smooth master curve. This strongly suggests that a linear shear imposed on Vicsek self-propelled particle systems washes away the nonequilibrium phase transition and changes it into a smooth crossover between a largely disordered and an ordered phase regime. This behavior is very different from that of sheared passive systems, such as that of the Ising or XY model, where shear changes the universality class of the transition but does not remove it altogether [24,25]. Our results may be qualitatively understood by recalling that at $\dot{\gamma} = 0$ the transition in the Vicsek model occurs via spontaneous symmetry breaking: A flock forms by selecting, via random

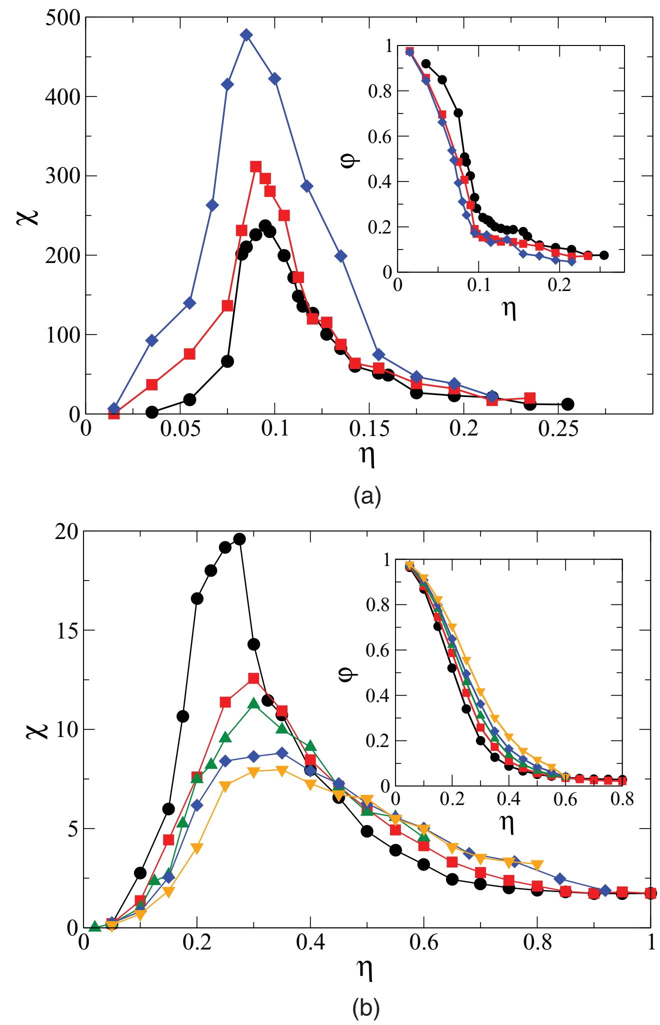

FIG. 1. (Color online) Plot of the order parameter φ (in the insets) and of its variance χ as a function of noise strength, for $\dot{\gamma} = 0$ (a) and $\dot{\gamma} = 5 \times 10^{-5}$ (b). Curves correspond to N equal to 1058 (circles), 2048 (squares), 3200 (upward triangles), 5000 (diamonds), and 10 082 (downward triangles). All data refer to a density $\rho \equiv N/L^2 = 1/8$. Typically averages have been performed over the last 5000 steps, after an equilibration of 10^5 time steps.

fluctuations, a direction for its motion, leading to a violation of total momentum conservation. One may then view a shear flow as an external symmetry-breaking field, akin to a magnetic field in a thermal Ising or XY model. Just as there is no finite-temperature transition in a bulk equilibrium Ising model with a field, this argument then suggests that there may be no flocking transition in the Vicsek model under shear, as the imposed laminar flow leads to a natural, preferred direction for the collective motion. These results were supported by a short-time dynamics [26] analysis of the initial evolution of the order parameter: No power-law behavior signaling the occurrence of a second-order transition was found.

Therefore, shear drastically alters the critical behavior of the Vicsek model in a steady state, *de facto* removing the disordered phase. But what is the effect of an external flow on the *dynamics* of flocking, e.g., at low values of the noise? For $\dot{\gamma} = 0$, the flocking, ordered state is characterized by clusters, which form and coarsen until eventually one single flock remains; the exact domain morphology strongly depends on parameters such as density and noise strength. In particular, due to the absence of surface tension in the original Vicsek model that we employ, there is a lot of transient breaking and reforming of the domains during coarsening. An example of coarsening dynamics at $\dot{\gamma} = 0$ is shown in Fig. 2I [snapshots (a)–(d)], for $\rho = 1$. Figure 2II [snapshots (a)–(d)] instead shows the corresponding coarsening when a linear shear is applied ($\dot{\gamma} = 10^{-4}$). While the very early stages are comparable to the quiescent state, it is apparent that the imposed shear fragments the domains and selects a well-defined size in steady state. Therefore even a *linear* velocity profile can arrest coarsening in suspensions of SP particles, at odds with what happens in passive systems such as binary fluids (see above).

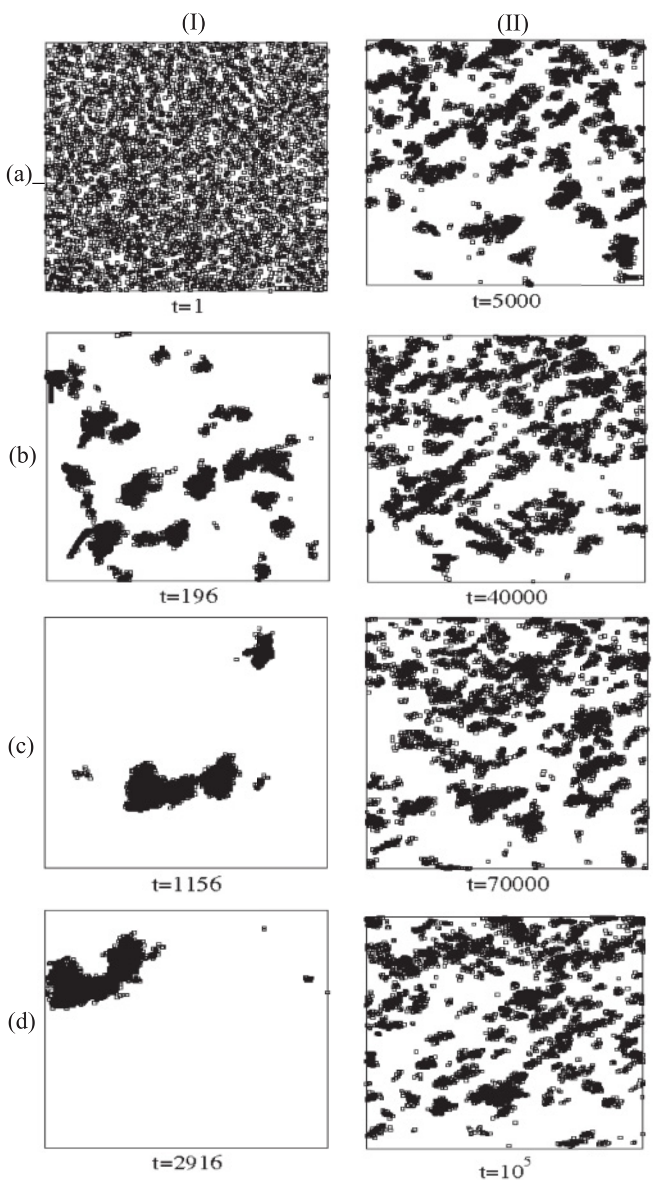

FIG. 2. (I) (a)–(d) Snapshots of the evolution of a system of 5184 Vicsek particles with $\dot{\gamma} = 0$, $\eta = 0.01$, and $\rho = 1$. (II) (a)–(d) Evolution of 5184 Vicsek particles with $\rho = 1$, $\eta = 0.01$, and $\dot{\gamma} = 10^{-4}$. For all snapshots, the corresponding time is also shown.

Figure 2 also suggests that for large enough shear the domains elongate and align at a small angle with respect to the shear. That the coarsening is actually arrested may be quantitatively proved by computing, e.g., the time series of the second moment of the instantaneous structure factor of the SP particle density; Fig. 3 shows its inverse for two values of the imposed shear. It can be seen that after a transient (of less than $\sim 10^4$ time steps), the coarsening is arrested, and the typical length scales of the domains (or flocks) do not increase with time anymore; rather, a statistical steady state is reached where domains fragment and coalesce, but remain with a well-defined and shear-dependent size. Figure 3 also shows that shear arrests domain growth in both directions, with the steady-state domain size larger along the shear direction. Moreover, the typical length scales decrease with shear rate [Fig. 3(a), inset], as in a passive system or binary fluids [22].

Another hallmark of active matter such as active nematics and suspensions of self-propelled particles is the existence of the so-called giant density fluctuations [13,27]. This term

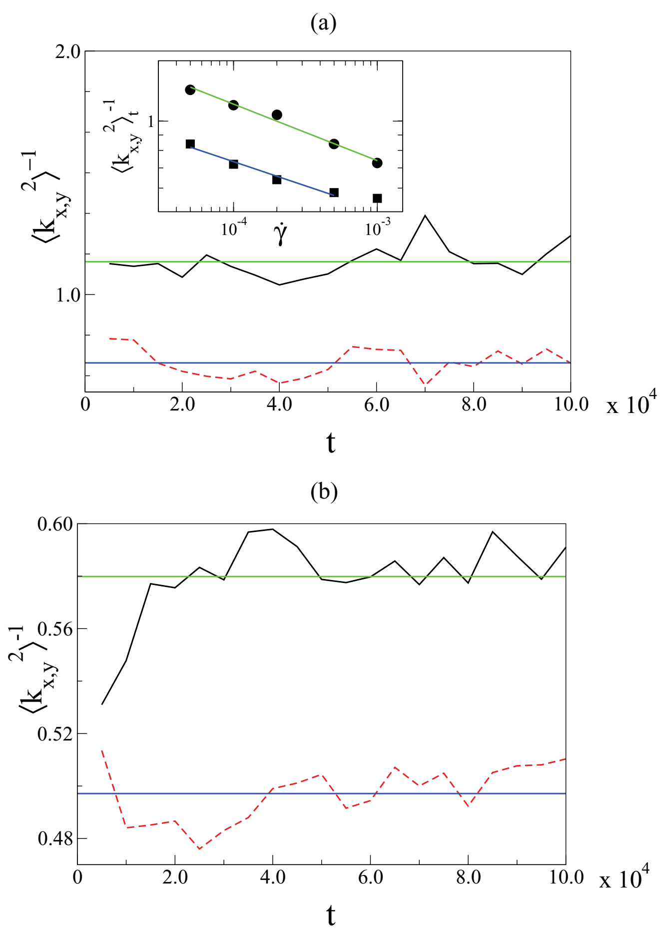

FIG. 3. (Color online) Time series of the inverse of the second moment of the structure factor, $\langle k^2_{x,y} \rangle$ giving the square of the typical domain length scales along x, the shear direction (top curves), and y, the velocity gradient direction (bottom curves). Parameters are $\eta = 0.01$, and: $\rho = 1$, $N = 5184$, and $\dot{\gamma} = 10^{-4}$ (a); $\rho = 1/8$, $N = 5000$, and $\dot{\gamma} = 5 \times 10^{-5}$ (b). In (a) the inset shows the dependence of the time average $\langle k^2_{x,y} \rangle^{-1}$ on shear rate.

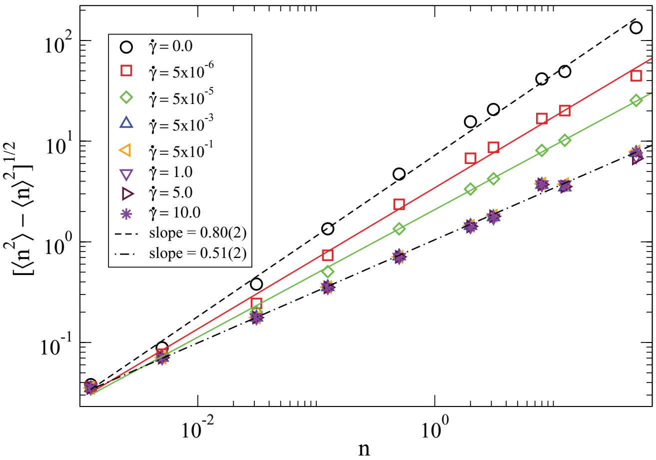

FIG. 4. (Color online) Plot of the fluctuations in number density as a function of the total number of particles, for a system with $\rho = 1/8$ ($N_p = 5000$, $L = 200$), and different value s of the shear rate, as is indicated in the legend. The upmost straight line is a linear regression of the data corresponding to $\dot\gamma = 0.0$, while the lowest straight line has a slope that is an average of the slopes (also obtained by linear regressions) of the data in the range $5 \times 10^{-3} \leqslant \dot\gamma \leqslant 10$.

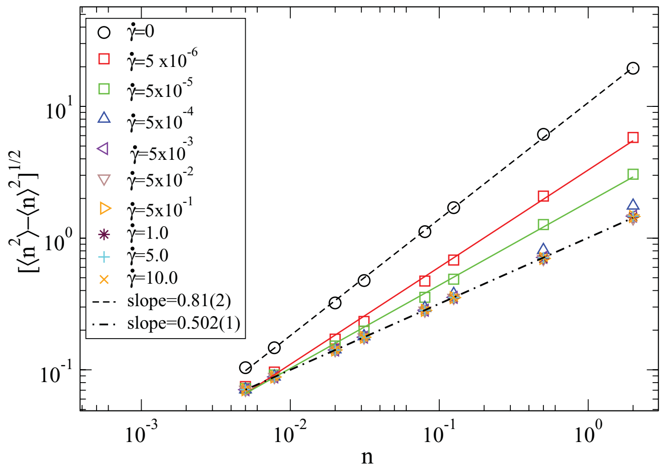

FIG. 6. (Color online) Plot of the fluctuations in number density as a function of the total number of particles, for different shear rates $\dot\gamma$ (indicated in the legend). The system's parameters are $N = 10\,082$, $\rho = 1/8$, $\eta = 0.05$, and $v_0 = 0.2$. The upmost straight line is a linear regression fits of the data corresponding to $\dot\gamma = 0.0$, while the lowest straight line has a slope that is an average of the slopes, obtained also by linear regression fits, of the data in the range $5 \times 10^{-5} \leqslant \dot\gamma \leqslant 10$.

refers to the observations of a nonstandard scaling of the fluctuations in local number of particles Δn as a function of the local average number of particles n, in dilute systems where the local density is allowed to vary a lot, such as in the Vicsek model. In a passive system in thermodynamic equilibrium the central limit theorem forces $\Delta n \sim n^{1/2}$, whereas in an active bath $\Delta n \sim n^{\alpha}$, with α larger than $1/2$, and, for instance, equal to $1/2 + d/2$ for active nematics [14]. In the Vicsek model,

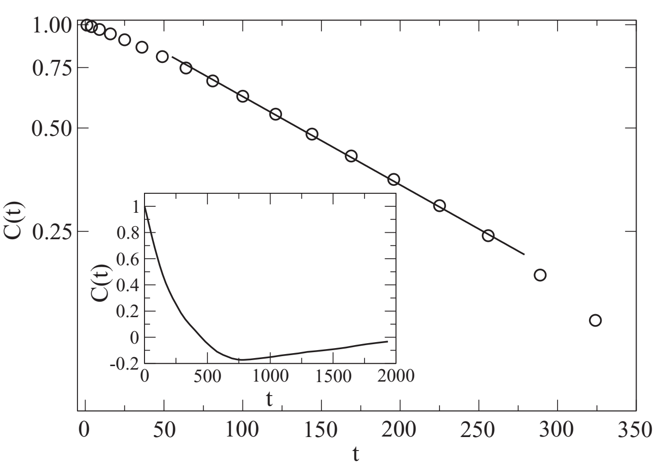

FIG. 5. Time correlation function in the Vicsek model ($\dot\gamma = 0$), defined as $C(t) = \langle(\varphi(t) - \langle\varphi\rangle)(\varphi(0) - \langle\varphi\rangle)\rangle$ (normalized to the value for $t = 0$) for the case with $N = 5000$, $L = 200$ ($\rho = 1/8$), $\eta = 0.05$, and $v_0 = 0.2$, and where $\langle\rangle$ denote temporal. The straight line is a regression fit with an exponential function that gives an estimate of the correlation time equal to $\tau = 171.49$. (A different estimate based on the first momentum of the correlation function gives $\tau \approx 150$.) The inset shows the complete evolution of the correlation function over a larger time interval. $C(t)$ was recorded after the system has reached the nonequilibrium steady states.

$\alpha \sim 0.8$ [9], although there remains no analytical predictions in quantitative agreement with the numerical results.

We have measured Δn in our simulations and found that it grows as a power law in n also in the presence of shear (see Fig. 4). Once more, we find that shear changes the universal properties of the Vicsek model. While we reobtain a scaling exponent $\alpha \sim 0.8 > 0.5$ in the quiescent case (in good agreement with Ref. [9]), we find that α tends to decrease with shear rate, reaching the "passive" value of $1/2$ for sufficiently large values of $\dot\gamma$. Our results therefore seem to suggest a slow transition from $\alpha \sim 0.8$ to $\alpha = 1/2$ at larger shear rates.

Interestingly, at the shear rate for which α approaches 0.5 the dimensionless "Deborah" number, $\tau\dot\gamma$, where τ, the

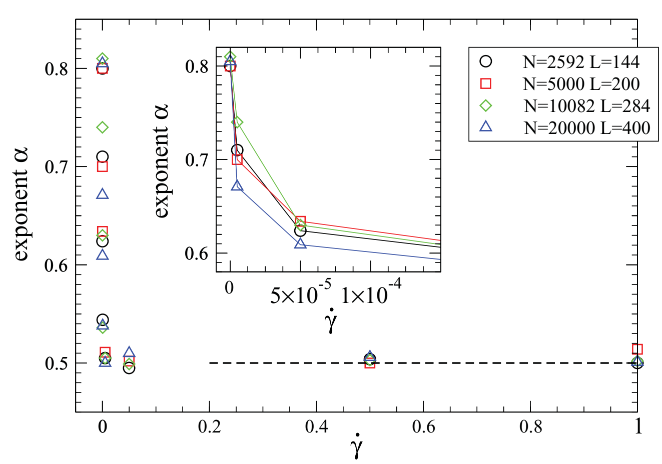

FIG. 7. (Color online) Plot of the giant density fluctuation exponent α as a function of the shear rate, for four different values of N indicated in the legend. Our data at small $\dot\gamma$ (see inset) support the scenario that α changes smoothly with shear rate, as there are no signs of a singular behavior at $\dot\gamma = 0$ in the thermodynamic limit. The dashed line is plotted to guide the eye and indicates $\alpha = 0.5$.

correlation time of the system, is very close to 1, as can be seen from Fig. 5. To investigate whether the crossover we observe may be a finite-size effect, we have performed simulations at values of L and N larger than those used in Fig. 4 (at fixed ρ): The results are shown in Fig. 6, which show a similar trend than the ones in Fig. 4. To further quantify size effects, we plot in Fig. 7 the effective exponent α as a function of shear rate, for different system sizes. While not being definitive, these data strongly suggest that α changes smoothly rather than discontinuously with shear rate. The shear-induced suppression of giant density fluctuations associated with activity we observe is to some extent reminiscent of what happens in sheared binary systems [24,25,28], where it is shown that the strength of thermal fluctuations decreases with shear. It would be interesting to study other particle-based models for active matter and assess whether this phenomenon is generic.

Besides providing an interesting fundamental model for nonequilibrium active matter under an external driving, the sheared Vicsek model could be relevant as a simplified model for a number of biological systems. For instance, it is nowadays possible to study the dynamical behavior of a suspension of microbial swimmers, such as algae, bacteria, or sperm cells, under an imposed shear [17]. As bacteria rarely flock in solutions, the relevant quiescent regime would be in the disordered phase, but arguably close to the transition as motion of bacteria shows long-range correlations. An imposed shear then should increase the average ordering in the bulk, and this effect may be observed in the lab. For instance, we find that at a moderate density $\rho = 1/8$ typical for experimentally used microbial suspensions [17,29], at a noise $\eta = 0.2 > \eta_c$ a seemingly small shear of 10^{-4} changes the order parameter from 0.01 to around 0.8. At a larger scale, animals such as birds, insects, and fish form flocks, swarms, or schools. In their natural environment, these may often be subjected to effective shear flows due to spatially varying winds or currents. Our results suggest that these should lead to a fragmentation of the flocks, which is qualitatively in agreement with the observations that insects are dispersed by strong winds, and that bird flocks decrease in size under windy conditions [30].

In conclusion, we have studied the effect of a linear shear flow on a suspension of Vicsek self-propelled particles. We have shown that shear dramatically reshapes the physics of the Vicsek model. First, it removes the order-disorder transition found at zero shear rate, $\dot{\gamma} = 0$. Second, shear arrests the coarsening of domains into a single flock, leaving a steady state made up of smaller clusters with a well-defined size. Finally, shear also suppresses the giant density fluctuations that characterize the Vicsek model in the ordered phase. Besides providing a model system for driven active matter, our results should be relevant to rheological experiments on suspensions of bacterial or microbial swimmers, and to the large-scale behavior of social animals, such as bird flocks and insect swarms, which are known to be fragmented or dispersed by strong winds.

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