

## Steady magnetic-field generation via surface-plasma-wave excitation

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The possibility of inducing a magnetic field via surface plasma-wave excitation is investigated with a simple nonrelativistic hydrodynamic model. A static magnetic field is predicted at the plasma surface, scaling with the square of the surface-wave field amplitude, and the influence of the electron plasma density is studied. In the case of resonant surface-wave excitation by laser this result can be applied to low intensities such that the electron quiver velocity in the field of the surface wave is less than its thermal velocity.

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The possibility of generating high magnetic fields during high-power laser matter interaction with a solid target has attracted considerable attention during the last decade [1–3] as it may have important consequences for the particle beams produced during the interaction. Typically in these studies a sharp-edged overdense plasma is created by an ultrashort ( $\tau < 100$  fs) intense ( $I\lambda^2 < 10^{15} \text{Wcm}^{-2} \mu\text{m}^2$ ) IR laser pulse. Electromagnetic energy is then partially absorbed ( $< 30\%$ ), and electrons are heated through collisionless mechanisms such as sheath inverse bremsstrahlung [4],  $\vec{J} \times \vec{B}$  heating [5], vacuum heating [6], and anomalous skin-layer heating [7].

Recently the idea of improving the laser absorption in an overdense plasma created by laser-solid interactions has been investigated considering structured targets [8–12] in order to enhance electrons acceleration to values in the range of keV to MeV. Of particular interest is the possibility of laser excitation of surface plasma waves (SPWs) [13] in structured systems in a large range of laser intensities, from low  $\sim 10^{15}$  to high  $10^{20} \text{Wcm}^{-2} \mu\text{m}^2$ . These waves are supported by a stepwise profile overdense plasma when the condition for resonant excitation is satisfied. SPWs propagate along the plasma-vacuum interface and are characterized by a localized, high-frequency, resonant electric field higher than the laser one. In a previous work [10], we have shown the possibility of resonant excitation of SPWs by an ultrashort (60 fs) high-intensity ( $10^{18} \text{Wcm}^{-2} \mu\text{m}^2$ ) laser pulse in overdense prestructured plasma ( $n_e = 25n_c$  where  $n_e$  is the electron density and  $n_c = \frac{\omega^2 m_e}{4\pi e^2}$  is the critical density,  $\omega$  being the laser frequency). A dramatic increase of both the laser absorption (up to the 70%), and the electron energy (several MeV) was obtained. A high local amplified electric field was also found that has interesting consequences on high electron bunch creation. In these 2D particle-in-cell (PIC) simulations, due to the high electron currents induced by the SPW at the plasma surface, a quasistatic self-generated magnetic field of 20MG was also observed.

We wish to present here an investigation of the role of SPWs on the creation of a steady magnetic field during laser

overdense plasma interaction. To this end we develop a simple nonrelativistic hydrodynamic model that gives an analytic expression for the self-generated magnetic field. The model shows a quadratic dependence of the magnetic field with the SPW electric field; the influence the electron plasma density on its intensity and location is also discussed. We remind readers that magnetic fields can also be generated on the plasma surface by mechanisms involving crossed density and temperature gradients [14]. However, since here we focus on plasmas generated by an ultrashort laser pulse, these mechanisms are not expected to play a significant role.

We use a standard collisionless hydrodynamical approach where the collective oscillation of the electron gas is described by the Maxwell equations and the linearized fluid equations. We consider two media separated by a planar interface at  $x = 0$ , such that the plasma fills the semispace  $x < 0$  and the vacuum fills the semispace at  $x > 0$ . In our geometry the plasma vacuum interface coincides with the  $y$  axis, and (the real part of) the wave vector of the surface wave will be along the  $y$  direction. Thus we have for the linearized fluid equations

$$\frac{\partial n_1}{\partial t} + n_0 \vec{\nabla} \cdot \vec{v}_1 = 0, \quad (1)$$

$$\frac{\partial \vec{v}_1}{\partial t} = -\frac{e\vec{E}_1}{m_e} - \frac{1}{n_0 m_e} \vec{\nabla} P_1, \quad (2)$$

where  $n = n_0 + n_1 + \dots$  is the electron density,  $P$  is the electron pressure, and subscripts 0 and 1 (here and in the following) refer to equilibrium and perturbation quantities (no index means the variable is not expanded yet). We assume that time variations are so fast that no significant heat exchange can take place in the plasma. Then the process can be considered adiabatic, so that the temperature-density relationship is  $T/T_0 = (n/n_0)^{\gamma-1}$ , where  $\gamma$  is the adiabatic index. Thus the linearized (first-order) moment equation (2) can be written as

$$\frac{\partial \vec{v}_1}{\partial t} = -\frac{e\vec{E}_1}{m_e} - \frac{\beta^2}{n_0} \vec{\nabla} n_1, \quad (3)$$

where  $\beta^2 = \frac{\gamma k_B T_0}{m_e}$  is a parameter expressing the equilibrium thermal speed of the electrons. The SPW fields are then derived from the linearized Maxwell equations looking for “H” waves and assuming a density perturbation of the form  $n_1 = g(x)e^{iky-i\omega t}$  ( $k$  and  $\omega$  being, respectively, the wave vector and the frequency of the SPW). The following expressions for the SPW fields inside the plasma are obtained after some algebra:

$$E_{1x} = E_{0x} \left( e^{-q_p x} - \frac{\omega_p^2}{\omega^2} e^{-q_t x} \right) e^{iky-i\omega t} + \text{c.c.}, \quad (4)$$

$$E_{1y} = E_{0x} \left( \frac{q_p}{ik} e^{-q_p x} + \frac{\omega_p^2}{\omega^2} \frac{ik}{q_t} e^{-q_t x} \right) e^{iky-i\omega t} + \text{c.c.}, \quad (5)$$

$$B_{1z} = -\frac{c}{i\omega k} (k^2 - q_p^2) E_{0x} e^{-q_p x} + \text{c.c.}, \quad (6)$$

where  $\omega_p^2 = \frac{4\pi e^2 n_e}{m_e}$  is the plasma frequency and

$$q_t = [k^2 + (\omega_p^2 - \omega^2)/\beta^2]^{1/2}, \quad (7)$$

$$q_p = [k^2 + (\omega_p^2 - \omega^2)/c^2]^{1/2} \quad (8)$$

are the reverse of the two evanescence lengths of the field inside the plasma.  $E_{0x}$  is a constant representing the SPW field at the surface.

It is to be noticed that these expressions agree with Ref. [13]. Namely, in the limit of a warm plasma ( $v_{\text{osc}} = eE_1/m_e\omega < \beta$ ), we can impose the boundary condition  $v_{1x} = 0$  at  $x = 0$  (as for the reflection of the electrons by a plasma sheath at the interface) and find the same dispersion relation as [13]

$$k^2 \omega_p^2 = q_t q_p \omega^2 + (\omega^2 - \omega_p^2) q_v q_t, \quad (9)$$

where  $q_v = [k^2 - \omega^2/c^2]^{1/2}$  is the reverse of the field evanescence length in the vacuum. We observe that, for the range of densities we are going to investigate (5–100 $n_e$ ) and an electron thermal velocity  $\beta < c$ , we have  $q_v \ll q_p < q_t$ . In the limit of  $\beta = 0$  we recover the well-known dispersion relation of the surface wave for cold plasmas:

$$k^2 = \frac{\omega^2}{c^2} \frac{\omega^2 - \omega_p^2}{2\omega^2 - \omega_p^2}. \quad (10)$$

We are now interested in the second-order fields in order to find the quasistatic magnetic field generated into the plasma by the SPW. The second-order momentum equation becomes

$$\begin{aligned} n_0 \frac{\partial \vec{v}_2}{\partial t} + n_0 (\vec{v}_1 \cdot \vec{\nabla}) \vec{v}_1 + n_1 \frac{\partial \vec{v}_1}{\partial t} \\ = -\frac{n_0 e}{m_e c} (c \vec{E}_2 + \vec{v}_1 \times \vec{B}_1) - \beta^2 \vec{\nabla} n_2 - \beta^2 \frac{T_1}{T_0} \vec{\nabla} n_1 - n_1 \frac{e}{m_e} \vec{E}_1, \end{aligned} \quad (11)$$

where the subscript 2 refers to the second-order perturbation quantities. Accordingly with the adiabatic temperature-density relationship, we have  $T_1 = \gamma T_0 (n_1/n_0)$ , which means at this approximation order we are neglecting any thermo-electric source, i.e., a term of the form  $\vec{\nabla} n \times \vec{\nabla} T$  that gives no contribution on the fast time scales of the SW oscillation.

Using (2) and Maxwell-Faraday equation, we can combine two terms in Eq. (11):  $[\frac{e}{m_e c} \vec{v}_1 \times \vec{B}_1 + (\vec{\nabla} \cdot \vec{v}_1) \vec{v}_1] = \vec{\nabla} v_1^2$ .

Moreover Eq. (11) will be manipulated as follows in order to obtain a relation for the second-order magnetic field  $\vec{B}_2$ : First, the time derivative of  $v_1$  is replaced using the first-order momentum equation (2); then we perform the time derivative of the obtained equation, where we write the time derivative of  $\vec{E}_2$  as a function of  $(\vec{B}_2, n_1, v_1, v_2)$ , via the Maxwell-Ampere equation at second order. Finally, by taking the curl of the resulting equation, we find the relation

$$\frac{\partial^2 \vec{B}_2}{\partial t^2} - c^2 \nabla^2 \vec{B}_2 + \omega_p^2 \vec{B}_2 = -\omega_p^2 \frac{m_e c}{en_0} \vec{\nabla} \times (n_1 \vec{v}_1). \quad (12)$$

As we are looking now for the slowly varying component of the electromagnetic field, we neglect the term  $\frac{\partial^2 \vec{B}_2}{\partial t^2}$ , and we perform a time average in order to eliminate the oscillating terms. The  $x$  component of the product  $\times(n_1 \vec{v}_1)$  is thus eliminated by the averaging procedure. We finally obtain the differential equation (valid for  $x > 0$ )

$$\begin{aligned} -c^2 \frac{\partial^2 \vec{B}_2}{\partial x^2} + \omega_p^2 \vec{B}_2 = 2c \frac{e}{m_e} E_{0x}^2 \frac{\omega_p^2}{\omega^2} \frac{k^2 - q_t^2}{q_t \omega} \\ \times \left[ \frac{-(q_p + q_t) q_p}{k} e^{-(q_p + q_t)x} + e^{-2q_t x} 2k \right]. \end{aligned} \quad (13)$$

In the vacuum side ( $x < 0$ ) the analogous of Eq. (13) reads  $\frac{\partial^2 \vec{B}_{2,v}}{\partial x^2} = 0$ . The condition that the field is zero far away from the surface imposes  $B_{2,v} = 0$ , leading to the solution

$$\vec{B}_2 = \hat{z} (A e^{-(q_p + q_t)x} + B e^{-2q_t x} + C e^{-\frac{\omega_p^2}{c} x}), \quad (14)$$

$$A = 2c \frac{e}{m_e} E_{0x}^2 \frac{\omega_p^2}{\omega^2} \frac{k^2 - q_t^2}{q_t \omega k} \frac{-(q_p + q_t) q_p}{\omega_p^2 - c^2 (q_p + q_t)^2}, \quad (15)$$

$$B = 2c \frac{e}{m_e} E_{0x}^2 \frac{\omega_p^2}{\omega^2} \frac{k^2 - q_t^2}{q_t \omega} \frac{2k}{\omega_p^2 - 4c^2 q_t^2}, \quad (16)$$

$$C = -(A + B). \quad (17)$$

The terms A and B can also be expressed as a function of the SPW field at the plasma-vacuum interface,  $E_{\text{sw}} = |E_{1x}(x = 0)| = (1 - \frac{\omega_p^2}{\omega^2}) E_{0x}$ , writing

$$A = \frac{2ce}{m_e} \frac{E_{\text{sw}}^2}{(1 - \frac{\omega_p^2}{\omega^2})^2} \frac{\omega_p^2}{\omega^2} \frac{k^2 - q_t^2}{q_t \omega k} \frac{-(q_p + q_t) q_p}{\omega_p^2 - c^2 (q_p + q_t)^2}, \quad (18)$$

$$B = \frac{2ce}{m_e} \frac{E_{\text{sw}}^2}{(1 - \frac{\omega_p^2}{\omega^2})^2} \frac{\omega_p^2}{\omega^2} \frac{k^2 - q_t^2}{q_t \omega} \frac{2k}{\omega_p^2 - 4c^2 q_t^2}. \quad (19)$$

Equation (14) shows that the surface wave gives rise to a second-order magnetic field proportional to the square of the SPW field  $E_{\text{sw}}$  and confined near the interface, whose strength depends on the source field, electron temperature, and density.

Two limits are now interesting to explore, in order to clarify the dependence of  $B_2$  on the involved parameters. First, in the limit of  $T_0 \rightarrow 0$  ( $q_t \rightarrow \infty$ ), that is, a cold plasma, we observe that  $B$  vanishes while  $A$  does not. Thus, using relations (14)

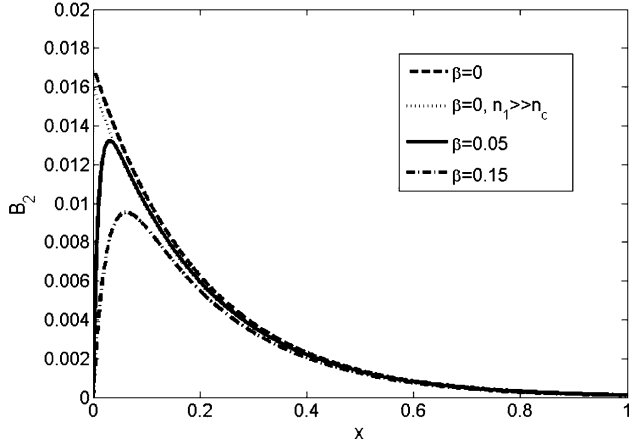


FIG. 1. Magnetic field (absolute value  $B_2 = |\vec{B}_2|$ ) as a function of the depth inside the plasma, given by Eq. (14), for  $E_{sw} = 0.2$  for  $n_e = 25n_c$  and different values of  $\beta$ . The values of the fields are given in units of  $B_0 = m_e\omega/e$ , and the depth is in units of  $k^{-1}$ .

and (17) we obtain the second-order magnetic field for a cold plasma at  $x > 0$  ( $B_2 = 0$  at  $x = 0$ ):

$$\vec{B}_{2,\text{cold}} = 2 \frac{e}{m_e c \omega} \frac{E_{sw}^2}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)^2} \frac{\omega_p^2}{\omega^2} \frac{q_p}{k} e^{-\frac{\omega_p}{c} x} \hat{z}. \quad (20)$$

It should be noticed that this result is hard to obtain directly from the cold plasma equations, due to the discontinuity of the electric field  $E_{1x}$  at  $x = 0$ .

Then, in the limit of strongly overdense plasma  $\frac{\omega_p}{\omega} \gg 1$ , we have  $k^2 \sim \omega^2/c^2$  from the dispersion relation (10), and Eq. (20) can be expressed as

$$\vec{B}_{2,\text{ovd}} = \text{sgn}(k) 2 \frac{e}{m_e c \omega} E_{sw}^2 \frac{\omega}{\omega_p} e^{-\frac{\omega_p}{c} x} \hat{z}. \quad (21)$$

Thus, in the case of extremely high density and low temperature, the model predicts a magnetic field having a rapidly vanishing amplitude inside the plasma, with a maximum value proportional to  $\sqrt{n_c/n_e}$ .

In Fig. 1 the dependence of the magnetic field given by the expression (14) on the electron thermal velocity (expressed by the term  $\beta$ ) is shown, for  $E_{sw} = 0.2m_e\omega/e$ ,  $\omega_{pe}^2/\omega^2 = 25$ , and three values of  $\beta/c$ : 0.15, 0.05, and 0. The magnetic field is peaked near the plasma surface for low electron thermal temperatures, while it becomes less localized when the electron thermal velocity is increased. This is consistent with the fact that increasing the electron thermal velocity the parameter  $1/q_t \sim \lambda_{De}$  (where  $\lambda_{De}$  is the Debye length) increases, while in the case of a cold plasma the evanescence length is simply given by the skin depth, which for our parameters is equal to  $c/\omega_{pe} = 0.2c/\omega \sim 0.2k^{-1}$ . For comparison, the curves corresponding to the limits of a cold ( $T_0 \rightarrow 0$ ) and dense plasma ( $\omega_{pe}/\omega > 1$ ) are also shown: The long dashed curve corresponds to Eq. (20), while the gray fine dashed curve correspond to Eq. (21). As expected a higher maximum strength of  $B_{2,\text{cold}} = 0.017m_e\omega/e$  is obtained in the limit of a cold plasma, while in the limit of very high density the peak decreases to  $B_{2,\text{ovd}} = 0.016m_e\omega/e$ .

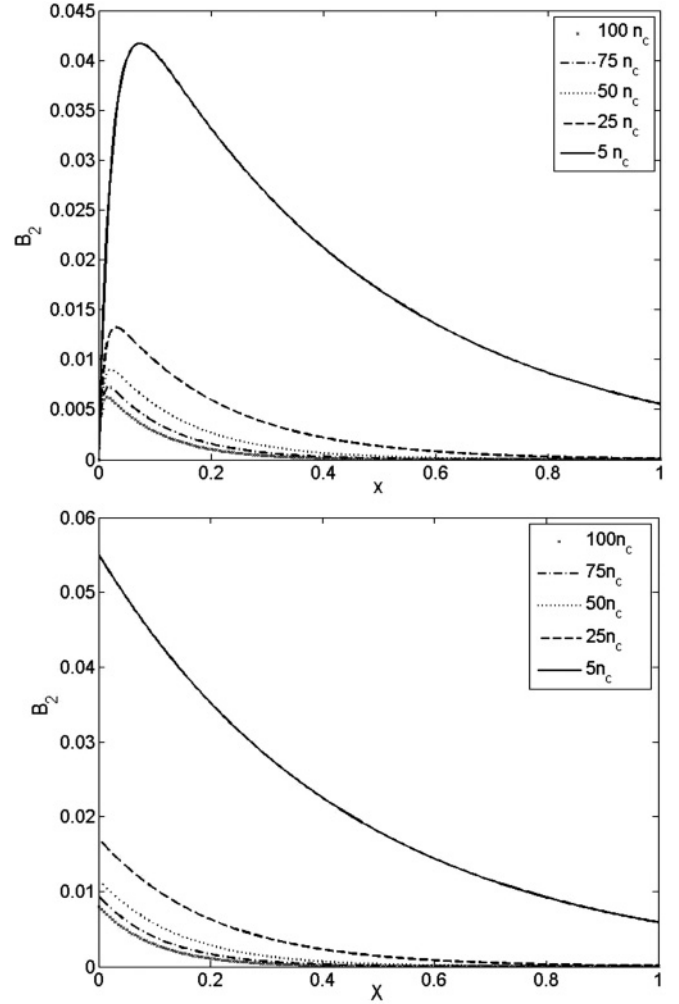


FIG. 2. Magnetic field (absolute value  $B_2 = |\vec{B}_2|$ ) as a function of the depth inside the plasma, given by Eq. (14), for  $E_{sw} = 0.2$  and different values of the  $n_e$ , up for  $\beta/c = 0.05$  and down in the limit of a cold plasma  $\beta = 0$ . The values of the fields are given in units of  $B_0 = m_e\omega/e$ , and the depth is in units of  $k^{-1}$ .

To complete the discussion we have reported in Fig. 2 the dependence of the magnetic field on the electron density for  $n_e = 5, 25, 50, 75$ , and  $100n_c$  for an electron thermal velocity  $\beta/c = 0.05$  (up) and in the cold plasma limit  $\beta = 0$  (down). As expected from the expression (21) in the limit of very high density, we observe that the field amplitude decreases for increasing electron density and becomes more peaked and close to the surface. This is consistent with the decreasing of the skin depth for increasing the plasma density. The same trend is observed in the cold plasma limit.

Thus, in the case of resonant excitation of the surface wave by a laser field the model predicts the presence of a quasistatic magnetic field in the plasma skin depth, induced by the SPW, whose amplitude scales with the square of  $E_{sw}$ . Since, according to Ref. [10],  $E_{sw}$  is proportional to the laser field, the self-generated magnetic field would scale with the laser intensity. We notice that the quasistatic magnetic field derived in this model is stronger than what was derived for the case of generation of a static magnetic field by a laser propagating

in underdense, cold, homogeneous plasma [15], where the perturbative analysis needed expansion to fourth order in order to prove the possibility of magnetic-field self-generation. Nevertheless, it should be mentioned here that the validity of this approach is limited to moderate laser intensity range where  $I\lambda^2 < \sim 10^{16} \text{Wcm}^{-2} \mu\text{m}^2$  because of the hypothesis  $v_{\text{osc}}/c = eE_{\text{sw}}/cm_e\omega < \beta$ . In particular, some limitations of the model should also be noticed, such as, for example, the transfer of energy from the wave to the particle via kinetic effects (such as Landau damping, wave breaking, and vacuum heating), creating hot tails in the electron distribution functions in directions parallel and perpendicular to the surface, which are not described in this fluid approach. However, in a higher laser intensity range, SPWs still exist [10] and can be an attractive method for quasistatic magnetic-field generation.

In conclusion, we predict, with a simple nonrelativistic hydrodynamic model, a new effect in a moderate intensity laser interaction regime  $I\lambda^2 < \sim 10^{16} \text{Wcm}^{-2} \mu\text{m}^2$ : the significant generation of quasistatic magnetic field by resonant excitation

of a surface wave on a plasma target. The magnetic-field intensity is shown to have a quadratic dependence with the SPW electric field, which can be related to the laser one, and to decrease with  $\omega/\omega_{\text{pe}}$  for increasing density. The development of such a quasistatic magnetic field in the vicinity of the laser-plasma interaction layer is known to have an important effect on the electron beams produced during the interaction [1, 16]. For this reason the knowledge of the process of magnetic-field generation is of great importance when seeking to control the divergence of electron beams, and this result should promote new experimental and numerical studies with grating targets. We remind readers that the surface plasma wave can also appear on flat (nonstructured) overdense plasmas by a decay process [17], and thus the associated generation of a quasistatic magnetic field is relevant for many situations of interest. In particular, a high-intensity laser pulse regime where kinetic and relativistic effects are dominant should be investigated. This point is under study with PIC simulations and will be the subject of a future publication.

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- [1] J. C. Adam, A. Heron, and G. Laval, *Phys. Rev. Lett.* **97**, 205006 (2006).
- [2] A. S. Sandhu, A. K. Dharmadhikari, P. P. Rajeev, G. R. Kumar, S. Sengupta, A. Das, and P. K. Kaw, *Phys. Rev. Lett.* **89**, 225002 (2002).
- [3] A. Gopal *et al.*, *Phys. Plasmas* **15**, 122701 (2008).
- [4] P. J. Catto and M. R. More, *Phys. Fluids* **20**, 704 (1997).
- [5] W. L. Kruer and K. Estabrook, *Phys. Fluids* **28**, 430 (1985).
- [6] F. Brunel, *Phys. Rev. Lett.* **59**, 52 (1987).
- [7] E. G. Gamaliy and R. Dragila, *Phys. Rev. A* **42**, 929 (1990).
- [8] J. C. Gauthier *et al.*, *Proc. SPIE* **2523**, 242 (1995).
- [9] J. Kupersztych, M. Raynaud, and C. Riconda, *Phys. Plasmas* **11**, 1669 (2004).
- [10] M. Raynaud, J. Kupersztych, C. Riconda, J. C. Adam, and A. Heron, *Phys. Plasmas* **14**, 092702 (2007).
- [11] G. Hu *et al.*, *Phys. Plasmas* **17**, 083102 (2010).
- [12] X. Lavocat-Dubuis and J.-P. Matte, *Phys. Rev. E* **80**, 055401(R) (2009); W.-M. Wang, Z.-M. Sheng, and J. Zhang, *Phys. Plasmas* **15**, 030702 (2008); G. Hu *et al.*, *ibid.* **17**, 033109 (2010).
- [13] P. K. Kaw and J. B. McBride, *Phys. Fluids* **13**, 1784 (1970).
- [14] D. W. Forslund and J. U. Brackbill, *Phys. Rev. Lett.* **48**, 1614 (1982); R. D. Jones, *ibid.* **51**, 1269 (1983); M. Y. Yu and L. Stenflo, *Phys. Fluids* **28**, 3447 (1985); A. S. Sandhu, A. K. Dharmadhikari, P. P. Rajeev, G. R. Kumar, S. Sengupta, A. Das, and P. K. Kaw, *Phys. Rev. Lett.* **89**, 225002 (2002).
- [15] L. Gorbunov, P. Mora, and T. M. Antonsen Jr., *Phys. Rev. Lett.* **76**, 2495 (1996); L. Gorbunov, P. Mora and T. M. Antonsen Jr., *Phys. Plasmas* **4**, 4358 (1997).
- [16] R. J. Mason, E. S. Dodd, and B. J. Albright, *Phys. Rev. E* **72**, 015401(R) (2005).
- [17] A. Macchi, F. Cornolti, F. Pegoraro, T. V. Liseikina, H. Ruhl, and V. A. Vshivkov, *Phys. Rev. Lett.* **87**, 205004 (2001).