Dissipation in a spin bath: Thermally induced coherent intensity and spectral splitting

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We have explored the role of dissipation in the scattered intensity and spectrum of resonance fluorescence from an excited two-level system in a spin bath. It has been shown that depending on the field strength a crossover temperature sets up a boundary between the coherent and incoherent intensity regimes, and at low field the scattered intensity may be coherent even at high temperature. We demonstrate the formation of a thermally induced Mollow triplet in the low-field regime.

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I. INTRODUCTION

The Langevin-Bloch equations describe the fundamental paradigm of interaction between a driven two-level system and a thermal bath [1,2]. The thermal bath is responsible for relaxation of the excited system as well as for introducing thermal fluctuation into the system at finite temperature. In general, the dissipation and the fluctuation are related through celebrated fluctuation-dissipation relation. From a microscopic point of view [2–5] the bath is comprised of many (practically infinite) degrees of freedom which, most often, correspond to those of harmonic oscillators, and hence the thermal bath is bosonic in nature. The underlying driven spin-boson model and the associated Bloch equations have captured an extraordinary rich dynamical behavior in a wide variety of phenomena in nuclear magnetic resonance [1] and quantum optics [2] over decades.

We begin with a simple question: What happens if the harmonic bath is replaced by a spin bath of two-level atoms? The immediate consequences emerge from two points. First, since the statistical character of the baths is different, resulting in a difference in the average thermal excitation number of a bosonic bath [6] (Bose distribution) and that of a spin bath (Fermi distribution), their thermal behavior is characteristically distinct. The origin essentially lies in the anticommutation rules followed by fermions in contrast to the commutation rules obeyed by bosons. Second, the dynamical evolution of the system is characterized by a nonlinear contribution of the system-reservoir interaction which gives rise to a temperature-dependent factor of hyperbolic tangent nature. The presence of this factor was noted earlier by Caldeira *et al.* [7] in their influence functional for describing the spin–spin-bath interaction within the framework of path integrals. This factor gets suppressed with increase in temperature so that the resulting effective damping (essentially the effective coupling) becomes weaker compared to the corresponding situation in a harmonic bath. Furthermore, expansion of the hyperbolic tangent factor at low frequency results in additional powers of frequency in the spectral density. Thus at relatively higher temperature the spin bath with fermionic nature favors coherent dynamics of the system. These considerations have been examined from several points of [8–21]. For example, a two-level system in the presence of a degenerate fermionic heat bath has been considered to treat spontaneous and electron-assisted tunneling [13]. The fermionic bath has been useful for understanding magnetic relaxation of molecular crystals [14] and in quantum decoherence measurements [15]. The particle–fermionic-reservoir interaction may generate an effective potential causing dynamic localization of the particle at low temperature [16]. The distinctive behavior of transport properties within the Feynman-Vernon formalism elucidated in the context of the spin–spin-bath model [17] has been experimentally investigated in some oxide systems [18].

From the aforesaid discussion it therefore follows that, although at zero temperature the spin–spin-bath model exhibits similar behavior as the spin–harmonic-bath model, the nature of the former becomes conspicuous at higher temperature because of the characteristic temperature dependence of the effective spectral density function, leading to a reduced effective coupling or friction favoring enhancement of coherence in the dynamics [19]. The focus of the present work is to explore this issue in a quantum optical context [2]. In what follows we consider a two-level system driven by a classical electromagnetic field and interacting with a spin bath of two-level atoms. The Langevin-Bloch equations are derived to examine the scattered intensity and resonance fluorescence from the excited two-level system. An examination of the coherent and incoherent components of intensity reveals that depending on the driving field a crossover temperature characterizes the coherent-incoherent boundary. We show that at low intensity of the driving field and low temperature the spectrum is characterized by a single peak which splits into a triplet with the increase of temperature as a result of reduction of effective width. This thermally induced Mollow triplet [22–28] is a direct manifestation of the suppression of decoherence with increase of temperature when the dissipative environment is a spin bath. In view of recent experiments [23–32] on resonance fluorescence and other quantum optical phenomena, single semiconductor quantum dots may serve as a good testing ground for observing thermally induced Mollow triplet at low field and the coherent-incoherent transition of scattered intensity.

The outline of the paper is as follows: In Sec II we introduce the model Hamiltonian of a two-level system in a spin bath of two-level atoms and construct the reduced dynamics for the system in terms of Langevin-Bloch equations within the Born-Markov approximation. The bath degrees of freedom

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are then described by c-number variables using spin coherent states. In the next section we apply the theoretical scheme for calculation of optical properties of the classically driven system. The scattered intensity and spectrum of resonance fluorescence are the two key quantities in this calculation. The paper is concluded in Sec. IV.

II. THE DRIVEN TWO-LEVEL SYSTEM IN A SPIN BATH

A. The model and the operator Langevin equation

We set up the problem of quantum dissipation of a two-level system in a spin bath of two-level atoms. The system is driven by a classical electromagnetic field $v(t)$. We consider the following Hamiltonian:

$$\hat{H} = \frac{1}{2}\hbar\Omega\hat{\sigma}_z + \frac{1}{2}\sum_k \hbar\omega_k\hat{\sigma}_{zk} + \hbar\sum_k (g_k\hat{\sigma}_k^\dagger\hat{\sigma} + g_k^*\hat{\sigma}^\dagger\hat{\sigma}_k) + \hbar[v(t)\hat{\sigma}^\dagger + v^*(t)\hat{\sigma}]. \tag{2.1}$$

Here the first term is the Hamiltonian for the system specified by the Pauli operators $\hat{\sigma}^\dagger$, $\hat{\sigma}$, and $\hat{\sigma}_z$. The second term corresponds to the reservoir Hamiltonian where the bath operators are denoted by the operators $\hat{\sigma}_k^\dagger$, $\hat{\sigma}_k$, and $\hat{\sigma}_{zk}$; the subscript k refer to the kth atom of the bath. The third term represents the interaction between the system and the bath. The fourth term refers to the interaction between the system and the classical field $v(t)$, which is given by $v(t) = V_2\exp[-i\omega_0 t]$, $V_2 = \frac{d\mathscr{E}}{2\hbar}$. d is the transition dipole moment and $\mathscr{E}$ is the complex classical field amplitude. ω_0 is the frequency of the field. The Pauli operators for the system follow the usual commutation relations as given below:

$$\begin{gathered}[\hat{\sigma}^\dagger,\hat{\sigma}] = \hat{\sigma}_z, \quad [\hat{\sigma}^\dagger,\hat{\sigma}_z] = -2\hat{\sigma}^\dagger, \quad [\hat{\sigma},\hat{\sigma}_z] = 2\hat{\sigma}, \\ \hat{\sigma}^\dagger\hat{\sigma} = \tfrac{1}{2}(1+\hat{\sigma}_z), \quad \hat{\sigma}\hat{\sigma}^\dagger = \tfrac{1}{2}(1-\hat{\sigma}_z).\end{gathered} \tag{2.2}$$

Similar relations hold good for the bath operators. In particular, using the commutation and anticommutation rules between the reservoir operators, we find

$$\hat{\sigma}_k^\dagger\hat{\sigma}_k - \hat{\sigma}_k\hat{\sigma}_k^\dagger = \hat{\sigma}_{zk}, \tag{2.3}$$

$$\hat{\sigma}_k^\dagger\hat{\sigma}_k + \hat{\sigma}_k\hat{\sigma}_k^\dagger = 1. \tag{2.4}$$

So $\hat{\sigma}_{zk} = 2\hat{n}_k - 1$ where $\hat{n}_k$ is the number operator $\hat{\sigma}_k^\dagger\hat{\sigma}_k$. A brief discussion of the approximations needed for derivation of the dynamical equations that follow may be pertinent at this point. The Hamiltonian (2.1) is based on the rotating wave approximation for coupling to the bath and to the classical field v. This amounts to neglecting the Bloch-Seigert frequency shift which for the optical domain is extremely small $[\sim O(10^{-10}\ \Omega)]$ and can be safely neglected. Second, almost in all physically relevant situations in quantum optics it is sufficient to work within weak coupling (Born approximation) for the two-level atom and the bath, which implies that the bath effectively behaves as a free field. However, the coupling between the two-level system and the driving field is arbitrary. The working range of intensity is such that the Rabi frequency is either larger than the damping rate for coherent regime or much smaller for the incoherent regime. Another important approximation that is almost always made is that the correlation time of the reservoir is very short (Markov approximation). Therefore, while the system-bath interaction is within the scope of the linear response regime, the strength of the system-field interaction is arbitrary and is well beyond the perturbation limit. For a practical realization of Hamiltonian (2.1) in a typical quantum optical setting, one may envisage exciting a semiconductor quantum dot of an artificial two-level "atom" by light while the atom is embedded in a sea of two-level quantum dots of varying size, i.e., of varying frequencies with a broad distribution [33]. Recent technical advances in molecular beam epitaxy have produced important results in this direction [23–32].

The Heisenberg equations of motion for the system operators can then be written as follows:

$$\dot{\hat{\sigma}}^\dagger(t) = i\left(\Omega\hat{\sigma}^\dagger(t) - \sum_k g_k\hat{\sigma}_k^\dagger(t)\hat{\sigma}_z(t) - v^*(t)\hat{\sigma}_z(t)\right), \tag{2.5}$$

$$\dot{\hat{\sigma}}_z(t) = 2i\left(\sum_k g_k\hat{\sigma}_k^\dagger(t)\hat{\sigma}(t) - \sum_k g_k^*\hat{\sigma}^\dagger(t)\hat{\sigma}_k(t) + v^*(t)\hat{\sigma}(t) - v(t)\hat{\sigma}^\dagger(t)\right), \tag{2.6}$$

and the adjoint of Eq. (2.5). The bath dynamics is given by

$$\dot{\hat{\sigma}}_k(t) = i\left[-\omega_k\hat{\sigma}_k(t) + g_k\hat{\sigma}_{zk}(t)\hat{\sigma}(t)\right], \tag{2.7}$$

$$\dot{\hat{\sigma}}_{zk}(t) = -2i[g_k\hat{\sigma}_k^\dagger(t)\hat{\sigma}(t) - g_k^*\hat{\sigma}^\dagger(t)\hat{\sigma}_k(t)], \tag{2.8}$$

and the adjoint of Eq. (2.7). After formally integrating Eq. (2.7) and its adjoint, $\hat{\sigma}_k(t)$ and $\hat{\sigma}_k^\dagger(t)$ are eliminated in the usual way. In the next step we consider rapidly varying time dependence of the system operators and introduce the slowly varying operators $\hat{S}(t), \hat{S}^\dagger(t)$, and $\hat{S}_z(t)$ such that $\hat{S}(t) = \hat{\sigma}(t)\exp(i\omega_0 t)$, $\hat{S}^\dagger(t) = \hat{\sigma}^\dagger(t)\exp(-i\omega_0 t)$, and $\hat{S}_z(t) = \frac{\hat{\sigma}_z(t)}{2}$. The dynamics of the system can then be written as follows:

$$\begin{aligned}\dot{\hat{S}}^\dagger(t) = {}& -2i\sum_k g_k\hat{\sigma}_k^\dagger(0)\hat{S}_z(t)\exp[-i(\omega_0-\omega_k)t] \\ & -2\left[\sum_k |g_k|^2\int_0^t dt'\hat{\sigma}_{zk}(t')\hat{S}^\dagger(t') \times\exp[-i(\omega_0-\omega_k)(t-t')]\right]\hat{S}_z(t) \\ & -2iV_2^*\hat{S}_z(t) + i\delta_0\hat{S}^\dagger(t),\end{aligned} \tag{2.9}$$

$$\begin{aligned}\dot{\hat{S}}_z(t) = {}& i\sum_k g_k\hat{S}(t)\hat{\sigma}_k^\dagger(0)\exp[-i(\omega_0-\omega_k)t] \\ & -i\sum_k g_k^*\hat{\sigma}_k(0)\hat{S}^\dagger(t)\exp[i(\omega_0-\omega_k)t] \\ & +\left[\sum_k |g_k|^2\int_0^t dt'\hat{\sigma}_{zk}(t')\hat{S}^\dagger(t') \times\exp[-i(\omega_0-\omega_k)(t-t')]\right]\hat{S}(t) \\ & +\hat{S}^\dagger(t)\left[\sum_k |g_k|^2\int_0^t dt'\hat{S}(t')\hat{\sigma}_{zk}(t')\right.\end{aligned}$$

$$\left.\times \exp[i(\omega_0 - \omega_k)(t - t')]\right]$$
$$+ iV_2^* \hat{S}(t) - iV_2 \hat{S}^\dagger(t), \tag{2.10}$$

and the adjoint of Eq. (2.9) where the detuning is defined as $\delta_0 = \Omega - \omega_0$.

Equations (2.9) and (2.10) are exact within the scope of the rotating wave approximation for the Hamiltonian (2.1). We now make a further approximation for the slow variables $\hat{S}(t) \approx \hat{S}(t')$. This description is good [2] so long as the variables are slow on the time scale over which the different k modes of the bath degrees freedom in the sum in Eqs. (2.9) and (2.10) interfere (Born-Markov approximation). In the same spirit, we approximate $\hat{\sigma}_{zk}(t') \approx \hat{\sigma}_{zk}(0)$ since by virtue of Eqs. (2.7) and (2.8) the time evolution of energy is slow in comparison to that for the polarization operators $\hat{\sigma}_k$ and $\hat{\sigma}_k^\dagger$. Taking $\hat{\sigma}_{zk}(0)$ out of the integrals and replacing it by $(2\hat{n}_k - 1)$, we obtain

$$\begin{aligned}\dot{\hat{S}}^\dagger(t) = &-2i\hat{S}_z(t)\sum_k g_k \hat{\sigma}_k^\dagger(0)\exp[-i(\omega_0-\omega_k)t]\\ &-\hat{S}^\dagger(t)\sum_k |g_k|^2[1-2\hat{n}_k(0)]\pi\delta(\omega_0-\omega_k)\\ &-2iV_2^*\hat{S}_z(t) + i\delta_0\hat{S}^\dagger(t),\end{aligned} \tag{2.11}$$

$$\begin{aligned}\dot{\hat{S}}_z(t) = &\, i\hat{S}(t)\sum_k g_k\hat{\sigma}_k^\dagger(0)\exp[-i(\omega_0-\omega_k)t]\\ &- i\hat{S}^\dagger(t)\sum_k g_k^*\hat{\sigma}_k(0)\exp[i(\omega_0-\omega_k)t]\\ &-\left[\hat{S}_z(t)+\frac{1}{2}\right]\left[\sum_k |g_k|^2[1-2\hat{n}_k(0)]\pi\delta(\omega_0-\omega_k)\right]\\ &-\left[\hat{S}_z(t)+\frac{1}{2}\right]\left[\sum_k |g_k|^2[1-2\hat{n}_k(0)]\pi\delta(\omega_0-\omega_k)\right]\\ &+iV_2^*\hat{S}(t) - iV_2\hat{S}^\dagger(t)\end{aligned} \tag{2.12}$$

and the adjoint of Eq. (2.11). In deriving the above equations we have used the commutation rules at equal time and the integral $\int_0^t dt' \exp[\pm i(\omega_0-\omega_k)(t-t')]$ is replaced by $\pi\delta(\omega_0-\omega_k)$. In doing this we have made $t \to \infty$ (i.e., $t \gg 1/\gamma_0$) so that we have $\int_0^t dt'(\cdots) = \pi\delta(\omega_0-\omega_k) \pm i\mathrm{P}\frac{1}{\omega_0-\omega_k}$, where P is the Cauchy principal part. The imaginary part of the integral contributes to the Lamb shift due to the coupling of the two-level system to the reservoir. The effect of the shift is to modify the resonant frequency ω_0 $(=\Omega)$ by a very small amount proportional to $|g_k|^2$. This is neglected. We now define the damping operator $\hat{\gamma}$ and noise operator $\hat{F}(t)$ as follows:

$$\hat{\gamma} = \sum_k |g_k|^2[1-2\hat{n}_k(0)]\pi\delta(\omega_0-\omega_k), \tag{2.13}$$

$$\hat{F}(t) = i\sum_k g_k^*\hat{\sigma}_k(0)\exp[i(\omega_0-\omega_k)t]. \tag{2.14}$$

With Eqs (2.13) and (2.14), the quantum Langevin-Bloch equations for the driven two-level system in contact with the spin bath can be rewritten as

$$\dot{\hat{S}}^\dagger(t) = -(\hat{\gamma}-i\delta_0)\hat{S}^\dagger(t) - 2iV_2^*\hat{S}_z(t) + 2\hat{S}_z(t)\hat{F}^\dagger(t), \tag{2.15}$$

$$\begin{aligned}\dot{\hat{S}}_z(t) = &-\hat{\Gamma}\left[\hat{S}_z(t)+\tfrac{1}{2}\right] + iV_2^*\hat{S}(t) - iV_2\hat{S}^\dagger(t)\\ &-\hat{S}(t)\hat{F}^\dagger(t) - \hat{S}^\dagger(t)\hat{F}(t),\end{aligned} \tag{2.16}$$

and the adjoint of Eq. (2.15). Here $\hat{\Gamma} = 2\hat{\gamma}$. Equations (2.15) and (2.16) show that the noise operators are multiplicative in nature.

The noise properties of the operators $\hat{F}(t)$ and $\hat{F}^\dagger(t)$ can be derived by using a suitable canonical thermal distribution of the bath operators at $t = 0$ as follows:

$$\langle\hat{F}(t)\rangle_{qs} = 0, \quad \langle\hat{F}^\dagger(t)\rangle_{qs} = 0, \tag{2.17}$$

$$\begin{aligned}&\mathrm{Re}\{\langle\hat{F}(t)\hat{F}^\dagger(t') - \hat{F}^\dagger(t)\hat{F}(t')\rangle_{qs}\}\\ &\quad= \sum_k |g_k|^2\tanh\left(\frac{\hbar\omega_k}{2kT}\right)\cos(\omega_0-\omega_k)(t-t').\end{aligned} \tag{2.18}$$

Here, $\langle\cdots\rangle_{qs}$ implies the quantum statistical average and is defined as

$$\langle\hat{A}\rangle_{qs} = \frac{\mathrm{Tr}\hat{A}e^{-\hat{H}_{\mathrm{bath}}/kT}}{\mathrm{Tr}\, e^{-\hat{H}_{\mathrm{bath}}/kT}} \tag{2.19}$$

for any operator $\hat{A}$, where $\hat{H}_{bath} = \frac{1}{2}\sum_k \hbar\omega_k\hat{\sigma}_{zk}$ at $t = 0$.

Equation (2.18) is the fluctuation-dissipation relation expressed in terms of noise operators appropriately ordered. The negative sign in the left-hand side of Eq. (2.18) carries the signature of the anticommutation relation for the spin-bath operators in contrast to the positive sign for the corresponding bosonic case. The temperature-dependent contribution $\tanh\left(\frac{\hbar\omega_k}{2kT}\right)$ originates from the following averages:

$$\langle\hat{n}_k\rangle_{qs} = \frac{1}{\exp(\hbar\omega_k/kT)+1} = \overline{n}_F(\omega_k) \tag{2.20}$$

and

$$\begin{aligned}\langle\hat{\sigma}_{zk}\rangle_{qs} &= 2\langle\hat{n}_k\rangle_{qs} - 1\\ &= -\tanh\left(\frac{\hbar\omega_k}{2kT}\right).\end{aligned} \tag{2.21}$$

Here $\overline{n}_F$ can be identified as the Fermi-Dirac distribution function denoting the average thermal excitation number of the bath.

B. Langevin-Bloch equations for c-number noise

Our object in this section is to construct a quantum Langevin equation with c-number noise of the bath degrees of freedom. We return to Eqs. (2.15) and (2.16) and as a first step carry out the quantum mechanical average $\langle\cdots\rangle$ to obtain

$$\begin{aligned}\langle\dot{\hat{S}}^\dagger(t)\rangle = &-(\langle\hat{\gamma}\rangle - i\delta_0)\langle\hat{S}^\dagger(t)\rangle - 2iV_2^*\langle\hat{S}_z(t)\rangle\\ &+2\langle\hat{S}_z(t)\rangle\langle\hat{F}^\dagger(t)\rangle,\end{aligned} \tag{2.22}$$

$$\begin{aligned}\langle\dot{\hat{S}}_z(t)\rangle = &-\langle\hat{\Gamma}\rangle\left[\langle\hat{S}_z(t)\rangle + \tfrac{1}{2}\right] + iV_2^*\langle\hat{S}(t)\rangle - iV_2\langle\hat{S}^\dagger(t)\rangle\\ &-\langle\hat{S}(t)\rangle\langle\hat{F}^\dagger(t)\rangle - \langle\hat{S}^\dagger(t)\rangle\langle\hat{F}(t)\rangle,\end{aligned} \tag{2.23}$$

and the adjoint of Eq. (2.22). Here the quantum mechanical average is taken over the initial product separable quantum states of the system and the spins at $t = 0$, $|\phi\rangle|\xi_1\rangle|\xi_2\rangle\cdots|\xi_k\rangle\cdots|\xi_N\rangle$. Here $|\phi\rangle$ denotes any arbitrary initial state of the two-level system and $|\xi_k\rangle$ corresponds to the initial coherent state of the kth spin. The existence of such coherent states (see Appendix A) was proved a couple of decades ago by Radcliffe [34]. Typically, for a spin-1/2 system, such a state is generated by the action of a creation operator on vacuum. The expectation value factorization in the dynamical equations (2.22) and (2.23) is based on the assumption that the system and the bath are uncorrelated at $t = 0$. The effect of correlation was considered earlier in a different context by other workers [10].

The idea behind using spin coherent states for quantum mechanical averages of the bath operators is to formulate Eqs. (2.22) and (2.23) as classical-looking Langevin equations for a two-level system kept in a two-level reservoir since $\langle \hat{F}(t)\rangle$ and $\langle \hat{F}^\dagger(t)\rangle$ behave as multiplicative noise variables in the dynamics. We then denote the resulting quantum mechanical averages as

$$\begin{gathered}\langle \hat{S}^\dagger(t)\rangle = S^*(t), \quad \langle \hat{S}_z(t)\rangle = S_z(t), \quad \langle \hat{S}(t)\rangle = S(t),\\ \langle \hat{F}(t)\rangle = f(t), \quad \langle \hat{F}^\dagger(t)\rangle = f^*(t), \quad \langle \hat{\gamma}\rangle = \gamma,\end{gathered} \tag{2.24}$$

where

$$\begin{aligned} f(t) &= i\sum_k g_k^* \langle \hat{\sigma}_k(0)\rangle \exp[i(\omega_0-\omega_k)t]\\ &= i\sum_k g_k^*\xi_k(0)\exp[i(\omega_0-\omega_k)t], \end{aligned} \tag{2.25}$$

$f^*(t)$ is the complex conjugate of $f(t)$, and

$$\begin{aligned} \gamma &= \sum_k |g_k|^2\langle[1-2\hat{n}_k(0)]\rangle \pi\delta(\omega_0-\omega_k)\\ &= \sum_k |g_k|^2 \mid \xi_k(0)\mid^2 \pi\delta(\omega_0-\omega_k). \end{aligned} \tag{2.26}$$

Here, $\xi_k(0)$ and $\xi_k^*(0)$ are the associated c numbers for the bath operators. Equations (2.22) and (2.23) may then be rewritten as

$$\dot{S^*}(t) = -(\gamma - i\delta_0)S^*(t) - 2iV_2^* S_z(t) + 2S_z(t)f^*(t), \tag{2.27}$$

$$\begin{aligned}\dot{S}_z(t) = &-\Gamma\left[S_z(t)+\tfrac{1}{2}\right] + iV_2^*S(t) - iV_2S^*(t)\\ &- S(t)f^*(t) - S^*(t)f(t),\end{aligned} \tag{2.28}$$

and the equation for the complex conjugate of $S^*(t)$. Now to realize $f(t)$ [and $f^*(t)$] as an effective c-number noise, we introduce the ansatz that $\xi_k(0)$ and $\xi_k^*(0)$ are distributed according to a thermal canonical distribution of Gaussian form as follows:

$$P_k\{\xi_k(0),\xi_k^*(0)\} = N\exp\left\{-\frac{|\xi_k(0)|^2}{2\tanh\left(\frac{\hbar\omega_k}{2kT}\right)}\right\}, \tag{2.29}$$

where N is the normalization constant. The width of the distribution is given by $\tanh(\frac{\hbar\omega_k}{2kT})$. This distribution is essentially the fermionic counterpart of the Wigner thermal canonical distribution function [35,36] for a harmonic bath. For any arbitrary quantum mechanical mean value of a bath operator $\langle \hat{A}_k\rangle$ which is a function of $\xi_k(0)$,$\xi_k^*(0)$, its statistical average $\langle\cdots\rangle_s$ can then be written down as

$$\langle\langle \hat{A}_k\rangle\rangle_s = \int \langle \hat{A}_k\rangle P_k(\xi_k(0),\xi_k^*(0))d\xi_k(0)d\xi_k^*(0). \tag{2.30}$$

The ansatz Eq. (2.29) and the definition of the statistical average Eq. (2.30) can be used to show that the c-number noise $f(t), f^*(t)$ satisfies the following relations:

$$\langle f(t)\rangle_s = 0 = \langle f^*(t)\rangle_s, \tag{2.31}$$

$$\begin{aligned}\mathrm{Re}\{\langle f(t)f^*(t')\rangle_s\} = \sum_k |g_k|^2\tanh\left(\frac{\hbar\omega_k}{2kT}\right)\\ \times\cos(\omega_0-\omega_k)(t-t').\end{aligned} \tag{2.32}$$

Care must be taken to distinguish the averages $\langle\cdots\rangle_{qs}$, $\langle\cdots\rangle$, and $\langle\cdots\rangle_s$. The physical averages are evaluated with a thermal state for the bath, and are denoted by $\langle\cdots\rangle_{qs}$ here. In the present scheme they are calculated in two steps, i.e., as statistical averages $\langle\cdots\rangle_s$ of quantum mechanical averages $\langle\cdots\rangle$. This gives the averages of interest as $\langle\cdots\rangle_{qs} = \langle\langle\cdots\rangle\rangle_s$.

Equations (2.31) and (2.32) imply that c-number noise $f(t)$ [$f^*(t)$] is such that it is zero centered and follows the fluctuation-dissipation relation as expressed in Eq. (2.18). Therefore Eqs. (2.18) and (2.32) are equivalent. However, a decisive advantage of the formulation of the c-number noise $f(t), f^*(t)$ as defined by Eqs. (2.31) and (2.32) is that one can easily borrow suitable techniques from classical nonequilibrium statistical mechanics for various purposes as shown elsewhere [11,20,36].

In order to quantify the properties of the thermal bath it is convenient to introduce, as usual, a spectral density function $J(\omega)$ associated with the system-bath interaction. With the help of $J(\omega)$ one may rewrite the expression for damping as

$$\begin{aligned}\langle\hat{\gamma}\rangle_{qs} = \langle\gamma\rangle_s &= \int d\omega|g(\omega)|^2 J(\omega)\tanh\left(\frac{\hbar\omega}{2kT}\right)\pi\delta(\omega_0-\omega)\\ &= \gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)\end{aligned} \tag{2.33}$$

and the fluctuation-dissipation relation as

$$\begin{aligned}&\mathrm{Re}\{\langle f(t)f^*(t')\rangle_s\}\\ &\quad= \mathrm{Re}\{\langle \hat{F}(t)\hat{F}^\dagger(t') - \hat{F}^\dagger(t)\hat{F}(t')\rangle_{qs}\}\\ &\quad= \int_0^\infty d\omega|g(\omega)|^2 J(\omega)\tanh\left(\frac{\hbar\omega}{2kT}\right)\cos(\omega_0-\omega)(t-t')\\ &\quad= \gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)\delta(t-t'),\end{aligned} \tag{2.34}$$

where $\gamma_0 = \pi|g(\omega_0)|^2 J(\omega_0)$. In arriving at the expression for γ_0 we have taken care of the following consideration. We first return to Eq (2.13). Since we assume that the reservoir modes are closely spaced with $J(\omega)d\omega$ the number of modes between ω and $\omega + d\omega$, we may change the summation to an integral, and also the random forces are δ correlated provided that the system is Markovian. Or in other words the reservoir becomes uncorrelated long before the system has changed much. Therefore the range of validity of γ_0 is restricted by the inequality $\gamma_0^{-1} \gg \Delta t \gg \tau_c$, i.e., the time for relaxation is much larger than the coarse-grained time scale (over which the system changes) and the correlation time of the noise. Such a constant γ_0 requires a broad distribution of energy. The possible realization of this condition depends on the viability of preparing a sea of two-level quantum dots of varying size which admit of a broad distribution of frequencies [33].

A little more digression at this point would be helpful for further discussions. It is apparent from Eqs. (2.33) and (2.34) that the spectral density function $J(\omega)$ is multiplied by a temperature-dependent hyperbolic tangent factor, which is characteristic of the spin bath. This makes the effective spectral density function, i.e., the coupling, temperature dependent. Such a factor was analyzed earlier by Caldeira *et al.* [7] in an influence functional for a spin bath within the path integral formalism. At $T = 0$ the hyperbolic tangent factor (as well as the hyperbolic cotangent factor) becomes unity and therefore the bosonic and spin baths behave identically. At higher temperature because of the hyperbolic cotangent factor the harmonic bath reaches the classical limit. On the other hand, for the spin bath the temperature-dependent factor differs appreciably from unity and an expansion of the hyperbolic tangent factor results in additional low frequencies modifying the spectral density in an effective way. The net effect is that an increase of temperature reduces the friction or coupling which induces coherence in the dynamical behavior of the system. The origin of this different behavior of the spin bath may traced to the anticommutation relations obeyed by spin-1/2 fermions, which have no classical analog.

We now make use of the above-mentioned statistical averages to rewrite Eqs. (2.27) and (2.28) as

$$\langle \dot{S^*}(t)\rangle_s = -\left[\gamma_0 \tanh\left(\frac{\hbar\omega_0}{2kT}\right) - i\delta_0\right]\langle S^*(t)\rangle_s - 2iV_2^*\langle S_z(t)\rangle_s, \tag{2.35}$$

$$\langle \dot{S_z}(t)\rangle_s = -\Gamma_0 \tanh\left(\frac{\hbar\omega_0}{2kT}\right)\left[\langle S_z(t)\rangle_s + \frac{1}{2}\right] + iV_2^*\langle S(t)\rangle_s - iV_2\langle S^*(t)\rangle_s, \tag{2.36}$$

and the complex conjugate of Eq. (2.35). Next, it is convenient to define the following quantities which are helpful for proceeding to the next section, i.e., the steady-state solutions of Eqs. (2.35) and (2.36) as given by

$$\langle S^*\rangle_s = -2iV_2^*\langle S_z\rangle_s\mathscr{D}_2^*, \tag{2.37}$$

$$\langle S_z\rangle_s = -\frac{1}{2}\frac{1}{(1+I_2\mathscr{L}_2)}, \tag{2.38}$$

and the complex conjugate of $\langle S^*\rangle_s$. Here, the dimensionless incident intensity $I_2 = \frac{4|V_2|^2}{\Gamma_0\gamma_0}$, $\mathscr{D}_2^* = [\gamma_0 \tanh\left(\frac{\hbar\omega_0}{2kT}\right) + i\delta_0]/\{\gamma_0^2 [\tanh\left(\frac{\hbar\omega_0}{2kT}\right)]^2 + \delta_0^2\}$, and $\mathscr{L}_2 = \gamma_0^2/\{\gamma_0^2[\tanh\left(\frac{\hbar\omega_0}{2kT}\right)]^2 + \delta_0^2\}$.

The steady-state values of the populations of the two levels are also useful. These are as follows:

$$\langle S_a\rangle_s = \frac{1}{2} + \langle S_z\rangle_s = \frac{\frac{1}{2}I_2\mathscr{L}_2}{1+I_2\mathscr{L}_2}, \tag{2.39}$$

$$\langle S_b\rangle_s = \frac{1}{2} - \langle S_z\rangle_s = \frac{1}{2} + \frac{1}{2}\frac{1}{(1+I_2\mathscr{L}_2)}. \tag{2.40}$$

Here, a and b denote the upper and lower levels of the two-level system, respectively.

III. RESONANCE FLUORESCENCE FROM THE DRIVEN ATOM IN A SPIN BATH

A. Scattered intensity of resonance fluorescence

We now proceed to examine the quantum optical properties of an excited two-level system in a dissipative environment of two-level atoms. To this end we employ the Langevin-Bloch equations derived in the last section for calculation of the scattered intensity and resonance fluorescence, primarily focusing on temperature-induced coherence effects.

First, we calculate the spectral intensity. Since the far field emitted by a dipole is proportional to the dipole, the scattered intensity is proportional to the single-time first-order correlation function of the dipole [2] and we have

$$I \propto \int_0^T dt\langle E^-(t)E^+(t)\rangle = \alpha\int_0^T dt\langle \hat{S}^\dagger(t)\hat{S}(t)\rangle_{qs} = \alpha T\langle S_a\rangle_s. \tag{3.1}$$

Here $E^+(t)$ is the positive frequency part of the field and is proportional to $\hat{\sigma}(t)$. $\hat{\sigma}^\dagger(t)$,$\hat{\sigma}(t)$ are related to the slowly varying operators $\hat{S}^\dagger(t)$,$\hat{S}(t)$ as defined after Eq. (2.8). α is a proportionality constant. We have also used the relation

$$\hat{S}^\dagger(t)\hat{S}(t) = |a\rangle\langle b||b\rangle\langle a| = |a\rangle\langle a| \equiv \hat{S}_a(t). \tag{3.2}$$

In Eq. (3.1) one further neglects the irrelevant constants and ignores, for simplicity, the vectorial character of the field. The spectrum can therefore be calculated by recognizing the operator nature of the system variables $\hat{S}^\dagger(t)$, $\hat{S}_z(t)$, and $\hat{S}(t)$, where $S^*(t)[\equiv\langle\hat{S}^\dagger(t)\rangle]$, $S_z(t)[\equiv\langle\hat{S}_z(t)\rangle]$, and $S(t)[\equiv\langle\hat{S}(t)\rangle]$ are the quantum mechanical mean values and $\delta\hat{S}^\dagger(t)$, $\delta\hat{S}_z(t)$, and $\delta\hat{S}(t)$ are the corresponding fluctuation operators [so that $\hat{S}^\dagger(t) = S^*(t) + \delta\hat{S}^\dagger(t)$, etc.]. By construction, $\langle\delta\hat{S}^\dagger(t)\rangle = \langle\delta\hat{S}_z(t)\rangle = \langle\delta\hat{S}(t)\rangle = 0$ and $[\delta\hat{S}^\dagger(t),\delta\hat{S}(t)] = 2\delta\hat{S}_z(t)$; $\{\delta\hat{S}^\dagger(t),\delta\hat{S}(t)\}_+ = 1$. Equation (3.1) therefore yields

$$I = \alpha\int_0^T dt|\langle S^*(t)\rangle_s|^2 + \alpha\int_0^T dt\langle\langle\delta\hat{S}^\dagger(t)\delta\hat{S}(t)\rangle\rangle_s \tag{3.3}$$

or

$$I \equiv I_{\text{coh}} + I_{\text{inc}}. \tag{3.4}$$

The scattered intensity consists of two terms. The first one originates from the mean motion of the dipole driven by the applied field, i.e., the coherently scattered intensity I_{coh}, while the incoherent contribution I_{inc} is due to the fluctuations of the dipole motion. Comparing (3.1) and (3.4), we find that the incoherent intensity is given by

$$I_{\text{inc}} = \alpha T(\langle S_a\rangle_s - |\langle S^*\rangle_s|^2). \tag{3.5}$$

Using the steady-state values we have

$$I_{\text{coh}} = \alpha T|\langle S^*\rangle_s|^2 = \alpha T\frac{\frac{1}{2}I_2\mathscr{L}_2\Gamma_0}{2\gamma_0(1+I_2\mathscr{L}_2)^2}, \tag{3.6}$$

$$I_{\text{inc}} = \alpha T\frac{\frac{1}{2}I_2\mathscr{L}_2}{1+I_2\mathscr{L}_2} - I_{\text{coh}}. \tag{3.7}$$

As usual, the coherently scattered intensity vanishes for a strong driving field $I_2 \gg 1$. This is because the coherent part is proportional to the squared magnitude of the steady-state dipole, which bleaches to zero in the strong

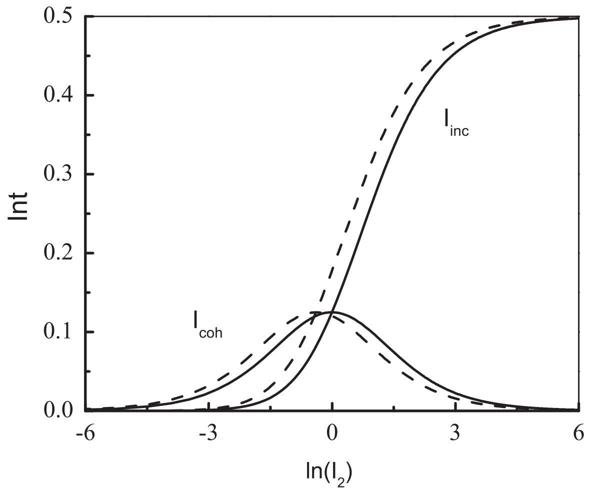

FIG. 1. Variation of coherent and incoherent intensity with dimensionless incident intensity for $\hbar\Omega = 20$ meV at two different temperatures $T = 30$ K (continuous line) and 100 K (dotted line) with zero detuning ($\delta_0 = 0$).

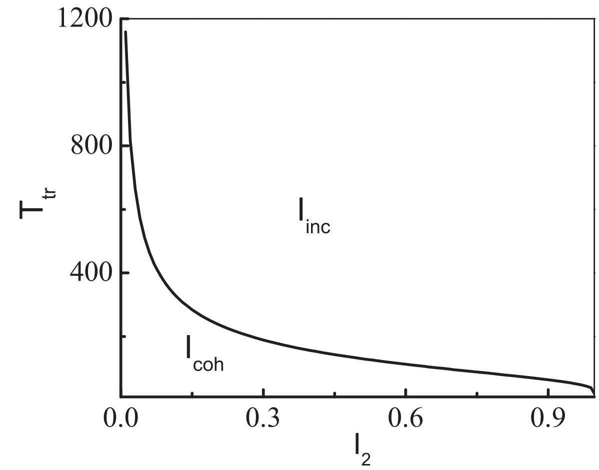

FIG. 2. Variation of T_{tr} for smaller value of dimensionless incident intensity I_2 with $\hbar\Omega = 20$ meV for zero detuning ($\delta_0 = 0$).

driving field limit. On the other hand, the incoherent part is due to spontaneous emission from the upper level, whose probability of occupation is approximately $\frac{1}{2}$ in a strong field. In the weak field limit, the atom remains essentially in the ground state, with no spontaneous emission taking place. These radiative properties of the two-level system closely correspond to the usual case of a harmonic bath and are illustrated in Fig. 1, which shows the coherent and incoherent contributions to the scattered light intensity as functions of the dimensionless incident intensity I_2. The signature of the spin bath, however, is markedly manifested when the coherent and incoherent scattered intensities are plotted at different temperatures as shown in Fig. 1. In the weak field limit, $I_{\rm coh}$ at higher temperature ($T = 100$ K; dotted line) is larger than that at lower temperature ($T = 30$ K; continuous line). This essentially indicates that increasing temperature assists coherence. A closer look at the expressions (3.6) and (3.7) for coherent and incoherent intensities further suggests that by equating them one may obtain a transition temperature T_{tr} for the lower value of I_2 as given by $I_2 = [\tanh(\frac{\hbar\Omega}{2kT_{tr}})]^2$ for zero detuning ($\delta_0 = 0$). In Fig. 2, T_{tr} is plotted against I_2 to illustrate the dividing line between coherent and incoherent regimes. It therefore follows that by tuning the temperature of the spin bath it is possible to realize a crossover between coherent and incoherent regimes at a particular strength of the driving field. This result is a further confirmation of an earlier observation that the spin–spin-bath model exhibits distinctly different physics at nonzero temperature as compared to the spin-boson model. The higher field masks the coherence effect due to power broadening.

The essential content of the above observation may be emphasized in the context of traditional quantum optics, where one deals with two-level systems in atomic beams and continuous electromagnetic field modes as a zero-temperature harmonic reservoir. It is observed that for large values of γ_0, the incoherent component of the intensity has weak field values comparable to or larger than those of its coherent component. Therefore the crossover between coherent and incoherent intensities depends on dissipation. Our result, on the other hand, reveals that when the harmonic bath is replaced by a spin bath, the crossover is determined by a transition temperature. A higher temperature enhances the coherent intensity. Thus although at 0 K the dissipative dynamics for the spin bath and the harmonic bath agrees well, their finite temperature behavior is markedly different at the very fundamental level.

B. Calculation of resonance fluorescence spectrum: Thermally induced splitting

For an ergodic and stationary process, the spectrum $\mathcal{S}(\varpi)$ [2] can be expressed following the Wiener-Khintchine theorem as

$$\mathcal{S}(\varpi) \propto \int_0^\infty d\tau \langle E^-(\tau)E^+(0)\rangle \exp(-i\varpi\tau) + \text{c.c.} \quad (3.8)$$

Then using the slowly varying operators $\hat{S}(t)$ and $\hat{S}^\dagger(t)$ as before one may rewrite Eq. (3.8) in the following form:

$$\mathcal{S}(\varpi) = \beta \int_0^\infty d\tau \langle \hat{S}^\dagger(\tau)\hat{S}(0)\rangle_{qs} \exp[i(\omega_0 - \varpi)\tau] + \text{c.c.}, \quad (3.9)$$

where β is a proportionality constant. We may decompose the light spectrum into coherent and incoherent parts as in the case of the scattered intensity:

$$\begin{aligned}\mathcal{S}(\varpi) &\equiv \mathcal{S}_{\rm coh}(\varpi) + \mathcal{S}_{\rm inc}(\varpi)\\ &= \beta|\langle S^*(t)\rangle_s|^2 \int_0^\infty d\tau \exp[i(\omega_0 - \varpi)\tau]\\ &\quad + \beta\int_0^\infty d\tau \langle\langle \delta\hat{S}^\dagger(\tau)\delta\hat{S}(0)\rangle\rangle_s \exp[i(\omega_0 - \varpi)\tau] + \text{c.c.}\end{aligned} \quad (3.10)$$

The coherent part, i.e., the Rayleigh peak can readily found to be

$$\begin{aligned}\mathcal{S}_{\rm coh}(\varpi) &= 2\pi\beta|\langle S^*\rangle_s|^2\delta(\omega_0 - \varpi)\\ &= \frac{\frac{1}{2}\pi\beta I_2\mathscr{L}_2\Gamma_0}{\gamma_0(1 + I_2\mathscr{L}_2)^2}\delta(\omega_0 - \varpi).\end{aligned} \quad (3.11)$$

It consists simply of a δ function centered at ω_0 ($= \Omega$ for $\delta_0 = 0$).

To evaluate the incoherent part, we need to estimate the two-time correlation functions of the atomic fluctuation operators, which can be calculated from the quantum correction equations. Therefore, we return to the quantum operator equations (2.15) and (2.16) and use Eqs. (2.27) and (2.28) to obtain

$$\delta\dot{\hat{S}}^{\dagger}(t) = -\delta\hat{\gamma}S^*(t) - (\hat{\gamma} - i\delta_0)\delta\hat{S}^{\dagger}(t) - 2iV_2^*\delta\hat{S}_z(t) + 2S_z(t)\delta\hat{f}^{\dagger}(t) + 2\delta\hat{S}_z(t)\hat{F}^{\dagger}(t), \quad (3.12)$$

$$\delta\dot{\hat{S}}_z(t) = -\delta\hat{\Gamma}\left[S_z(t) + \tfrac{1}{2}\right] - \hat{\Gamma}\delta\hat{S}_z(t) + iV_2^*\delta\hat{S}(t) - iV_2\delta\hat{S}^{\dagger}(t) - S(t)\delta\hat{f}^{\dagger}(t) - \delta\hat{S}(t)\hat{F}^{\dagger}(t) - S^*(t)\delta\hat{f}(t) - \delta\hat{S}^{\dagger}(t)\hat{F}(t), \quad (3.13)$$

and the adjoint equation of (3.12), where the fluctuation operators are

$$\delta\hat{\gamma} = \hat{\gamma} - \gamma, \quad \delta\hat{f}(t) = \hat{F}(t) - f(t), \quad \delta\hat{f}^{\dagger}(t) = \hat{F}^{\dagger}(t) - f^*(t), \quad \delta\hat{\Gamma} = \hat{\Gamma} - \Gamma. \quad (3.14)$$

After calculating first the quantum mechanical averages and then their statistical averages, one obtains after a little bit of algebra the following linear equation:

$$\frac{d}{d\tau}\begin{pmatrix}\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}_z(\tau)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}(\tau)\delta\hat{S}(0)\rangle\rangle_s\end{pmatrix} = \begin{pmatrix}-\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right) + i\delta_0 & -2iV_2^* & 0\\ -iV_2 & -\Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right) & iV_2^*\\ 0 & 2iV_2 & -\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right) - i\delta_0\end{pmatrix}\begin{pmatrix}\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}_z(\tau)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}(\tau)\delta\hat{S}(0)\rangle\rangle_s\end{pmatrix}. \quad (3.15)$$

To calculate the spectrum we need the two-time correlation function $\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s$. A pertinent point regarding (3.15) may be noted here. This equation follows the quantum regression theorem. This provides a consistency check for the present scheme for the calculation of the correlation function in terms of quantum correction equations and c-number noise.

By solving Eq. (3.15) we obtain $\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s$, which can then be substituted into Eq. (3.10) to find the inelastic part of the scattering spectrum. The spectral quantity we need in Eq. (3.10) is actually the Laplace transform of $\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s$ denoted by $\delta\mathcal{S}^*(s)$, which is explicitly given by

$$\delta\mathcal{S}^*(s) = \int_0^{\infty} d\tau\, \exp(-s\tau)\langle\langle\delta\hat{S}^{\dagger}(\tau)\delta\hat{S}(0)\rangle\rangle_s, \quad (3.16)$$

and similarly for the others. s is the Laplace transform variable identified as $-i(\omega_0 - \overline{\omega}) = -i\Delta$.

Explicit evaluation of the above quantity (see Appendix B) for central pump detuning ($\delta_0 = 0$) yields the three-peaked incoherent spectrum:

$$\mathcal{S}(\Delta)|_{\text{inc}} \simeq \frac{\beta}{2\gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)}\left[\frac{\gamma_0{}^2\left[\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]^2}{\gamma_0{}^2\left[\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]^2 + \Delta^2}\right] + \frac{\beta}{2\gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)} \times\left[\frac{\gamma_0}{\Gamma_0 + \gamma_0}\frac{\frac{1}{4}(\Gamma_0 + \gamma_0)^2\left[\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]^2}{(\mathscr{R}_0 \pm \Delta)^2 + \frac{1}{4}(\Gamma_0 + \gamma_0)^2\left[\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]^2}\right]. \quad (3.17)$$

The incoherent part of the scattered spectrum consists of three peaks, as expected from the dressed atom picture. The central peak centered at $\overline{\omega} = \Omega$ has a width $\gamma_0\tanh(\frac{\hbar\Omega}{2kT})$, while the sidebands centered about the frequency $\overline{\omega} = \Omega \pm \mathscr{R}_0$ have a width $\frac{1}{2}(\Gamma_0 + \gamma_0)\tanh(\frac{\hbar\Omega}{2kT})$. $\mathscr{R}_0$ is the Rabi flopping frequency defined by $\frac{d\mathscr{E}}{\hbar}$. Although the zero-temperature situation is completely identical for both bosonic and spin heat baths, we obtain an interesting result for the spin bath at nonzero finite temperature which deviates greatly from its bosonic counterpart. The substantial reduction of linewidth with rise of temperature in quantum dot two-level systems as reported in a number of photoluminescence experiments [29–32] by several groups lends support to this observation. Figure 3 shows that at low field, i.e., for small Rabi frequency $\mathscr{R}_0$, one may obtain a three-peak Mollow triplet by tuning the temperature to a higher value. This result is counterintuitive since it demonstrates that with increase in temperature the

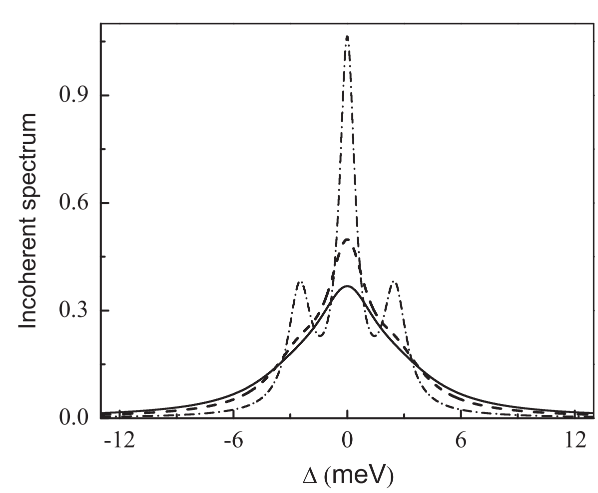

FIG. 3. Thermally induced Mollow triplet: At low field the single peak splits into three by increase of temperature; $T = 70$ (continuous line), 160 (dashed line), and 460 K (dash-dotted line) for $I_2 = 0.78$ and $\hbar\Omega = 20$ meV for zero detuning.

single peak splits into three. Increasing temperature may thus induce a Mollow triplet in a low driving field regime, because of the emergence of coherence behavior as a result of suppression of width at the relatively higher temperature in a spin bath. This result is a strong departure from the situation one observes in conventional quantum optics. At low intensity when the Rabi frequency is far lower than the decay rates, the spectrum is characterized by a single Rayleigh line. By increasing the intensity of the exciting light when the Rabi frequency far exceeds the decay rate, the single peak splits into a triplet, the separation between the peaks being determined by the Rabi frequency. At higher temperature the separation between the spectral components of the conventional Mollow triplet is masked by thermal broadening, resulting in a single peak with a large width. The triplet observed in the present study, on the other hand, is essentially a spin-bath-induced coherence effect whereby increase in temperature results in reduction of width, so that the three-peak spectrum is realized at higher temperature even at a low field strength.

For suitable experimental realization of the above scheme, the coherent-incoherent transition of intensity, and thermally induced splitting it may be possible to use quantum dots serving as artificial two-level systems. Measurement of the scattered intensity and resonance fluorescence following the resonance excitation of a single two-level quantum dot at several temperatures at relatively low field is expected to yield the desired splitting, provided the low-frequency phonon modes of the environment are suitably eliminated.

IV. CONCLUSION

In this paper we have explored the role of a spin bath of two-level atoms in the quantum optical properties of an excited two-level system. Based on the spin-coherent state representation of the noise operators and a canonical thermal distribution of the associated c numbers the dissipative dynamics of the system has been formulated in terms of Langevin-Bloch equations. We have calculated the scattered intensity and resonance fluorescence from the classically driven two-level system. The main conclusions can be summarized below.

(1) While at zero temperature the behaviors of the spin bath and a harmonic bath are almost identical, they begin to differ at finite temperature. Since the spectral density function undergoes modification due to the temperature dependence through a hyperbolic tangent factor, increase in temperature results in an effective reduction of friction. This causes suppression of linewidth, and therefore a rise in temperature favors splitting of spectral lines even at low driving field. We have demonstrated how Mollow triplet in resonance fluorescence can be realized as a thermally induced process, rather than as a field-induced effect as observed in traditional quantum optics.

(2) The temperature dependence of the incoherent and coherent components of the scattered intensity is distinctly different. While the incoherent intensity is always higher at higher temperature between saturation and the zero level of intensity, irrespective of the strength of the driving field, the coherent intensity is lower at higher temperature in the high-field regime. An important offshoot of this calculation is a crossover temperature which sets the boundary between coherent and incoherent regimes. It is possible to observe the coherence effect at low field even at high temperature.

(3) That temperature favors coherence is a consequence of the nature of a spin bath of two-level atoms due to the effective reduction of coupling. This is consistent with the earlier observations on dissipative tunneling [9] or decoherence dynamics [19] in a spin bath and can be traced to the limited ability of the two-level bath to damp tunneling oscillations or coherence, in contrast to a harmonic bath for which the options for excitations are unlimited.

Our finding in this paper may be worth further investigation from the standpoint of experimental quantum optics with quantum dots. Since resonance fluorescence has already been observed in two-level quantum dots [23–28], the preparation of a suitable two-level bath with appropriate spectral density is expected to throw more light in future in this direction.

ACKNOWLEDGMENTS

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APPENDIX A: COHERENT STATES OF SPIN-$\frac{1}{2}$ SYSTEMS

In analogy to the coherent states of a linear harmonic oscillator, Radcliffe [34] in the early 1970s introduced spin coherent states. In this appendix we give a brief outline of the coherent states of a spin-$\frac{1}{2}$ system and some of their main properties relevant to our discussion.

Before we define the spin-coherent states, it will be useful to discuss the coherent states of a one-dimensional harmonic oscillator. These coherent states are the eigenstates of an annihilation operator $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ and are functions of the variable α, which runs over the entire complex plane. They are explicitly given by

$$\begin{aligned}|\alpha\rangle &= \pi^{-1/2}\exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n\rangle \\ &= \pi^{-1/2}\exp\left(-\frac{|\alpha|^2}{2}\right)\exp\left(\alpha\hat{a}^\dagger\right)|0\rangle, \qquad \text{(A1)}\end{aligned}$$

where $|n\rangle$ is the nth energy eigenstate of the oscillator and $\hat{a}^\dagger$ is the usual creation operator. These states are minimum uncertainty product states such that $\Delta q\,\Delta p = \hbar$ and form a complete set, in the sense that

$$\int d^2\alpha|\alpha\rangle\langle\alpha| = \sum_{n=0}^{\infty}|n\rangle\langle n| = 1. \qquad \text{(A2)}$$

Analogous to this harmonic oscillator coherent state the spin coherent state is defined as

$$|\mu\rangle = (1+|\mu|^2)^{-S}\exp\left(\mu\hat{S}^\dagger\right)|0\rangle. \qquad \text{(A3)}$$

The state $|\mu\rangle$ forms a complete set, although it is necessary to include a weight function $m(|\mu|^2) \geqslant 0$ in the integral, i.e.,

$$\int d^2\mu|\mu\rangle m(|\mu|^2)\langle\mu| = 1. \qquad \text{(A4)}$$

These spin coherent states reduce to harmonic oscillator coherent states in the limit $S \gg 1$, which is the high-spin limit of the Holstein-Primakoff transformation, i.e.,

$$\hat{S}^\dagger \longrightarrow (2S)^{1/2}\hat{a}^\dagger \tag{A5}$$

and

$$\mu \longrightarrow \alpha/(2S)^{1/2}. \tag{A6}$$

The normalized states $|\alpha\rangle_{(S)}$ are then given by

$$|\alpha\rangle_{(S)} = \left(1+\frac{|\alpha|^2}{2S}\right)^{-S}\exp\left(\alpha\hat{a}^\dagger\right)|0\rangle, \tag{A7}$$

since we have

$$\lim_{S\to\infty}\left(1+\frac{|\alpha|^2}{2S}\right)^{-S} = \exp\left(\frac{|\alpha|^2}{2}\right) \tag{A8}$$

and therefore

$$\lim_{S\to\infty}|\alpha\rangle_{(S)} = \exp\left(-\frac{|\alpha|^2}{2}\right)\exp\left(\alpha\hat{a}^\dagger\right)|0\rangle, \tag{A9}$$

which apart from normalization is precisely the harmonic oscillator coherent state.

Now, using operator algebra for the spin-$\frac{1}{2}$ particle as given in Sec. II A and the definition of the spin coherent state Eq. (A3), one may calculate the matrix elements of the Pauli operators (e.g., $\langle\mu|\hat{\sigma}|\mu\rangle \equiv \langle\hat{\sigma}\rangle$ and others) with respect to the spin coherent state $|\mu\rangle$. These are given by

$$\langle\hat{\sigma}\rangle = \mathcal{N}\mu\ (\equiv \xi), \tag{A10}$$

$$\langle(1-2\hat{n}_k)\rangle = \mathcal{N}^2|\mu|^2\ (\equiv |\xi|^2), \tag{A11}$$

and the complex conjugate of Eq. (A10). $\mathcal{N}$ includes the normalization constant and other numerical factors. ξ_k and ξ_k^* are the c-number variables corresponding to $\hat{\sigma}_k$ and $\hat{\sigma}_k^\dagger$ for the kth bath operator as used in the main text.

APPENDIX B: SOME DETAILS OF CALCULATION OF THE SPECTRUM

Laplace transform of Eq. (3.15) reduces the coupled differential equations to a set of algebraic equations given by

$$(sI-B)\begin{pmatrix}\delta\mathcal{S}^*(s)\\ \delta\mathcal{S}_z(s)\\ \delta\mathcal{S}(s)\end{pmatrix} = \begin{pmatrix}\langle\langle\delta\hat{S}^\dagger(0)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}_z(0)\delta\hat{S}(0)\rangle\rangle_s\\ \langle\langle\delta\hat{S}(0)\delta\hat{S}(0)\rangle\rangle_s\end{pmatrix}, \tag{B1}$$

where I is the identity matrix and B is the (3×3) matrix in Eq. (3.15). The matrix $sI-B$ is given by

$$sI-B = \begin{pmatrix}\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)-i\delta_0+s & 2iV_2^* & 0\\ iV_2 & \Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s & -iV_2^*\\ 0 & -2iV_2 & \gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+i\delta_0+s\end{pmatrix}. \tag{B2}$$

The general form of the initial condition on the right-hand side of Eq. (B1) is given by

$$\langle\langle\delta\hat{S}_i(0)\delta\hat{S}(0)\rangle\rangle_s = \langle\langle\hat{S}_i(0)\hat{S}(0)\rangle\rangle_s - \langle S_i(0)\rangle_s\langle S(0)\rangle_s \tag{B3}$$

with the explicit form

$$\langle\langle\delta\hat{S}^\dagger(0)\delta\hat{S}(0)\rangle\rangle_s = \langle\langle\hat{S}_a(0)\rangle\rangle_s - |\langle S^*(0)\rangle_s|^2, \tag{B4}$$

$$\langle\langle\delta\hat{S}_z(0)\delta\hat{S}(0)\rangle\rangle_s = -\langle S_a(0)\rangle_s\langle S(0)\rangle_s, \tag{B5}$$

$$\langle\langle\delta\hat{S}(0)\delta\hat{S}(0)\rangle\rangle_s = -\langle S(0)\rangle_s^2. \tag{B6}$$

So the incoherent spectrum $\mathcal{S}_{\rm inc}$ is given by

$$\mathcal{S}_{\rm inc} = \beta\delta\mathcal{S}^*(-i\Delta) + \text{c.c.} \tag{B7}$$

and explicit evaluation results in

$$\delta\mathcal{S}^*(s) = \frac{\begin{vmatrix}\langle\langle\hat{S}_a(0)\rangle\rangle_s-|\langle S^*(0)\rangle_s|^2 & 2iV_2^* & 0\\ -\langle S_a(0)\rangle_s\langle S(0)\rangle_s & \Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s & -iV_2^*\\ -\langle S(0)\rangle_s^2 & -2iV_2 & \gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+i\delta_0+s\end{vmatrix}}{D} \tag{B8}$$

where D is given by

$$D = \left[\Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+i\delta_0+s\right]\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)-i\delta_0+s\right]+4|V_2|^2\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]. \tag{B9}$$

Therefore,

$$\delta\mathcal{S}^*(s) = \frac{[\langle\langle\hat{S}_a(0)\rangle\rangle_s-|\langle S^*(0)\rangle_s|^2]\left\{[\gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)+i\delta_0+s][\Gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)+s]+2|V_2|^2\right\}}{\left[\Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]\left\{[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s]^2+\delta_0^2\right\}+4|V_2|^2\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]}$$

$$+\frac{2iV_2^*\langle S_a(0)\rangle_s\langle S(0)\rangle_s\left[\gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)+s\right]-2|V_2|^2\langle S(0)\rangle^2}{\left[\Gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]\left\{\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]^2+\delta_0^2\right\}+4|V_2|^2\left[\gamma_0\tanh\left(\frac{\hbar\omega_0}{2kT}\right)+s\right]}. \tag{B10}$$

To find the three-peaked spectrum we consider central pump tuning ($\delta_0 = 0$) and large Rabi flopping, so ω_0 can be replaced by the system frequency Ω, i.e., $I_2 \gg [\tanh(\frac{\hbar\Omega}{2kT})]^2$ or equivalently $4|V_2|^2 \gg \gamma_0\Gamma_0[\tanh\left(\frac{\hbar\Omega}{2kT}\right)]^2$. Again, $4|V_2|^2 = \mathscr{R}_0^2$, since the semiclassical Rabi flopping frequency $\mathscr{R}_0 = \frac{d\mathscr{E}}{\hbar}$. First, we consider the condition of resonance, i.e., the region around $s = -i\Delta \simeq 0$, where D reduces to $\mathscr{R}_0^2[\gamma_0\tanh(\frac{\hbar\Omega}{2kT}) + s]$, neglecting terms $O(\gamma_0\Gamma_0[\tanh\left(\frac{\hbar\Omega}{2kT}\right)]^2)$. The numerator of Eq. (B10) reduces to $\frac{1}{4}|\mathscr{R}_0|^2$, so that it becomes

$$\delta\mathcal{S}^*(s\simeq 0)|_{\rm inc} \simeq \frac{1}{4\left[\gamma_0\tanh\left(\frac{\hbar\Omega}{2kT}\right)+s\right]}. \tag{B11}$$

For $s \simeq \pm i\mathscr{R}_0$, i.e., in the vicinity of the sidebands, the numerator of Eq. (B10) reduces to $-\frac{1}{4}|\mathscr{R}_0|^2$, and the denominator D reduces approximately to $s\left[\mathscr{R}_0^2 + s^2 + s(\gamma_0+\Gamma_0)\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]$. Substituting $s = -i\Delta$ in the ratio and noting that $\mathscr{R}_0^2 - \Delta^2 = (\mathscr{R}_0+\Delta)(\mathscr{R}_0-\Delta)$, we find

$$\begin{aligned}\delta\mathcal{S}^*(-i\Delta \simeq \pm i\mathscr{R}_0) &\\ \simeq -&\frac{\frac{1}{4}i|\mathscr{R}_0|^2}{\Delta\left[\mathscr{R}_0^2-\Delta^2-i\Delta(\gamma_0+\Gamma_0)\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]}\\ \simeq \mp&\frac{i}{8\left[\mathscr{R}_0\pm\Delta\mp\frac{1}{2}i(\gamma_0+\Gamma_0)\tanh\left(\frac{\hbar\Omega}{2kT}\right)\right]}.\end{aligned} \tag{B12}$$

Substituting this along with Eq. (B11) into Eq. (B7), we find Eq. (317).

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