Nonlinear wave propagation in a strongly coupled collisional dusty plasma

Samiran Ghosh
Department of Applied Mathematics, University of Calcutta, 92, Acharya Prafulla Chandra Road, Kolkata 700 009, India

Mithil Ranjan Gupta
Department of Instrumentation Science, Jadavpur University, Kolkata 700 032, India

Nikhil Chakrabarti
Saha Institute of Nuclear Physics, 1/AF Bidhannagar Calcutta 700 064, India

Manis Chaudhuri
Max-Planck Institut für extraterrestrische Physik, D-85741 Garching, Germany
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The propagation of a nonlinear low-frequency mode in a strongly coupled dusty plasma is investigated using a generalized hydrodynamical model. For the well-known longitudinal dust acoustic mode a standard perturbative approach leads to a Korteweg-de Vries (KdV) soliton. The strong viscoelastic effect, however, introduced a nonlinear forcing and a linear damping in the KdV equation. This novel equation is solved analytically to show a competition between nonlinear forcing and dissipative damping. The physical consequence of such a solution is also sketched.

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I. INTRODUCTION

The physics of strongly coupled plasmas (SCPs), in which the average potential energy per particle dominates over the average kinetic energy, is of great interest because of its possible applications to a large number of physical systems, for example, complex (dusty) plasmas [1], charged particles in cryogenic traps, electrons trapped on the surface of liquid He, different astrophysical systems such as the ion liquid in white dwarf interiors [2], neutron star crusts, supernova cores, and giant planetary interiors, condensed matter systems such as molten salts and liquid metals, as well as degenerate electron or hole liquids in two-dimensional or layered semiconductor nanostructures [3,4]. The formation of SCPs in normal two-species was predicted by Ikezi [5] for the first time due to the presence of additional finite-size particles. About a decade later it was found experimentally in different laboratories [6–8]. In recent years complex plasmas, a mixture of ions, electrons, and highly charged micro/nanoparticles, are being considered as a major interdisciplinary research field for exploring fundamental physics of strongly coupled Coulomb and Yukawa systems [9–11]. Presently complex plasma is also being considered as the plasma state of soft condensed matter [12].

One of the fundamental characteristics of a many-particle interacting system is the coupling parameter Γ defined as the ratio of the potential energy of interaction between neighboring particles to their kinetic energy. In complex plasmas the negatively charged micro-particles interact with each other via the isotropic Debye-Hückel (Yukawa) repulsive potential, with the screening determined by the plasma electrons and ions. In this case the coupling parameter is defined as $\Gamma = [q_d^2/(k_B a T_d)]\exp(-\kappa)$, where $\kappa = a/\lambda_{\mathrm{D}}$ is the screening parameter. Here $q_d\ (= Ze)$ is the charge on each dust particle, Z is the number of elementary electronic charge e, $a\ (\simeq n_d^{-1/3})$ is the interdust particle distance, n_d and T_d are the dust density and temperature, λ_{D} is the plasma Debye length, and k_B is the Boltzmann constant. Experimental findings [13] suggest the existence of liquid- or solid-like behavior of the dusty plasma medium in the strong correlation regime characterized by $\Gamma \gg 1$. In the regime $1 \ll \Gamma < \Gamma_c$ (where Γ_c is the critical value of coupling parameter beyond which the system becomes crystalline), the dusty plasma behaves like a mixture of a liquid and solid; i.e., both viscosity and elasticity play equally important roles. These properties together are known as viscoelasticity, and the dust grains are said to be in a quasicrystalline state [14,15]. In the regime $\Gamma > \Gamma_c$, viscosity disappears, and only elasticity dominates over the system. The dust grains exhibit crystalline structure and support various dust lattice modes [16]. Therefore, the low-frequency collective modes involving the motion of dust grains in SCPs should be greatly influenced by the strong correlation effects in the dust component [17].

A number of authors have investigated the effects of dust-dust correlation on the low-frequency collective behavior of linear dust acoustic waves (DAWs) [18] in strongly coupled quasicrystalline dusty plasma ($1 \ll \Gamma < \Gamma_c$) by using various theoretical models such as the quasilocalized charge approximation [19], static local field correction [20], and generalized hydrodynamic (GH) model [14,15,21]. In the GH model [22], there are two regimes: the "hydrodynamic regime" ($\omega\tau_m \ll 1$) and "kinetic regime" ($\omega\tau_m \gg 1$), where τ_m is the memory (viscoelastic) relaxation time and ω^{-1} is the typical time scale of the wave under consideration. In the "hydrodynamic limit," the viscoelastic relaxation is instantaneous, and one has the usual hydrodynamic equation. In this case dust grains support only the longitudinal dust acoustic wave (LDAW), which suffers only viscous dissipation [14,15]. On the other hand, in the "kinetic regime" the viscoelastic relaxation is not instantaneous, and the dusty plasma supports both the LDAW as well as the transverse shear wave [14,15,23,24].

In weakly coupled dusty plasma ($\Gamma \ll 1$), the nonlinear coherent structures (such as shock and soliton) of the DAW are well investigated both experimentally [25–27] and theoretically [28–30]. But the nonlinear propagation characteristics of theLDAW in strongly coupled quasicrystalline dusty plasma ($1 \ll \Gamma < \Gamma_c$) are not well investigated. Recently, the nonlinear wave propagation characteristics of LDAWs have been investigated theoretically [31] in strongly coupled dusty plasma using the GH equation in the "hydrodynamic regime," and it has been shown that the nonlinear wave is governed by "Korteweg-de Vries (KdV)-Burger's" equation due to viscous damping. But the nonlinear wave propagation characteristics have not been well investigated in the "kinetic regime" of strongly coupled dusty plasma. Therefore, in this paper we investigate the nonlinear propagation characteristics of a small but finite-amplitude LDAW in the "kinetic regime" of strongly coupled dusty plasma. The effects of dust-neutral collision and finite strain relaxation are also taken into account. It is shown in addition to a weak dissipation due to dust-neutral collision [32], that the nonzero value of $1/\tau_m\omega$ also produces a weak dissipation. The memory-dependent strong correlation produces an extra nonlocal nonlinear forcing term in the nonlinear evolution of LDAWs. As a consequence of these, the dynamics of the nonlinear LDAWs are governed by a modified form of the forced-damped KdV equation.

The paper is organized in the following manner: Section II contains the theoretical model and basic equations. The modified form of the damped KdV equation describing the nonlinear wave propagation is derived in Sec. III. The analytical solution of the modified form of the damped KdV equation is investigated in Sec. IV. Section V deals with the physical interpretations of the results. Finally, a brief discussion of our results and concluding remarks are given in Sec. VI.

II. MODEL AND BASIC EQUATIONS

An unmagnetized strongly coupled dusty plasma whose constituents are electrons, ions, and negatively fixed charged, massive dust grains are considered. Thus at equilibrium the quasineutrality condition is $n_{e0} + Zn_{d0} = n_{i0}$, where $n_{e0}\,(n_{d0},n_{i0})$ is the electron (dust, ion) equilibrium number density and Z is the number of the charge residing on the dust grains. The dust grains are strongly correlated to each other (strongly coupled) due to their larger electric charge and lower temperature, whereas both electrons and ions are weakly coupled because of their smaller electric charges and higher temperatures. So the dynamics of nonlinear LDAWs in strongly coupled dusty plasma can be modeled by the GH equation. Since we are looking at the low-frequency mode ($\omega \ll kv_{th\,e(i)}$), it can be assumed that electrons and ions can form a Boltzmann distribution. Thus the number densities of electrons and ions can be expressed as

$$n_e = n_{e0}\exp\left(\frac{e\varphi}{T_e}\right);\;\; n_i = n_{i0}\exp\left(-\frac{e\varphi}{T_i}\right), \tag{1}$$

where φ is the electrostatic potential and $T_{e(i)}$ is the electron (ion) temperature.

For the description of the dust fluid, we have considered the GH, model which is given by the following equation [14,22]:

$$\left(1+\tau_m\frac{d}{dt}\right)\left(m_d n_d\frac{dv_d}{dt} - q_d n_d\frac{\partial\varphi}{\partial x} + \frac{\partial p_d}{\partial x} + m_d n_d \nu_{dn} v_d\right) = \left(\frac{4}{3}\eta+\zeta\right)\frac{\partial^2 v_d}{\partial x^2}, \tag{2}$$

where $d/dt = \partial/\partial t + v_d\partial/\partial x$, and we have taken only one spatial dimension. Generalization to three dimensions is trivial and straightforward. In Eq. (2) the parameters η and ζ are shear and bulk viscosity coefficients, and p_d, v_d, and ν_{dn} are the dust pressure, dust fluid velocity, and dust-neutral collision frequency, respectively. It is always convenient to write equations in the dimensionless form; for this, here we introduce the following dimensionless variables: $x' = x/\lambda_d$, $t' = \omega_{pd}t$, $n = n_d/n_{d0}$, $\phi = e\varphi/T_e$, and $v = v_d/v_{td}$. Here $\lambda_d(=\sqrt{T_d/4\pi n_{d0}q_d^2})$ and $v_{td}(=\sqrt{T_d/m_d})$ are the dust Debye length and dust thermal velocity, respectively. In dimensionless form Eq. (1) can be rewritten as

$$\left[\frac{1}{\tau_m\omega_{pd}} + \left(\frac{\partial}{\partial t'} + v\frac{\partial}{\partial x'}\right)\right]\left[n\left(\frac{\partial}{\partial t'} + v\frac{\partial}{\partial x'}\right)v - n\sigma_d\frac{\partial\phi}{\partial x'} + \mu_d\frac{\partial n}{\partial x'} + \left(\frac{\nu_{dn}}{\omega_{pd}}\right)nv\right] = \bar{\eta}\frac{\partial^2 v}{\partial x'^2}, \tag{3}$$

where $\sigma_d = ZT_e/T_d$, Z is the fixed number of the charge on the dust particle, and μ_d is the compressibility factor for the dust fluid defined as

$$\mu_d = \frac{1}{T_d}\frac{\partial p_d}{\partial n_d} = 1 + \frac{u(\Gamma)}{3} + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma},$$

where the function $u(\Gamma)$ is a measure of the excess internal energy of the system, which is related to the correlation energy and can be written as $u(\Gamma) = a(\kappa)\Gamma + b(\kappa)\Gamma^{\frac{1}{3}} + c(\kappa) + d(\kappa)\Gamma^{-\frac{1}{3}}$ [19]. Note that, in the above GH equation, the normalized viscoelastic relaxation time τ_m is given by [33]

$$\tau_m\omega_{pd} = 3\eta^*\Gamma\left[1 - \gamma_d\mu_d + \frac{4}{15}u(\Gamma)\right]^{-1},$$

where γ_d is the adiabatic index, $\eta^* = (\frac{4}{3}\eta+\zeta)/m_d n_{d0}\omega_{pd}a_d^2$ is the viscosity coefficient, $a_d = ae^{-a/2\lambda_D}$ is the effective interaction distance of dust grains, and $\bar{\eta}$ is given by

$$\bar{\eta} = \frac{\eta^*}{\tau_m\omega_{pd}}\left(\frac{a_d}{\lambda_d}\right)^2 = \left[1 - \gamma_d\mu_d + \frac{4}{15}u(\Gamma)\right]. \tag{4}$$

The above GH equation (3) is coupled with the continuity equation

$$\frac{\partial n}{\partial t'} + \frac{\partial}{\partial x'}(nv) = 0 \tag{5}$$

and Poisson's equation

$$\sigma_d(\delta-1)\frac{\partial^2\phi}{\partial x'^2} = \exp(\phi) - \delta\exp\left(-\frac{\phi}{\sigma_i}\right) + (\delta-1)n, \tag{6}$$

where $\sigma_i = T_i/T_e$ and $\delta = n_{i0}/n_{e0}$. Therefore Eqs. (3), (5), and (6) will be studied in the next section for nonlinear propagation of longitudinal mode in a strongly coupled plasma.

III. MODIFIED FORM OF THE DAMPED KORTEWEG-DE VRIES EQUATION

To study the nonlinear propagation characteristics of a LDAW in the "kinetic regime" $[1/(\tau_m\omega_{pd}) < 1]$ of strongly coupled dusty plasma, the reductive perturbation technique has been employed, and the following stretched coordinate has been introduced:

$$\xi = \epsilon^{\frac{1}{2}}(x' - Mt');\ \tau = \epsilon^{\frac{3}{2}}t', \tag{7}$$

where M is the phase velocity of the mode normalized by the dust thermal speed and ϵ measures the order of smallness of the perturbations. The dynamical variables n,v, and ϕ are expanded in power series of ϵ as

$$f = f^{(0)} + \sum_{i=1}^{\infty}\epsilon^i f^{(i)}, \tag{8}$$

where $f = n,v,\phi$, $f^{(0)} = 1$ for n and $f^{(0)} = 0$ for $f = v,\phi$.

Now, to introduce the effects of finite strain relaxation (under the assumption that $1/\tau_m\omega_{pd}$ is small but finite) and dust-neutral collision (under the assumption that ν_{dn}/ω_{pd} is small but finite), and to make the nonlinear perturbation consistent with that of (7) and (8), the following scalings are assumed: $1/\tau_m\omega_{pd} \sim \epsilon^{\frac{3}{2}}$ and $\nu_{dn}/\omega_{pd} \sim \nu_c\epsilon^{\frac{3}{2}}$ where $\nu_c \approx O(1)$.

Finally substitution of (7) and (8) into the dynamical equations (2), (5), and (6) yields the following relations in lowest powers of ϵ:

$$v^{(1)} = Mn^{(1)},\quad (M^2 - \bar{\eta})v^{(1)} = M(\mu_d n^{(1)} - \sigma_d\phi^{(1)}),$$
$$n^{(1)} = -\frac{\delta+\sigma_i}{\sigma_i(\delta-1)}\phi^{(1)}. \tag{9}$$

These relations can reproduce the linear dispersion relation when M is replaced by ω/k. From these relations we obtain

$$M = \sqrt{\bar{\eta} + \mu_d + \frac{\sigma_d\sigma_i(\delta-1)}{\delta+\sigma_i}}.$$

Note that for a simple case $\bar{\eta} = 0 = \mu_d$, one recovers the well-known expression for the dispersion relation of usual DAW in a dusty plasma [18]:

$$\omega^2 = \frac{k^2C_d^2(Zn_{d0})}{n_{e0} + \frac{T_e}{T_i}n_{i0}}, \tag{10}$$

where $C_d = \sqrt{ZT_e/m_d}$. Next, dynamical equations in the next higher powers of ϵ are obtained as

$$\frac{\partial n^{(1)}}{\partial\tau} + 2Mn^{(1)}\frac{\partial n^{(1)}}{\partial\xi} = M\frac{\partial n^{(2)}}{\partial\xi} - \frac{\partial v^{(2)}}{\partial\xi}, \tag{11}$$

$$\frac{\partial}{\partial\xi}\left[2M\frac{\partial v^{(1)}}{\partial\tau} + \sigma_d\frac{\partial\phi^{(1)}}{\partial\tau} - \mu_d\frac{\partial n^{(1)}}{\partial\tau}\right.$$
$$\left. + (Mv^{(1)} + \sigma_d\phi^{(1)} - \mu_d n^{(1)}) + M\nu_c v^{(1)}\right]$$
$$= \frac{\partial}{\partial\xi}\left[(M^2-\bar{\eta})\frac{\partial v^{(2)}}{\partial\xi} + M\sigma_d\frac{\partial\phi^{(2)}}{\partial\xi} - M\mu_d\frac{\partial n^{(2)}}{\partial\xi}\right.$$
$$\left. -\bar{\eta}Mn^{(1)}\frac{\partial n^{(1)}}{\partial\xi} + M\sigma_d n^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}\right] + \frac{\bar{\eta}}{M}\left(\frac{\partial v^{(1)}}{\partial\xi}\right)^2, \tag{12}$$

$$\sigma_d(\delta-1)\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = \left(\frac{\delta+\sigma_i}{\sigma_i}\right)\phi^{(2)}$$
$$+ \left(\frac{\sigma_i^2-\delta}{2\sigma_i^2}\right)\phi^{(1)2} + (\delta-1)n^{(2)}. \tag{13}$$

Finally, elimination of $n^{(2)}$, $\phi^{(2)}$, and $v^{(2)}$ from Eqs. (11)–(13) and using Eq. (9), the following modified form of the KdV equation in the "kinetic regime ($\omega\tau_m \gg 1$)" of strongly coupled dusty plasma is obtained:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial n^{(1)}}{\partial\tau} + \alpha\, n^{(1)}\frac{\partial n^{(1)}}{\partial\xi} + \beta\,\frac{\partial^3 n^{(1)}}{\partial\xi^3} + \nu\, n^{(1)}\right] = \gamma\left(\frac{\partial n^{(1)}}{\partial\xi}\right)^2, \tag{14}$$

where coefficients α, β, ν, and γ can be written in the following simplified form:

$$\alpha = \frac{1}{2M}\left\{\bar{\eta} + 2\mu_d + \left[\frac{\sigma_i\sigma_d(\delta-1)}{\sigma_i+\delta}\right]\left[3 + \frac{(\delta-1)(\sigma_i^2-\delta)}{(\delta+\sigma_i)^2}\right]\right\};$$

$$\beta = \frac{1}{2M}\left[\frac{\sigma_i\sigma_d(\delta-1)}{\sigma_i+\delta}\right]^2,$$

$$\nu = \frac{\nu_c}{2} + \frac{1}{2}\frac{\bar{\eta}}{M^2};\quad \gamma = \frac{\bar{\eta}}{2M}.$$

Equation (14) is the modified form of the KdV equation. Modification was due to the viscoelastic effect, as can be seen from the expression sν and γ. It should be mentioned that, even if the usual dust-neutral collision is absent, i.e., $\nu_c = 0$ in the expression of ν above, then the effective collision is introduced due to the viscoelastic effect through $\bar{\eta}$. Therefore the viscoelastic effect introduced new physics in the KdV equation, namely, nonlinear forcing and collisional diffusion. Not only does viscoelasticity also fortify the usual nonlinear KdV term as can be seen from the expression α above, but the dispersion term β is also unaffected due to the viscoelastic effect.

Equation (14) can be further simplified by integrating with respect to ξ in the interval $(-\infty,\xi]$ and using the boundary conditions $n^{(1)},\partial n^{(1)}/\partial\xi \to 0$ as $\xi \to -\infty$. Applying these we have the following equation:

$$\frac{\partial n^{(1)}}{\partial\tau} + \alpha n^{(1)}\frac{\partial\, n^{(1)}}{\partial\xi} + \beta\,\frac{\partial^3 n^{(1)}}{\partial\xi^3} + \nu\, n^{(1)} = \gamma\int_{-\infty}^{\xi}\left(\frac{\partial n^{(1)}}{\partial\xi'}\right)^2 d\xi'. \tag{15}$$

Equation (15) shows that in the absence of dust correlation, i.e., for $\bar{\eta} = 0$ and collision, i.e., for $\nu_c = 0$, we get the usual KdV equation. Thus, in the "kinetic regime" viscoelasticity introduces dissipation and an extra nonlinear force in the strongly correlated dust fluid.

IV. ANALYSIS FOR SOLUTION

An exact analytical solution of Eq. (15) does not seem possible. However, we can find an approximate time-dependent solution of (15). In order to study the effects of the collision and viscoelastic effect that is responsible for changing the character of the KdV equation, we first find that in the

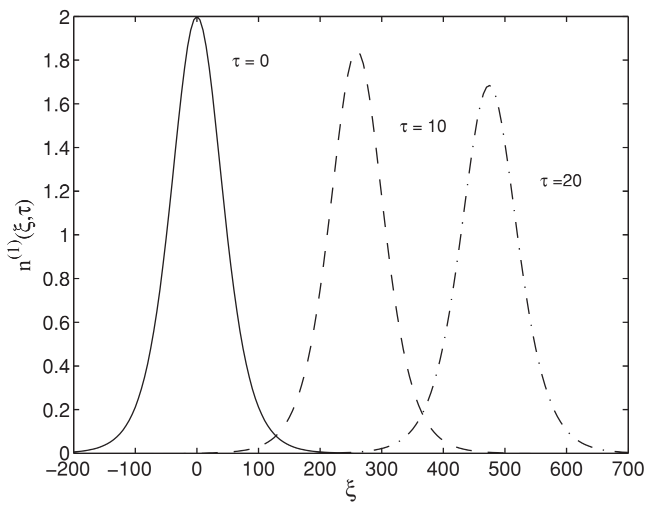

FIG. 1. Propagating solitary wave solution of the modified KdV equation at different times at initial amplitude is below a critical value (here $\sim$3.4). This shows that the amplitude of the solitary wave decreases with time in the presence of collisions.

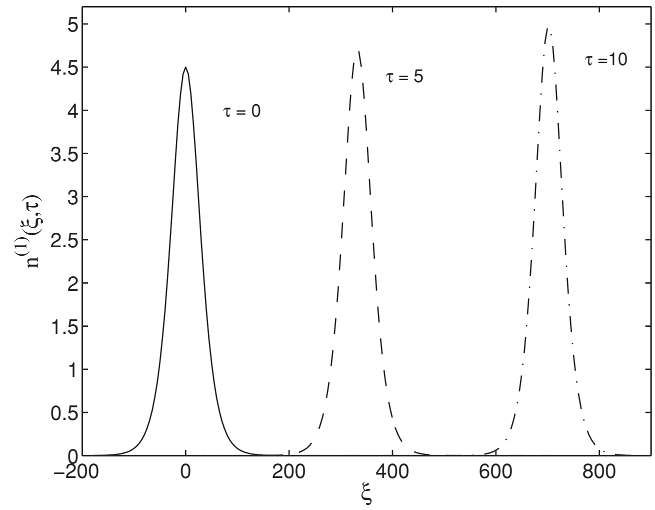

FIG. 2. Propagating solitary wave solution of the modified KdV equation at different times when initial amplitude exceeds a critical value as mentioned above. The figure shows that the amplitude of the solitary wave increases with time due to the nonlinear effect.

absence of collision and viscosity ($\nu_c = 0, \bar{\eta} = 0$), Eq. (15) is a well-known KdV equation for which the solitary wave solution is given by

$$n^{(1)}(\xi,\tau) = U \mathrm{sech}^2\left[\sqrt{\frac{\alpha U}{12\beta}}\left(\xi - \frac{\alpha U}{3}\tau\right)\right], \tag{16}$$

where U is the normalized speed in which the solitary wave moves. The KdV equation (15) with $\nu_c = 0, \bar{\eta} = 0$ has an infinite set of conservation laws. To determine the effects of the collision and viscoelastic effect on the solution given by (15), we consider a "momentum" conservation law. In the presence of collision and dust correlation this results in

$$\frac{dI}{d\tau} = -2\nu I + \gamma \int_{-\infty}^{\infty} n^{(1)} \left[\int_{-\infty}^{\xi} \left(\frac{\partial n^{(1)}}{\partial \xi}\right)^2 d\xi\right] d\xi, \tag{17}$$

where $I = \frac{1}{2}\int_{-\infty}^{\infty} {n^{(1)}}^2 d\xi$. For small γ and ν perturbation theory [34,35] provides the following approximate method of finding an analytical solution of Eq. (15). Following the above mentioned references, we allow the free parameter U in Eq. (16) to be time dependent, i.e.,

$$n^{(1)}(\xi,\tau) = U(\tau)\, \mathrm{sech}^2\left\{\sqrt{\frac{\alpha\, U(\tau)}{12\beta}}\left[\xi - \frac{\alpha\, U(\tau)}{3}\tau\right]\right\}. \tag{18}$$

Substituting Eq. (18) in Eq. (17) and solving, we get

$$\frac{U(\tau)}{U(0)} = \left[\frac{2\nu\tau_0}{1 + (2\nu\tau_0 - 1)\, e^{2\nu\tau}}\right]^{\frac{2}{3}}, \tag{19}$$

where $U(0)$ is the value of U at $\tau = 0$ and

$$\tau_0 = \frac{5}{4\gamma}\sqrt{\frac{12\beta}{\alpha U(0)^3}}.$$

This solution shows that if $2\nu\tau_0 > 1$, i.e., for sufficiently strong damping (ν_c large) and a small dust correlation effect ($\bar{\eta} \to 0$):

$$U(\tau) \sim U(0) \exp\left(-\frac{4}{3}\nu_c\tau\right). \tag{20}$$

On the other hand, for an increasingly strong correlation effect, i.e., for increasing $\bar{\eta}$ (the case of a strongly coupled plasma), if the condition $2\nu\tau_0 < 1$ is satisfied, then the solution (18) is strongly affected in both amplitude and phase. Near a critical time $\tau_c \sim \ln[1/(1 - 2\nu\tau_0)]/2\nu$ the amplitude of solitary wave $U(\tau)$ in Eq. (19) becomes very large; consequently $\mathrm{sech}^2[\sqrt{U(\tau)}]$ is also very small, and therefore the solitary wave solution [Eq. (18)] remains finite. Due to the strong viscoelastic effect the amplitude of the solution increases, whereas due to the same effect the dissipative effect is also enhanced as a result; finally the character of the solution remains as shown in Figs. 1 and 2.

V. SUMMARY AND DISCUSSION

In this paper the propagation characteristics of small but finite-amplitude longitudinal dust acoustic waves are investigated in the "kinetic regime" of strongly coupled dusty plasma. In this work we have included the dust-neutral collision and finite strain relaxation effect. The evidence of the wave dispersion in the "kinetic regime" has been reported by a molecular dynamics (MD) simulation [23], in which the parameters used are as follows: $1/\tau_m\omega_{pd} \approx 0.22, 0.13, 0.12$ (with notations $\tau_m = \tau_R$ and $\omega_{pd} = \omega_p$) and $\Gamma \gg 1$. Thus the MD simulation parameters justify our assumption and the scaling of the different parameter used in this work. The modified form of the forced-damped KdV equation has been solved with the perturbation technique. The outcomes of this solution are as follows.

(1) The analytical solutions [Eqs. (18) and (19)] show that for a solitary wave we must have $\alpha U(\tau)/12\beta > 0$. This implies that $\alpha > 0$ $U(\tau) > 0$ and therefore $n^{(1)}(\xi,\tau) > 0$. The variation

of α, the coefficient of the nonlinear term with respect to the coupling parameter Γ, shows that $\alpha(>0)$ decreases with the increase of Γ. Thus LDASWs are compressive in nature with the dust density enhancement. (2) The analytical solution [Eq. (18)] of the modified form of KdV equation shows that the nonlinear LDAW amplitude decays exponentially slowly with time [Eq. (19)] due to the nonzero values of $(\omega_{pd}\tau_m)^{-1}$ and dust-neutral collision for $2\gamma\tau_0 > 1$. (3) The large memory $\tau_m \gg \omega_{pd}^{-1}$ and finite value of $\bar{\eta}$ lead to (a) a rigidity effect even in the linear approximation and (b) in lowest order a nonlinear approximation to a nonlinear forcing term at the expense of correlation energy due to a change in the order of the arrangement of the dust particles with increased correlation effect.

To interpret the solution more physically, one should note that to include the strong coupling effect consequent to increased correlation among the dust particles, the hydrodynamic equation of the dust has been generalized [Eq. (3)] by replacing the viscosity factor $\bar{\eta}$ by the operator $\bar{\eta}(1+\omega_{pd}\tau_m\frac{d}{dt'})^{-1}$, which takes into account the memory-dependent nonlocal viscoelastic strain effect in addition to the irreversible fluidity effect. Here τ_m denotes the "dust viscoelastic relaxation time" [14]. If $\tau_m = 0$, i.e., relaxation is instantaneous, we have the usual hydrodynamic equation. In the case of the "hydrodynamic limit," $\omega_{pd}\tau_m \ll 1$ and the operator can be expanded in powers of $\omega_{pd}\tau_m\frac{d}{dt'}$; in this case also the fluid behavior persists retaining the approximate influence of memory and nonlocality. However, in this paper we have considered the opposite case, i.e., the "kinetic regime" where $\omega_{pd}\tau_m \gg 1$, i.e., the strain relaxation time $\tau_m \gg$ the dust fluid dynamic time (here dust acoustic time), which gives rise to very different physical behavior. The viscoelastic effects of the stress involve certain complications due to concomitant variation of the structure of the fluid. This consists of a change in the order of the arrangement of the dust particles. The change in the degree of order must in general lag with respect to the state of strain [22]. There arise two essentially different manifestations of this effect. We now discuss these.

(1) Under the condition $\omega_{pd}\tau_m \gg 1$ and that the viscosity coefficient is large enough so that $\bar{\eta}$ is a finite quantity, which makes the transition from the fluid to the solid state, the general hydrodynamic equation (3):

$$\left[\frac{1}{\omega_{pd}\tau_m} + \left(\frac{\partial}{\partial t'} + v\frac{\partial}{\partial x'}\right)\right]F = \bar{\eta}\frac{\partial^2 v}{\partial x'^2}, \qquad (21)$$

where

$$F = n\left[\left(\frac{\partial}{\partial t'} + v\frac{\partial}{\partial x'}\right)v + \sigma_d\frac{\partial\phi}{\partial x'} + \mu_d\frac{\partial n}{\partial x'} + \frac{\nu_{dn}}{\omega_{pd}}v\right],$$

reduces to [neglecting $O(\omega_{pd}\tau_m)^{-1}$]

$$\left(\frac{\partial}{\partial t'} + v\frac{\partial}{\partial x'}\right)F = \bar{\eta}\frac{\partial^2 v}{\partial x'^2}. \qquad (22)$$

Using the linear version of the above equation and the equation of linear continuity and the quasicharge neutrality condition, we find

$$\frac{\partial^2 v}{\partial t'^2} - \left[\frac{\sigma_i\sigma_d(\delta-1)}{\sigma_i+\delta} + \bar{\eta} + \mu_d\right]\frac{\partial^2 v}{\partial x'^2} + \left(\frac{\nu_{dn}}{\omega_{pd}}\right)\frac{\partial v}{\partial t'} = 0. \qquad (23)$$

Thus the viscoelastic stress effect represented by $\bar{\eta}\frac{\partial^2 v}{\partial x'^2}$ plays in this case the role of a restoring force rather than a dissipative one and yields the expression for phase velocity u, which can be obtained from the expression of M given above. It helps to sustain the wave motion negating the possibility of the appearance of a viscosity-dependent dissipative term. This and a similar outcome were shown to be true for transverse waves [14], $\bar{\eta}\nabla^2 v$ displaying the rigidity effect.

(2) Next we attend to the nonlinear effect. In addition to the rigidity effect displayed in the linear approximation. the finite time of relaxation of the activation energy stored in the process of rearrangement of the dust grains associated with correlation effect in the strained state results in the first-order nonlinear approximation to the appearance of the term

$$\int_{-\infty}^{\xi}\left(\frac{\partial v^{(1)}}{\partial\xi}\right)^2 d\xi = \frac{\partial}{\partial\xi}\int_{-\infty}^{\xi}(\xi-\xi)\left(\frac{\partial v^{(1)}}{\partial\xi}\right)^2 d\xi, \qquad (24)$$

which is proportional to the nonlocal forcing term in Eq. (15).

Here we demonstrate the genesis of the nonlocal contribution. We express the GH equation (21) in the following form:

$$F = \frac{1}{\sqrt{\epsilon}}\left[(-M+\epsilon v^{(1)})\frac{\partial}{\partial\xi}\right]^{-1}\left[\bar{\eta}\epsilon^2\frac{\partial^2}{\partial\xi^2}(\epsilon v^{(1)} + \epsilon^2 v^{(2)} + \cdots) - \epsilon^{3/2}\left(1+\frac{\partial}{\partial\tau}\right)F\right]. \qquad (25)$$

Now, retaining only terms $O(\epsilon^{5/2})$, using the inversion of the operator (25), we obtain

$$\frac{1}{\sqrt{\epsilon}}\left[(-M+\epsilon v^{(1)})\frac{\partial}{\partial\xi}\right]^{-1}\left[\bar{\eta}\epsilon^2\frac{\partial^2}{\partial\xi^2}(\epsilon v^{(1)} + \epsilon^2 v^{(2)} + \cdots)\right] = -\epsilon^{3/2}\frac{\bar{\eta}}{M}\int_{-\infty}^{\xi}\left(\frac{\partial^2 v^{(1)}}{\partial\xi^2} + \frac{\epsilon}{M}v^{(1)}\frac{\partial^2 v^{(1)}}{\partial\xi^2} + \frac{\epsilon}{M}\frac{\partial^2 v^{(2)}}{\partial\xi^2}\right)d\xi. \qquad (26)$$

As indicated above the first term corresponds in the linear approximation to the restoring force (rigidity effect), and the second one

$$\epsilon^{5/2}\frac{\bar{\eta}}{M^2}\int_{-\infty}^{\xi}\left(\frac{\partial v^{(1)}}{\partial\xi}\right)^2 d\xi$$

yields the nonlocal term appearing on the right-hand side (RHS) of (15) (using $v^{(1)} = un^{(1)}$).

Finally, it is to be noted that the RHS of Eq. (15) varies quadratically with $n^{(1)}$ while the damping is linear. Consequently there should exist a critical value of the initial intensity below which damping dominates, while above it there occurs amplification at the expense of the correlation energy. The results in the previous section demonstrate this.

We must mention that we have not encounter experimental observations of LDASWs yet. It would be very interesting to look at this mode in a laboratory. However, we hope that in the future a such type of a nonlinear longitudinal dust acoustic wave could be observed in a strongly coupled dusty plasma experiment.

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