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Effect of self-steepening on optical solitons in a continuous wave background

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We present an analytic method to generate solutions for the optical fiber soliton system that reveals self-steepening effects on solitons coupled to a continuous wave. Exact soliton solutions are obtained by adopting a universal Lax pair technique that solves simultaneously the nonlinear Schrödinger (NLS) equation and the derivative NLS equation. We find that, in the presence of a self-steepening term, the bright type NLS equation with abnormal group velocity dispersion is related to the dark type NLS equation with normal group velocity dispersion and, accordingly, exact soliton solutions of the bright type NLS equation describe both bright and dark solitons depending on the strength of the continuous wave. The self-steepening effect on solitons and possible applications of a continuous wave for the control of solitons are explained.

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I. INTRODUCTION

The self-steepening of an optical fiber pulse arises when the group velocity of a pulse depends on the intensity [1]. This causes an asymmetry in pulse shape and eventually leads to the formation of shock waves in the absence of dispersion. However, when both dispersion and the Kerr nonlinear effect are present, solitons can be found that are robust against the shock formation. The effect of self-steepening on optical solitons is an important issue in the optical fiber communication system when the soliton pulse width becomes ultrashort ($<$100 fs) and subsequently the higher-order effects can not be neglected [2]. The self-steepening of optical pulses is governed by the nonlinear Schrödinger (NS) equation with the additional self-steepening term,

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} + |U|^2U + is\frac{\partial}{\partial \tau}(|U|^2U) = 0, \tag{1}$$

where the normalized parameter s measures the magnitude of self-steepening. Particular solutions of this equation that describe shock or soliton pulse have been obtained using the ansatz method [3,4]. More elaborate solution techniques exist since this equation is directly related to the derivative nonlinear Schrödinger (DNLS) equation,

$$i\partial_X \Psi = -\partial_T^2 \Psi + 2i\partial_T(|\Psi|^2\Psi), \tag{2}$$

through the coordinate change and the field redefinition,

$$X = \xi, \quad T = -\sqrt{2}\left(\tau - \frac{\xi}{s}\right) \quad \text{and}$$
$$U \equiv \frac{2^{1/4}}{\sqrt{s}}\Psi \exp\left[i\frac{\tau}{s} - i\frac{\xi}{2s^2}\right]. \tag{3}$$

The DNLS equation is a well-known integrable equation that also governs the evolution of Alfvén waves in plasma physics [5,6]. The integrability of the DNLS equation was shown by Kaup and Newell [7] and soliton solutions have been found using various techniques such as the inverse scattering method [7,8], the Hirota method [9], and the Darboux transformation [10,11]. Though these techniques provide a rather comprehensive set of bright and dark soliton solutions of the DNLS equation, there has been a lack of understanding in the exact solutions of the nonlinear Schrödinger equation with a self-steepening term given in Eq. (1). Theoretically, solutions of Eq. (1) can be obtained directly from solutions of the DNLS equation in a one-to-one correspondence through the transform in Eq. (3). However, the apparent singularity of the transformation rule in Eq. (3) at the vanishing self-steepening ($s = 0$) makes it difficult to understand the effect of self-steepening on a conventional NLS soliton for which $s = 0$. Moreover, the integrability of the DNLS equation proven by Kaup and Newell is inherently different from that of the NLS equation. While the linear equation associated with the NLS equation is a second-order polynomial in the spectral parameter λ, the DNLS case is a fourth-order polynomial. Therefore, despite the fact that the NLS equation is essentially a special case ($s = 0$) of the DNLS equation at the equation level, the inverse scattering or the Darboux transformation of these two equations are quite different [8,11] so as to forbid the limit $s \to 0$.

In this paper, we resolve this issue by introducing a universal integrability condition that applies to the NLS and the DNLS equations simultaneously. This admits the limit $s \to 0$ and allows us to have the same type Darboux transformation in constructing soliton solutions of the NLS and DNLS equations. Using this method, we construct a general solution for bright or dark solitons of the DNLS equation with nonvanishing asymptotic fields. We find that, in the presence of a self-steepening term, the bright type NLS equation with abnormal group velocity dispersion is related to the dark type NLS equation with normal group velocity dispersion and accordingly exact soliton solutions of the bright type NLS equation describe both bright and dark solitons depending on the strength of the continuous wave. In particular, we present an exact expression for the optical fiber solitons that reveals

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the self-steepening effect on solitons coupled to a continuous wave. Various reductions to known soliton solutions including the $s = 0$ limit are demonstrated.

II. UNIVERSAL INTEGRABILITY

We first present the linear matrix equation that accounts for the integrability of the NLS and the DNLS equations simultaneously. Consider the following overdetermined linear matrix equation:

$$L_T(\lambda)\Phi \equiv \partial_T \Phi + U(\lambda)\Phi = 0,\quad L_X(\lambda)\Phi \equiv \partial_X \Phi + V(\lambda)\Phi = 0, \tag{4}$$

where

$$U(\lambda) = \begin{pmatrix} i\lambda/2 & r \\ q & -i\lambda/2 \end{pmatrix},\quad V(\lambda) = \begin{pmatrix} irq + i\lambda^2/2 & i\partial_T r + \lambda r \\ -i\partial_T q + \lambda q & -irq - i\lambda^2/2 \end{pmatrix}. \tag{5}$$

The consistency of the linear equation, $\partial_T \partial_X \Phi = \partial_X \partial_T \Phi$, requires that $[L_T(\lambda), L_X(\lambda)] = 0$. If this holds for any λ, the consistency condition is equivalent to

$$i\partial_X r = -\partial_T^2 r + 2qr^2 \quad \text{and} \quad i\partial_X q = \partial_T^2 q - 2q^2 r. \tag{6}$$

These coupled equations in r and q admit consistent reductions through relating r and q. An immediate example is the NLS reduction $r = \pm q^*$, which reduces Eq. (6) to

$$i\partial_X q = \partial_T^2 q \mp 2|q|^2 q. \tag{7}$$

This is the NLS equation with normal (upper sign) and abnormal (lower sign) group velocity dispersions that govern optical fiber solitons. A less obvious example is the DNLS reduction, which is one of the main results of the present paper. The DNLS reduction arises from the following implicit relation between r and q:

$$q = \Psi^* e^{2i\mu} \quad \text{and} \quad r = e^{-2i\mu}(|\Psi|^2\Psi + i\partial_T \Psi). \tag{8}$$

Here, the phase variable μ is defined through the equation

$$\partial_T \mu = |\Psi|^2 \quad \text{and} \quad \partial_X \mu = -i(\Psi \partial_T \Psi^* - \Psi^* \partial_T \Psi) + 3|\Psi|^4. \tag{9}$$

One can readily check that the integrability condition in Eq. (6) reduces consistently to the DNLS equation in Eq. (2). Note that the phase variable μ in Eq. (9) is also overdetermined. However, the integrability of $\mu(\partial_T \partial_X \mu = \partial_X \partial_T \mu)$ once again becomes the DNLS equation showing that the DNLS reduction is indeed consistent. Thus, we have established a unifying picture for the integrability of the NLS and the DNLS equations.

One advantage of the present formalism is that we can apply the same Darboux transformation to find exact solutions both for the NLS equation and the DNLS equation. In particular, this allows us to find exact soliton solutions coupled to continuous waves admitting the limit $s \to 0$. To do so, we first choose the continuous wave solution Ψ_{cw} as a seed solution of the DNLS equation and apply the Darboux transformation [12–14], where

$$\Psi_{cw} = a e^{iw[T + (2a^2 - w)X]}. \tag{10}$$

In terms of the variable U in Eq. (3), this corresponds to $U_{cw} = A e^{iB\tau + iC\xi}$ where

$$A = \frac{2^{1/4} a}{\sqrt{s}},\quad B = \frac{1}{s} - \sqrt{2}w,\quad C = -\frac{1}{2s^2} + \frac{\sqrt{2}w}{s} - w^2 + 2a^2 w. \tag{11}$$

In terms of q and r in Eq. (8), we have

$$r_{cw} = a(a^2 - w)e^{i\Delta_{cw}},\quad q_{cw} = a e^{-i\Delta_{cw}},\quad \Delta_{cw} = -(2a^2 - w)T - (6a^4 - 6wa^2 + w^2)X. \tag{12}$$

Consider two-dimensional complex vectors $\vec{s}$ and $\vec{t}$ satisfying the linear equation

$$\partial_T \vec{s} + U_{cw}(\eta)\vec{s} = 0,\quad \partial_X \vec{s} + V_{cw}(\eta)\vec{s} = 0 \tag{13}$$

and

$$-\partial_T \vec{t}^{\,T} + \vec{t}^{\,T} U_{cw}(\eta^*) = 0,\quad -\partial_X \vec{t}^{\,T} + \vec{t}^{\,T} V_{cw}(\eta^*) = 0, \tag{14}$$

where the superscript T and $*$ denote transpose and complex conjugation, respectively. $U_{cw}(\eta)$ and $V_{cw}(\eta)$ are U and V matrices in Eq. (5) evaluated at $r = r_{cw}$, $q = q_{cw}$, and $\lambda = \eta$. Then a direct application of the Darboux transformation provides a new solution describing a soliton in terms of $\vec{s}$ and $\vec{t}$ such that [12–14]

$$r_s = a(a^2 - w)e^{i\Delta_{cw}} + \frac{i(\eta - \eta^*)s_1 t_2}{s_1 t_1 + s_2 t_2},\quad q_s = a e^{-i\Delta_{cw}} - \frac{i(\eta - \eta^*)s_2 t_1}{s_1 t_1 + s_2 t_2}. \tag{15}$$

Vectors $\vec{s}$ and $\vec{t}$ are obtained by integration to yield

$$\begin{aligned} s_1 &= e^{i\Delta_{cw}/2}(\Gamma_+ u_1 e^{\Delta_2} + \Gamma_- u_2 e^{-\Delta_2}) \\ s_2 &= e^{-i\Delta_{cw}/2}(u_1 e^{\Delta_2} + u_2 e^{-\Delta_2}) \\ t_1 &= e^{-i\Delta_{cw}/2}(\Gamma_+^* v_1 e^{\Delta_2^*} + \Gamma_-^* v_2 e^{-\Delta_2^*})/(w - a^2) \\ t_2 &= e^{i\Delta_{cw}/2}(v_1 e^{\Delta_2^*} + v_2 e^{-\Delta_2^*}), \end{aligned} \tag{16}$$

where u_1, u_2, v_1, v_2 are constants of integration and

$$\Delta_2 = \frac{1}{2}\sqrt{4\eta a^2 - (\eta + w)^2}[T + (2a^2 - w + \eta)X],\quad \Gamma_\pm = -\frac{1}{2a}(-i\eta + 2ia^2 - iw \pm \sqrt{4\eta a^2 - (\eta + w)^2}). \tag{17}$$

In order to simplify the notation, we introduce real parameters β and γ such that

$$\eta + (w - 2a^2) \equiv 2ia\sqrt{w - a^2}\cosh(\beta + i\gamma). \tag{18}$$

Without loss of generality, we confine constants of integration to

$$u_1 = e^{\delta_1},\quad u_2 = e^{-\delta_1},\quad v_1 = e^{\delta_2},\quad v_2 = e^{-\delta_2}, \tag{19}$$

where δ_1,δ_2 are complex numbers. A critical observation is that the DNLS reduction in Eq. (8) can be realized simply by relating these constants of integration. A lengthy but straightforward calculation shows that the DNLS reduction can be made successfully if

$$\delta_1^* - \delta_2 \equiv -\kappa = \frac{1}{2}\ln\left(\frac{w + a^2[\exp(2\beta - 2i\gamma) - 1]}{w - 2a\sqrt{a^2 - w}\sinh(\beta - i\gamma)}\right). \tag{20}$$

For a more compact expression, we treat two cases, $w \geqslant a^2$ and $w < a^2$, separately. For $w \geqslant a^2$, we can rewrite r_s and q_s by

$$\begin{aligned} r_s &= -a(a^2 - w)e^{i\Delta_{\rm cw}}\frac{\cosh\beta\cosh(P + 2i\gamma) + \cos\gamma\cosh(M + 2\beta)}{\cosh\beta\cosh P + \cos\gamma\cosh M} \\ q_s &= -ae^{-i\Delta_{\rm cw}}\frac{\cosh\beta\cosh(P - 2i\gamma) + \cos\gamma\cosh(M - 2\beta)}{\cosh\beta\cosh P + \cos\gamma\cosh M}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} P &\equiv \delta_1 + \delta_1^* + \Delta_2 + \Delta_2^* + \beta + \kappa \\ M &\equiv \delta_1 - \delta_1^* + \Delta_2 - \Delta_2^* + i\gamma - \kappa. \end{aligned} \tag{22}$$

For $w < a^2$, or equivalently $1 < sB + sA^2$, we can rewrite r_s and q_s by

$$\begin{aligned} r_s &= -a(a^2 - w)e^{i\Delta_{\rm cw}}\frac{\sinh\beta\sinh(P + 2i\gamma) + i\sin\gamma\sinh(M + 2\beta)}{\sinh\beta\sinh P + i\sin\gamma\sinh M} \\ q_s &= -ae^{-i\Delta_{\rm cw}}\frac{\sinh\beta\sinh(P - 2i\gamma) + i\sin\gamma\sinh(M - 2\beta)}{\sinh\beta\sinh P + i\sin\gamma\sinh M}, \end{aligned} \tag{23}$$

where P and M are given in Eq. (22).

We now find the solution of the nonlinear Schrödinger equation with a self-steepening term given in Eq. (1) using the transformation in Eq. (3) and rewriting parameters a and w of the background continuous wave in terms of A and B as given in Eq. (11) and $\beta \equiv \ln(\sigma/A)$. The resulting solution is

$$U_{\rm sol}(\xi,\tau) = -e^{i\Theta}W^*, \tag{24}$$

where

$$\begin{aligned} \Theta &= 2\mu + (B + 2sA^2)\tau + \left(-\frac{1}{2}B^2 + A^2 - 3sA^2B - 3s^2A^4\right)\xi \\ W &= \frac{(\sigma^4e^{-iN} + A^4e^{iN})\cos\gamma + (A\sigma^3 + A^3\sigma)\cosh(P - 2i\gamma)}{(\sigma^3 + A^2\sigma)\cosh P + 2A\sigma^2\cos\gamma\cos N}, \quad 1 > sB + s^2A^2 \\ &= \frac{i(\sigma^4e^{-iN} - A^4e^{iN})\sin\gamma + (A^3\sigma - A\sigma^3)\sinh(P - 2i\gamma)}{(-\sigma^3 + A^2\sigma)\sinh P + 2A\sigma^2\sin\gamma\sin N}, \quad 1 < sB + s^2A^2 \end{aligned} \tag{25}$$

and we suppress an explicit expression of μ. For convenience, we introduce a new set of parameters,

$$\begin{aligned} \kappa &= \frac{1}{2}\ln\left(\frac{\epsilon + s^2A^2 + s\sqrt{-\epsilon}(A^2\sigma^{-1}e^{i\gamma} - \sigma e^{-i\gamma})}{\epsilon + s^2\sigma^2e^{-2i\gamma}}\right) \equiv \kappa_R + i\kappa_I, \\ \epsilon &\equiv 1 - sB - s^2A^2, \end{aligned} \tag{26}$$

where κ_R and κ_I denote the real and imaginary part of κ. In terms of these new parameters, the arguments P and $N(= -iM)$ in Eq. (25) are given by

$$P \equiv \Delta_+ + i\kappa_I, N \equiv \Delta_- + i\kappa_R, \tag{27}$$

where

$$\begin{aligned} \Delta_+ &\equiv -\sqrt{\epsilon}\cos\gamma(\sigma - A^2\sigma^{-1})\left[\tau - \left(B + 2sA^2 - \frac{\sqrt{\epsilon}\sin\gamma(\sigma^4 + A^4)}{\sigma(\sigma^2 - A^2)}\right)\xi\right], \quad \epsilon \geqslant 0 \\ &\equiv \sqrt{-\epsilon}\sin\gamma(\sigma + A^2\sigma^{-1})\left[\tau - \left(B + 2sA^2 - \frac{\sqrt{-\epsilon}\cos\gamma(\sigma^4 + A^4)}{\sigma(\sigma^2 + A^2)}\right)\xi\right], \quad \epsilon < 0 \end{aligned}$$

$$\begin{aligned}
\Delta_- &\equiv -\sqrt{\epsilon}\sin\gamma(\sigma + A^2\sigma^{-1})\left[\tau - \left(B + 2sA^2 + \frac{\sqrt{\epsilon}\cos 2\gamma(\sigma^2 - A^2)}{2\sigma\sin\gamma}\right)\xi\right], \quad \epsilon \geqslant 0\\
&\equiv -\sqrt{-\epsilon}\cos\gamma(\sigma - A^2\sigma^{-1})\left[\tau - \left(B + 2sA^2 - \frac{\sqrt{-\epsilon}\cos 2\gamma(\sigma^2 + A^2)}{2\sigma\cos\gamma}\right)\xi\right], \quad \epsilon < 0 \qquad (28)\\
\frac{\partial\mu}{\partial\tau} &= -s|W|^2\\
\frac{\partial\mu}{\partial\xi} &= \frac{i}{2}s\left(W^*\frac{\partial W}{\partial\tau} - W\frac{\partial W^*}{\partial\tau}\right) + s(B + 2sA^2)|W|^2 - \frac{1}{2}s^2|W|^4.
\end{aligned}$$

For simplicity, we have chosen that $\delta_1 = -(\kappa_R + \beta)/2 + i(\kappa_I - \gamma)/2$, which trivially shifts the location of the soliton. Finally, the validity of the solution in Eqs. (24)–(28) has been confirmed explicitly by using the MAPLE computer algebra system.

III. SOLITON SOLUTIONS

The exact solution given in Eqs. (24)–(28) describes an optical soliton coupled to a continuous wave in the presence of a self-steepening term. We note that the self-steepening parameter s divides the solution into two distinct types: (i) the bright soliton type ($1 > sB + s^2A^2$) and (ii) the dark soliton type ($1 < sB + s^2A^2$), which is absent if $s = 0$. When $1 = sB + s^2A^2$, we have $\Delta_+ = \Delta_- = 0$ and our solution simply reduces to the continuous wave solution. Now, we discuss these two types separately.

A. Bright soliton type

The continuous wave coupled to a bright soliton beats with the soliton despite the nonlinear nature of the coupling. The continuous wave controls the velocity of the soliton which can be read easily from the argument Δ_+ in Eq. (28) and can even stop the soliton [15]. In the absence of a continuous wave so that $A = 0$, $B = 0$, The solution simplifies to

$$U_{\rm sol}(\tau,\xi) = -\sigma\,\exp(\tilde{\kappa}_R + i\Sigma)\mathrm{sech}(i\tilde{\kappa}_I + \Omega), \qquad (29)$$

where

$$\begin{aligned}
\Omega &= \sigma(\tau - v\xi)\\
\Sigma &= v\tau - \frac{1}{2}(v^2 - \sigma^2)\xi - \frac{4s\sigma\exp(2\tilde{\kappa}_R)}{|\sin 2\tilde{\kappa}_I|}\tan^{-1}[\tan\tilde{\kappa}_I\tanh\Omega]
\end{aligned} \qquad (30)$$

and parameters $\tilde{\kappa}_R$ and $\tilde{\kappa}_I$ are the real and imaginary part of $\tilde{\kappa}$,

$$\tilde{\kappa} \equiv \tilde{\kappa}_R + i\tilde{\kappa}_I = -\tfrac{1}{2}\ln[1 - s(v - i\sigma)]. \qquad (31)$$

Note that in the absence of the self-steepening term ($s = 0$), κ vanishes and the soliton solution reduces to the well-known one-soliton solution of the NLS equation.

In order to see the self-steepening effect on bright solitons more explicitly, we expand $U_{\rm sol}$ in Eq. (29) in terms of s. Up to the first order, it is given by

$$\begin{aligned}
U_{\rm sol} &= -\sigma\frac{e^{i\Sigma_0}}{\cosh\Omega}\left[1 + \frac{1}{2}s(v + 9i\sigma\tanh\Omega)\right]\\
\Sigma_0 &= v\tau - \frac{1}{2}(v^2 - \sigma^2)\xi, \quad \Omega = \sigma(\tau - v\xi).
\end{aligned} \qquad (32)$$

This clearly shows that the soliton is robust against self-steepening, that is, it avoids the shock formation by maintaining the secant hyperbolic type pulse profile except for the symmetric distortion of amplitude and the antisymmetric phase shift across the pulse. It is remarkable that the profile distortion depends on soliton velocity v explicitly as depicted in Fig. 2. The background continuous wave in general makes soliton pulse asymmetric as can be seen in Fig. 1 though the shock formation is still absent in this case.

B. Dark soliton type

If the continuous wave is dominant so that $1 < sB + s^2A^2$, the soliton solution coupled to a continuous wave behaves like a dark type soliton beating with a continuous wave. This is rather surprising since the NLS equation we solved has an abnormal group velocity dispersion whereas dark solitons are known to arise for the NLS equation having a normal group velocity dispersion. Explicitly, the dark soliton type NLS equation has the form

$$i\frac{\partial U}{\partial\xi} - \frac{1}{2}\frac{\partial^2 U}{\partial\tau^2} + |U|^2U + is\frac{\partial}{\partial\tau}(|U|^2U) = 0, \qquad (33)$$

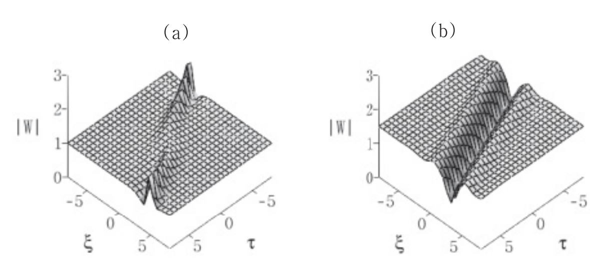

FIG. 1. (a) Bright soliton type and (b) dark soliton type solutions. Parameters are $\sigma = 2.5$, $s = 0.7$, $\gamma = 0.1$, $A = 1$, and $B = 0.5$ for the bright soliton type and $\sigma = 2$, $s = 0.7$, $\gamma = 0.1$, $A = 1.5$, and $B = 0.5$ for the dark soliton type.

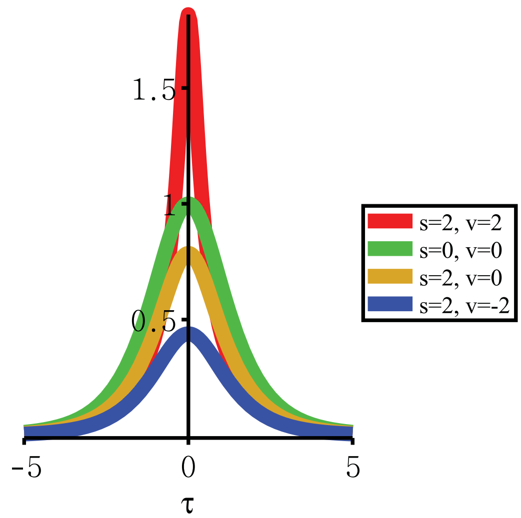

FIG. 2. (Color online) Effect of the self-steepening term on a one-soliton solution. Curves are with (i) $s = 2$, $v = 2$, (ii) $s = 0$, $v = 0$, (iii) $s = 2$, $v = 0$, and (iv) $s = 2$, $v = -2$, respectively (peak values in decreasing order).

where the sign of the second time derivative term is changed in comparison with the abnormal case in Eq. (1). A remarkable fact is that the dark type NLS equation can be also mapped to the DNLS equation via the following change of coordinates and field redefinition:

$$X = -\xi, \quad T = \sqrt{2}\left(\tau + \frac{\xi}{s}\right) \quad \text{and}$$
$$U \equiv \frac{2^{1/4}}{\sqrt{s}}\Psi \exp\left[i\frac{\tau}{s} + i\frac{\xi}{2s^2}\right]. \tag{34}$$

This in turn shows that the bright type and the dark type NLS equations are directly related if the self-steepening term is present. If we denote ξ_a, τ_a, U_a and ξ_n, τ_n, U_n for the coordinates and fields for the abnormal and normal cases, respectively, the correspondence between the bright type and the dark type NLS equations are given by

$$\xi_n = -\xi_a, \quad \tau_n = -\tau_a + \frac{2\xi_a}{s},$$
$$U_n = U_a \exp\left(-\frac{2i\tau_a}{s} + \frac{i\xi_a}{s^2}\right). \tag{35}$$

This remarkable property implies that our solution in Eqs. (24)–(28) could describe dark solitons as well. In fact, the $1 < sB + s^2A^2$ case, which we call as a dark type, indeed includes dark solitons. Note that, in the limit $\sigma \to A$, the solution in Eqs. (24)–(28) for the $1 < sB + s^2A^2$ case reduces to

$$\Delta_- = 0, \quad \kappa_R = 0,$$
$$\Delta_+ = 2A\sqrt{-\epsilon}\sin\gamma[\tau - (B + 2sA^2 - A\sqrt{-\epsilon}\cos\gamma)\xi]$$
$$W = A\frac{\sinh(\Delta_+ + i\kappa_I - 2i\gamma)}{\sinh(\Delta_+ + i\kappa_I)}, \tag{36}$$

which is a dark type soliton without a beating behavior, but not completely dark in general and thus occasionally referred to as a gray soliton. It becomes a completely dark soliton if we further restrict parameters by taking $\gamma = \pi/4, B = 1/s - sA^2/2$ so that $\kappa_I = \pi/2, \epsilon = -2$ and

$$|W|^2 = A^2\tanh^2(\Delta_+),$$
$$\Delta_+ = 2A\left[\tau - \left(\frac{1}{s} + \frac{3sA^2}{2} - A\right)\xi\right], \tag{37}$$

which is the well-known dark soliton.

IV. CONCLUSION

In this paper, we presented a universal integrability scheme where the NLS and the DNLS equations have the same Lax pair and the distinction between them is given only through the reduction procedure. The NLS equation arises as a twofold Z_2 reduction whereas the reduction for the DNLS equation is rather implicit. Despite the implicit nature of the DNLS reduction in Eq. (8), it is likely that there exists a hierarchy of reductions involving higher-order derivatives where the NLS and the DNLS are the first two cases of the hierarchy. Extension of our work to the multicomponent vector NLS and DNLS equations is also possible.

The exact soliton solution we found in this paper could have important physical applications. Ultrashort optical pulses necessarily require the self-steepening term in the governing equation. Our solution describing the motion of a soliton coupled to continuous wave can also describe the scattering between narrow and broad solitons where the broad soliton is approximated as a continuous wave. This raises the possibility of controlling solitons parametrically using a continuous wave or other solitons. The coexistence of bright and dark type solitons in the presence of a self-steepening term also raises the possibility of parametric conversion between bright and dark type solitons by controlling continuous waves.

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