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Suppression of noise in magnetic layers by a periodic spin torque

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The simultaneous effect of thermal noise and time-periodic spin torques on magnetic multilayers are treated in this work. Using two commonly studied magnetic systems with multiple stable states at zero temperature as examples, we show that periodic spin torques can enhance the stability of the system and suppress the noise due to interwell transitions. In the case of weak periodic spin torques, stochastic resonance, which is usually associated with ac magnetic fields, is also manifested for nonconservative torques. In more complex systems with a relatively low energy barrier, it is shown that high-frequency spin torques can inhibit interwell transitions and in effect suppress the telegraph noise due to the switching between neighboring states.

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I. INTRODUCTION

The application of stochastic resonance (SR) [1] to increase the signal-to-noise ratio (SNR) is now well established in many physical systems [2] and has also been shown to increase the stability of a system [3]. Particle production in a quantum field can also be enhanced in the presence of noise [4]. The SNR is enhanced by adding noise to the system as it is being driven by an external periodic force or vice versa. Hence the presence of noise in a device is not necessarily detrimental to its operational use and understanding its effects on the dynamic of the system can, in many cases, help us figure out the underlying physics in a device [5]. However, until now, mostly one-dimensional systems with additive noise have been studied and shown to have this interesting property of increasing the SNR with the addition of noise as it is being driven by a weak periodic force. In systems subjected to strong periodic forces, it has been shown that SR becomes frequency dependent [6]. For linear systems, SR is absent with additive noise, but it was shown that, if multiplicative colored noise is added to the system, SR becomes possible [7]. Multiplicative white noise has also been shown to give rise to SR in nonlinear one-dimensional systems [8] and in systems where both multiplicative and additive noise are present [9]. In this work, we will be mainly interested in multiplicative noise in a magnetic structure of current interest in spintronics and information storage [10].

This counterintuitive physical effect, i.e., SR, is inherently related to the nonlinearities present in these systems. One might therefore ask if a similar behavior can persist in higher dimensional systems, in particular magnetic systems, subjected to periodic nonconservative fields. Such an application is lacking in the current literature. Here we expand the study to this kind of system. In particular, we treat a spin valve with one free and one fixed magnetic layer separated by a normal conductor and with a current crossing the whole stack [10].

In the spin-valve geometry, the magnetization can be in-plane or perpendicular to the plane. In this paper, we will treat both cases with the magnetization taken to be nonuniform in the former and uniform in the latter. Since in this configuration the current traversing the structure is perpendicular to the plane (CPP), an additional torque between the layers exists called the spin torque [11,12]. This torque is not derivable from a potential and is usually taken static in current experiments [10]. Because of the constant drive to make every device smaller, thermal fluctuations are also becoming a bigger problem and hence new solutions to suppress this noise are needed. To address this problem in spin valves, we suggest the addition of an ac component to the current. This in turn will give rise to a periodic torque, which will be present in addition to thermal fluctuations in the spin valve. Hence our motivation to study SR-related physics in these systems.

In this paper, we will use both analytical calculations and numerical simulations to study the suppression of noise in magnetic systems with various energy landscapes using periodic spin torques. In the first part of the paper, we apply the Kramers-Brown theory [13] to calculate transition rates in a uniform magnetic layer with uniaxial anisotropy along the direction of current and under the simultaneous action of thermal fluctuations and a periodic spin torque with the current polarized along the easy axis. Brown [13] considered only the action of conservative fields, but in our case we will be dealing with fields that are not derivable from a potential and, in addition, are time dependent. To be able to carry out the calculations, we restrict ourselves to the adiabatic limit. Moreover, we will use the two-state approximation and the theory developed by Gardiner [14] to calculate the probability distribution of the orientations of the magnetization. This will then enable us to use the theory of McNamara and Wiesenfeld [15] to calculate the signal-to-noise ratio (SNR) in the system and show that it is enhanced by a weak periodic spin torque and hence SR is fully exhibited by this system. As far as we know, stochastic resonance (SR) has not been exploited in magnetic systems. The work by Mantega *et al.* [16] provides the only application we are aware of; it applies SR to the measurement of hysteresis loops in magnetic systems described by the Preisach model using periodic magnetic fields. Our calculation therefore shows that, while spin torques are usually used to switch the magnetization, they can also be exploited to induce SR in a magnetic system and help the suppression of telegraph noise observed in spin valves with perpendicular media [10].

In the second half of the paper, we relax the assumptions of uniform magnetization, high energy barrier, and low frequency

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used in the first part to discuss a more complex energy landscape. A solution of the noise-periodic spin torque physics in this system requires a numerical solution. In this system, the Kramers approximation is no longer valid. Moreover, the current we use is of a frequency comparable to that of the system and hence the adiabatic approximation cannot be applied either. The solution in this case requires a full numerical micromagnetic solution. We use the stochastic Landau-Lifshitz equation [13] to solve for the spectral densities of the magnetic state from which we can extract information about the signal and the background noise in the magnetic structure. In the presence of thermal fluctuations, the system is unstable. The magnetization is constantly switching between two states. We show that, depending on the frequency of the spin torque, the noise due to the interwell transitions can be suppressed in this case and hence the system becomes more stable. This proves the potential of using ac currents in spin valves in realistic systems, since most practical configurations will fall in between the two extremes we study here.

II. STOCHASTIC RESONANCE IN THE PRESENCE OF A PERIODIC SPIN TORQUE

In this section, we consider a relatively simple, but important, magnetic system of a uniform magnetization with uniaxial anisotropy along the z axis traversed by a current polarized by another pinned magnetic layer. The pinned layer has a magnetization in the z direction along the current. Both layers are usually separated by a normal conductor. Such a spin-valve configuration has possible applications in high-density information storage devices and was recently shown to exhibit telegraph noise [10] due to creation and annihilation of domain walls. This telegraph noise is due to both the thermal fluctuations in the system and the static spin current. This kind of noise has already been predicted in the simulations on single domain particles discussed in Ref. [17].

A. Perpendicularly magnetized nanoparticle with a time-independent spin torque

The dynamic of the averaged magnetization in a particle with unit volume is well described by the Landau-Lifshitz (LL) equation [18]

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \gamma \frac{\alpha}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}), \qquad (1)$$

where γ is the gyromagnetic ratio, α is the damping constant, and M_s is the constant magnitude of the magnetization vector $\mathbf{M}$. The effective field $\mathbf{H}_{\text{eff}}$ is derivable from an energy functional $E(\mathbf{M})$, $\mathbf{H}_{\text{eff}} = -\frac{\delta E}{\delta \mathbf{M}}$. To account for thermal agitation, a white noise stochastic field ξ with zero average is usually added to the effective field. However, in this section, we use the Fokker-Planck approach, since it is easier to find an analytical solution to the magnetic noise with this method [19].

For a single domain particle with uniaxial anisotropy, the energy is given by

$$E(\mathbf{M}) = U(\theta) = -\frac{1}{2} H_K M_s \cos^2 \theta, \qquad (2)$$

where H_K is the strength of the anisotropy field along the z axis. This energy has two stable states along the z axis, $\mathbf{M}_\pm = \pm M_s \mathbf{z}$, and one unstable state, $\mathbf{M}_0$, in the plane perpendicular to the anisotropy field, Fig. 1. We are interested in the most practical case, where the energy barrier, $E_b = E(\mathbf{M}_0) - E(\mathbf{M}_\pm)$, is much larger than $k_B T$. Hence, in this case, thermal fluctuations cause very few transitions between the two stable states. To make the system more stable against this thermal noise, a much higher anisotropy is needed; however, this is not desirable for practical reasons, since we still want to switch between the two stable states in a predictive manner with as small fields as possible. One recent idea is to use polarized currents rather than fields to switch the nanoparticles. It has been shown that a current polarized in the direction of the magnetization of the pinned layer, $\hat{\mathbf{z}}$, induces an additional field on the magnetization $\mathbf{M}$ in the free layer [11,12]. The field associated with this spin momentum transfer (SMT) is

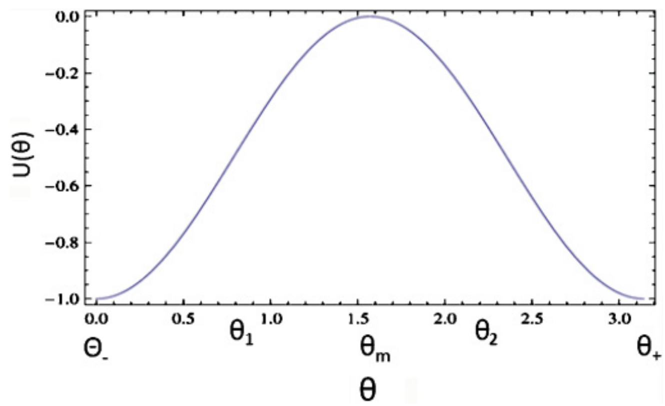

FIG. 1. (Color online) Potential of a uniaxial thin film with high anisotropy along the direction of the polarized current. The stable states are at $\theta_- = 0$ and $\theta_+ = \pi$. The unstable point $\theta_m \approx \pi/2$. The energy barrier is assumed much higher than $k_B T$, the thermal energy.

$$\mathbf{H}_{\text{SMT}} = \frac{a_s}{M_s} \hat{\mathbf{z}} \times \mathbf{M}, \qquad (3)$$

where the constant a_s is proportional to the current intensity and degree of polarization. This result is a by-product of angular momentum conservation between the conduction electrons and the local moments. This spin-transfer field, unlike the anisotropy field, is not derivable from an energy functional and requires a modified Kramer's theory [20] to calculate escape rates in its presence [14,21,22]. Therefore, for a uniaxial single domain nanopillar with polarized current applied perpendicular to the plane, the total field acting on the magnetization cannot, in general, be written in terms of an energy functional:

$$\mathbf{H}_{\text{total}} = \frac{H_K}{M_s} M_z \mathbf{z} + \mathbf{H}_{\text{SMT}}. \qquad (4)$$

To study the effect of noise on the stability of the magnetic state and calculate the SNR, we use the Fokker-Planck (FP) equation in angular form (θ,ϕ), since the magnetization has constant magnitude. This problem can be solved analytically in the high energy barrier case. The FP expresses the evolution in time of the magnetization probability density $P(\mathbf{M})$ [13,23]

$$\frac{\partial P}{\partial t} + \nabla_{\mathbf{M}} \cdot \mathbf{J} = 0, \qquad (5)$$

where $\mathbf{J}$ is the magnetization current density

$$\mathbf{J} = \frac{1}{M_s}\frac{d\mathbf{M}}{dt}P - D\nabla a_{\mathbf{M}}P. \tag{6}$$

The diffusive part of the current corresponds to the dissipation term in the LL equation, Eq. (1), in the absence of the spin torque term. At equilibrium, the solution of Eq. (6) must be a Boltzmann distribution $P = N\exp[-U(\theta)/k_BT]$, with N a normalization constant. This requirement gives an expression for the diffusion coefficient $D = \gamma\alpha k_BT/M_s$ for $\alpha \ll 1$. At room temperature, the operational temperature of the system, the spin shot noise is negligible compared to the thermal noise and hence the diffusion coefficient D is unchanged in the presence of spin torques [24]. In this case, a Boltzmann-like solution is still possible for this particular two-state system, but with a modified potential. For our purposes, it is enough to consider only θ-dependent effective potentials due to the symmetry of the system. In this case, the FP equation becomes simply

$$\frac{\partial P}{\partial t} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left[\frac{\gamma\alpha}{M_s}\left(\frac{\partial U}{\partial\theta}P - \frac{a_sM_s}{\alpha}\sin\theta P\right) + \frac{\gamma\alpha k_BT}{M_s}\frac{\partial P}{\partial\theta}\right]\right\}. \tag{7}$$

Now if we define an effective potential U_{eff}

$$U_{\mathrm{eff}}(\theta) = -H_KM_s\left[\frac{1}{2}\cos^2\theta - \frac{a_s}{a_c}\cos\theta\right], \tag{8}$$

with $a_c = \alpha H_K$, then it is easy to see that the distribution

$$P_s = Ne^{-U_{\mathrm{eff}}(\theta)/k_BT} \tag{9}$$

is a stationary solution of the FP equation, Eq. (7). In the absence of spin currents, $a_s = 0$, the original Boltzmann distribution is recovered.

B. Escape rate in the presence of a periodic spin torque

So far we have assumed that the spin torque is time-independent. In the rest of the paper, the spin torque will have a small periodic component:

$$a_s(t) = a_s^0 + b_s\sin\Omega t. \tag{10}$$

The magnitude of the time-dependent part and the frequency Ω will be taken small in this section so that an adiabatic approximation is valid. In this case, the foregoing discussion is still valid but now with time-dependent potential instead [22]. The form of the spin torque in the presence of ac currents still has the same form as in the Slonczewski equation. Zhu *et al.* [25] calculated the ac spin torque in magnetic tunnel junctions and found it to be similar in form to the static case. Among other things, they found that, at low frequencies, the total spin torque can become less than the spin torque of the static component of the current. Hence an ac current can introduce a partial self-cancellation of the spin torque and reduces any undesired consequences from its presence.

To calculate the escape rate from one well to the next, we apply Kramers theory [20]. The magnetization is constant in magnitude; therefore, the equation of motion is best written in angular form $\mathbf{M}(\theta,\phi)$. However, since the energy is only θ dependent, we will need its evolution only as a function of this angle. From the equation of motion of the z component $M_z = M_s\cos\theta$ of the magnetization, Eq. (1), we can write the corresponding equation for θ:

$$\dot\theta = -\frac{\alpha\gamma}{2}H_K\sin 2\theta + \gamma a_s(t)\sin\theta. \tag{11}$$

In the following, we ignore rotational effects on the transition rates since the energy does not depend on the angle ϕ and the damping is small in such typical systems: $\alpha \approx 0.01$. This effectively reduces the problem to one dimension. This approximation is valid in current systems of practical interest [26]. To calculate the transition rates between the stable states, we will need to calculate the current at the top of the potential.

The magnetization current of interest to us is the θ component [27],

$$J_\theta = \left(-\frac{\alpha\gamma}{2}H_K\sin 2\theta + \gamma a_s\sin\theta\right)P - k_BT\frac{\gamma\alpha}{M_s}\frac{\partial P}{\partial\theta}, \tag{12}$$

which when averaged over the azimuthal angle gives the total probability current, I, that flows between the states $\theta_\pm = 0,\pi$. Since the convective part of the current is given by a derivative of the potential with respect to θ, we see from this expression that, in the presence of small damping and small spin currents, an effective potential for the nonconservative system can be written in the form

$$U_{\mathrm{eff}}(t) = H_KM_s\left[\frac{1}{2}\sin^2\theta + \frac{a_s(t)}{a_c}\cos\theta\right]. \tag{13}$$

The unstable state at the top of the barrier is then approximately unchanged from the case with no spin current, i.e., $\sin\theta_m \simeq 1$ or $\theta_m \simeq \pi/2$, but the value of the potential and its curvature are slightly different:

$$U_{\mathrm{eff}}(\theta_m) = \frac{H_KM_s}{2}\left[1 + \left(\frac{a_s}{a_c}\right)^2\right], \tag{14}$$

$$U''_{\mathrm{eff}}(\theta_m) = H_KM_s\left[\left(\frac{a_s}{a_c}\right)^2 - 1\right]. \tag{15}$$

The sign of curvature is negative, as it should be for the unstable state θ_m close to $\theta = \pi/2$. If the probability density P is normalized to the total number, n, of particles in the system and since the barrier is high, we have $n = n_- + n_+$ and $I = \frac{dn_-}{dt} = -\frac{dn_+}{dt}$. The population distribution in each well, $n_\pm$, is then given by

$$P_\pm(\theta) = P(\theta_\pm)\exp\{-[U_{\mathrm{eff}}(\theta) - U_{\mathrm{eff}}(\theta_\pm)]/k_BT\}, \tag{16}$$

where, from Eq. (8), we have

$$U_{\mathrm{eff}}(\theta_\pm) = \pm H_KM_s\frac{a_s}{a_c}, \tag{17}$$

$$U''_{\mathrm{eff}}(\theta_\pm) = H_KM_s\left(1 \mp \frac{a_s}{a_c}\right). \tag{18}$$

The normalizations

$$\int_0^{\theta_1} P\,d\Omega = n_-, \tag{19}$$

$$\int_{\theta_2}^{\pi} P\,d\Omega = n_+, \tag{20}$$

and the expansions of the potential $U_{\rm eff}$ around the respective minimums $\theta_\pm$, keeping only leading terms, give the respective populations in both wells:

$$n_\pm = \frac{2\pi k_B T P(\theta_\pm)}{U''_{\rm eff}(\theta_\pm)}. \tag{21}$$

Integration of the current equation, Eq. (12), around the top of the potential between orientations θ_1 and θ_2 (see Fig. 1) gives

$$P(\theta_2)e^{U_{\rm eff}(\theta_2)/k_BT} - P(\theta_1)e^{U_{\rm eff}(\theta_1)/k_BT} = -\frac{M_s I e^{U_{\rm eff}/k_BT}}{2\pi k_B T\alpha\gamma}\sqrt{\frac{-2\pi k_B T}{U''_{\rm eff}(\theta_m)}}\frac{e^{U_{\rm eff}(\theta_m)/k_BT}}{\sin\theta_m}, \tag{22}$$

where $I = 2\pi \sin\theta J_\theta$ is the total current at the top. To get this result, we made a Taylor expansion of the effective potential around the unstable state and integrated only up to quadratic terms in θ. As a consequence of the continuity of the current and since $P(\theta_-)e^{U_{\rm eff}(\theta_-)} = P(\theta_1)e^{U_{\rm eff}(\theta_1)}$ and $P(\theta_+)e^{U_{\rm eff}(\theta_+)} = P(\theta_2)e^{U_{\rm eff}(\theta_2)}$, the rate of change of the population in each well is then

$$\dot n_\pm = W_\pm n_\mp - W_\mp n_\pm, \tag{23}$$

with the transition rates given by

$$W_\pm = \frac{\alpha\gamma}{M_s}\sqrt{\frac{-U''_{\rm eff}(\theta_m)}{2\pi k_B T}}U''_{\rm eff}(\theta_\mp)\sin\theta_m e^{-[U_{\rm eff}(\theta_m)-U_{\rm eff}(\theta_\mp)]/k_BT}. \tag{24}$$

This result is valid for high barriers and small spin currents. As the temperature is lowered toward zero, the transition rates vanish, which is the expected classical result.

C. Signal-to-noise ratio in the high barrier approximation

Now that we have calculated the transition rates between the stable states, we can use the two-state approximation to calculate the noise. The high barrier approximation also allows us to write the probability distribution in the form

$$P(\theta,t) = n_-(t)\delta(\theta-\theta_-) + n_+(t)\delta(\theta-\theta_+), \tag{25}$$

where $n_\pm$ is the population in each well. The probability density is almost zero at the top of the well and hence it is neglected. This is the two-state approximation. Therefore, solving the rate equations, Eq. (23), is equivalent to finding the distribution P. From the solution of the distribution P, the spectral density is then calculated as shown below. If we normalize the total $n = 1$, the general solution of the rate equations is [15]

$$n_+(t) = g^{-1}(t)\left(n_+(t_0)g(t_0) + \int_{t_0}^{t} W_+(t')g(t')dt'\right), \tag{26}$$

where $g(t) = \exp[\int^t W_+(t') + W_-(t')dt']$. For our system, we find that for an arbitrary initial state θ_0 at t_0

$$n_+(t;t_0,\theta_0) = \exp[-a_0(t-t_0)]\left(\delta_{\theta_0,\theta_+} - \frac{a_{01}}{a_0} + \frac{a_{11}}{(\Omega^2+a_0^2)^{1/2}}\cos(\Omega t_0+\varphi)\right) + \frac{a_{01}}{a_0} - \frac{a_{11}\cos(\Omega t+\varphi)}{(\Omega^2+a_0^2)^{1/2}}, \tag{27}$$

with

$$a_0 = 2\alpha H_K\sqrt{\frac{1}{2\pi}\frac{M_sH_K}{k_BT}}e^{-M_sH_K/2k_BT}, \tag{28}$$

$$a_{01} = \frac{a_0}{2}\left(1-\frac{a_s^0}{a_c}\right), \tag{29}$$

$$a_{02} = \frac{a_0}{2}\left(1+\frac{a_s^0}{a_c}\right), \tag{30}$$

$$a_{11} = -a_{12} = -\frac{a_0}{2}\left(\frac{b_s}{a_c}\right), \tag{31}$$

and $\varphi = \tan^{-1}(\frac{a_0}{\Omega})$. The function $n_+(t;t_0,\theta_0)$ is an expression for the conditional probability that the magnetization is in the direction θ_+ given that it was in the direction θ_0 at time t_0. The spectral density is now found by first calculating the correlation functions of the angle θ of the magnetization $\langle\theta(t+\tau)\theta(t)\rangle$ [2,15],

$$\langle\theta(t+\tau)\theta(t)\rangle = \lim_{t_0\to-\infty}\langle\theta(t+\tau)\theta(t);\ \theta_0,t_0\rangle \tag{32}$$

$$= \lim_{t_0\to-\infty}\theta_+(t+\tau)\theta_+(t)n_+(t+\tau;\theta_+,t)n_+(\theta_0,t_0), \tag{33}$$

where the cross terms vanish, since $\theta_- = 0$. To recover a translational invariant expression for the correlation function, i.e., an expression independent of t, this is averaged over a period $T_p = 2\pi/\Omega$. Therefore, the power spectrum is given by

$$\begin{aligned} S(\omega) &= \int_{-\infty}^{\infty} d\tau\left(\frac{1}{T_p}\int_0^{T_p} dt\langle\theta(t+\tau)\theta(t)\rangle\right)e^{i\omega\tau} \\ &= \sqrt{2\pi}\frac{a_{01}^2}{a_0^2+\Omega^2}\left(1+\frac{\Omega^2}{a_0^2}\right)\delta(\omega) \\ &\quad + \frac{\sqrt{2\pi}}{4}\frac{a_{11}^2}{a_0^2+\Omega^2}[\delta(\omega-\Omega)+\delta(\omega+\Omega)] \\ &\quad + \frac{1}{\sqrt{2\pi}a_0}\frac{2a_{01}a_{02}\Omega^2 - a_0^2a_{11}^2 + 2a_0^2a_{01}a_{02}}{(a_0^2+\omega^2)(a_0^2+\Omega^2)}. \end{aligned} \tag{34}$$

The spectral density has a broadband component, which represents the noise and the signal part that is proportional to the delta functions. The SNR is the ratio of these latter two components. The component of $S(\omega)$ that is proportional to $\delta(\omega)$ is due to the interwell transitions. Assuming positive frequency Ω, the SNR is measured at $\omega = \Omega$:

$$\mathrm{SNR} = \frac{\pi}{2}\frac{a_0a_{11}^2(a_0^2+\Omega^2)}{2a_{01}a_{02}\Omega^2 - a_0^2a_{11}^2 + 2a_0^2a_{01}a_{02}}. \tag{35}$$

This function is shown in Fig. 2. It has the familiar form for a system that exhibits stochastic resonance. The maximum of the SNR is achieved at nonzero thermal fluctuations. It is also curious to see that as the magnitude of the periodic spin torque approaches that of the constant term, the SNR increases. However, the dependence of SNR on frequency is almost flat for large Ω. Our calculation is, however, strictly not applicable in these limits. The system we explore next is

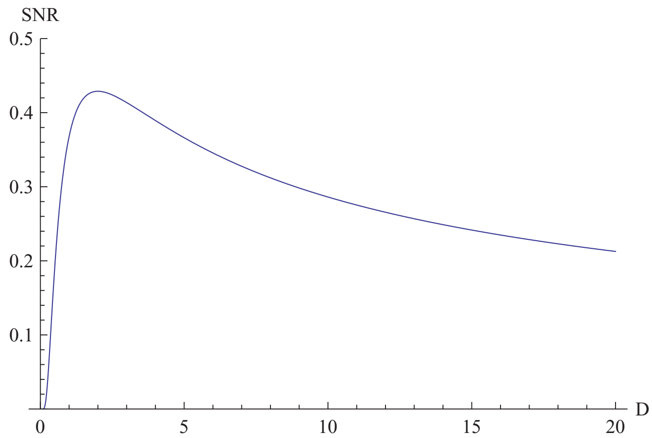

FIG. 2. (Color online) Signal-to-noise ratio in granular perpendicular media with a polarized current along the anisotropy axis (arbitrary units). $D = \gamma\alpha k_B T/M_s$ is the strength of noise. The SNR is maximum at nonzero noise.

more complex and most of the approximations used above are not valid anymore. In spite of the simplicity of this model, it is, however, of great interest to work on perpendicular media that tend to be granular with high anisotropy. Therefore, the results presented here are relevant to the suppression of noise in those systems that were observed lately [10].

III. SUPPRESSION OF TELEGRAPH NOISE IN A SPIN VALVE WITH NONUNIFORM MAGNETIZATION

A. Noise in a nonuniform magnetic state

We now study numerically a spin valve with current perpendicular to the plane (CPP) but with in-plane magnetization. The spin valve is made of two magnetic layers (Fig. 3): one with magnetization $\mathbf{S}_p$ ($\mathbf{M}_p = \gamma\mathbf{S}_p/V$) pinned along the x axis [reference layer (RL)] and the other (FL) with free magnetization $\mathbf{S}_f$ separated by a thin normal conductor layer. The static magnetic states of the geometry discussed in this section have been studied in Ref. [28] and are highly nonuniform due to a biasing field. In the simulations, we take account of both layers with a small interlayer exchange. The pinned layer is fixed with a large local magnetic field. This system has bistable nonuniform magnetic states separated by an energy barrier E_b comparable to the thermal energy in the system: $E_b \approx 2k_B T$ at $T = 373$ K. It has natural frequencies in the GHz regime, but with the power spectrum having a strong component at much lower frequencies. This low-frequency component is due to switching between two nearby states. An application of Kramers theory, which requires two well-separated states, is therefore not applicable to this system. We are also interested in time-dependent spin torques with a magnitude comparable to the dc part. To solve for the noise, we have to numerically integrate the stochastic LL equation. This spin-valve structure is widely used in magnetic recording and, very often, nonuniformities in the magnetization lessen their appeal for applications. However, we have shown in earlier work that nonuniform magnetic states may be advantageous when it comes to using them in sensors, since they appear to be easier to control by weaker spin torques than layers with uniform magnetization [29].

The Landau-Lifshitz equation [23] in the presence of a time-dependent spin torque and thermal fluctuations is

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times(\mathbf{H}_{\rm eff}+\mathbf{H}_{\rm SMT}+\xi) - \gamma\frac{\alpha}{M_s}\mathbf{M}\times[\mathbf{M}\times(\mathbf{H}_{\rm eff}+\mathbf{H}_{\rm SMT})]. \qquad (36)$$

The effective magnetic field $\mathbf{H}_{\rm eff}$ has four components made of the exchange field, the demagnetization field, the anisotropy field, and an in-plane external field [27]

$$\mathbf{H}_{\rm eff}(\mathbf{r}) = \frac{-2A}{M_s^2}\nabla^2\mathbf{M} + \int d^3r'\frac{\mathbf{M}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} + \mathbf{H}_{\rm an} + \mathbf{H}_{\rm ext}. \qquad (37)$$

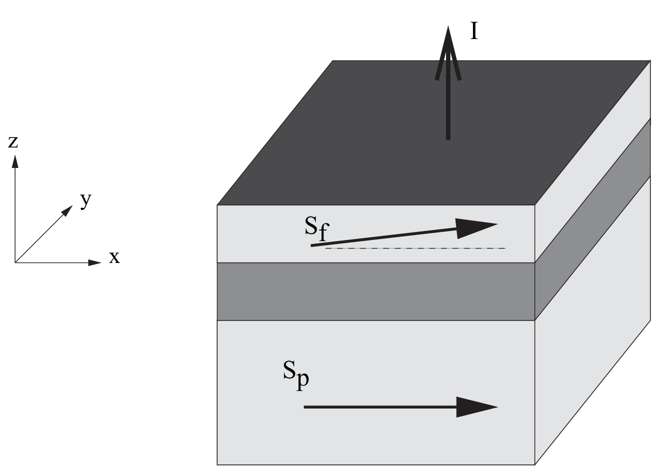

FIG. 3. Trilayer geometry of the spin valve used in this section. The bottom magnetic layer is supposed to be very thick and is pinned along the x axis. The top layer is also magnetic but free. Both layers are separated by a thin 0.8 nm normal conductor and traversed by an ac and a dc current as shown.

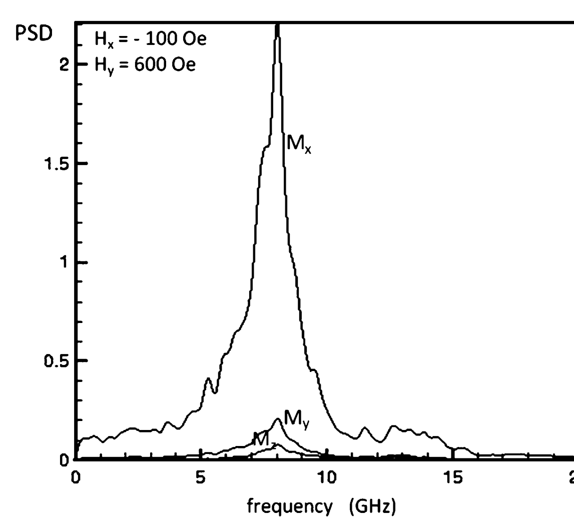

FIG. 4. Spectral densities (in arbitrary units) at zero current of the different components of the magnetization. The FMR peak of the system is around 7.0 GHz in the absence of a spin torque.

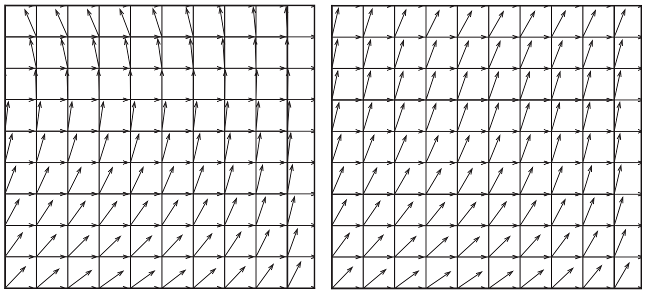

FIG. 5. Two metastable states exhibited by the spin-valve system at room temperature: a C state and an S state. The horizontal arrows are those of the magnetization of the bottom layer, which points along the x axis.

The effective field for the spin torque is

$$\mathbf{H}_{\rm SMT} = -\eta \frac{1}{d} \frac{\frac{J}{|e|}\frac{\hbar}{2}}{4\pi M_s^2} \mathbf{M} \times \mathbf{u}_p, \tag{38}$$

where η is the degree of polarization of the current taken to be equal to 0.5, J is the current density, d is the thickness of the free layer, and e is the charge of the electron. $\mathbf{u_p}$ is directed along the pinned magnetization, which is now directed along the x axis. The physical parameters we use are those for permalloy NiFe. The magnetization has magnitude $M_s = 1400$ emu/cc, with anisotropy $H_K = 50$ Oe, and an external bias field of 600 Oe applied along the y direction. The current traversing the spin valve has a dc part and ac part: $I_s = J/V = I + i\,\sin\Omega t$. The last term in the LL equation is the dissipation term, where $\alpha = 0.02$. By the fluctuation dissipation theorem [13], the random field ξ satisfies the correlation functions

$$\langle \xi_i(t)\rangle = 0, \quad \langle \xi_i(t)\xi_j(t')\rangle = 2\frac{\alpha k_B T}{\gamma M_s V}\delta_{ij}\delta(t-t') \tag{39}$$

in the white-noise approximation.

The magnetic system we study in this section cannot be reduced to a one-dimensional problem, has strong multiplicative noise, and does not clearly exhibit stochastic resonance as in the simple model presented above with weak spin torques. Instead, we will show that strong periodic spin torques can selectively suppress frequencies from the noise spectrum and enhance the ferromagnetic resonance (FMR) peak.

B. Results

Now we present and discuss the results of the solution of the nonlinear Landau-Lifshitz equation in the spin-valve model at $T = 373$ K and in the presence of ac currents. This system shows effects similar to those present in the system studied in Sec. II, but only in the lower part of the spectrum that is associated with telegraph noise or switching between two states.

Figure 4 shows the power spectral densities (PSD) of the three components of the magnetization in the absence of current. The PSD's have been normalized the same way in all the figures. The noise in the z component is smallest due to the action of the demagnetization field, which keeps the magnetization in-plane. The major source of noise in the system is due to thermal fluctuations that activate switching between two states and hence it is intrinsic to the system. To get a complex energy surface with more than one local minimum, the spin valve is being biased with an external field that is close to being perpendicular to the fixed magnetization of the RL and takes into account spin momentum transfer effects [11] between the two layers of the spin valve. The y component of the external field is kept at 600 Oe, while an additional one along $\mathbf{S}_p$ is kept around -100 Oe. The layers have dimensions $100 \times 100 \times 3$ nm^3, with the easy axis along the x axis and anisotropy $H_k = 50$ Oe. The magnetization in the pinned layer is fixed by a large bias field and is considered static. The magnetization in this structure shows two configurations (Fig. 5) that are nonhomogeneous and a result of the simultaneous action of the demagnetization field, the current field, the interaction between the layers, and the geometry of the structure [28]. Spin-valve structures are useful components of giant magnetoresistance (GMR) sensor devices, which often operate in the MHz regime usually below the FMR peak. Hence the manifestations of stochastic resonance in these systems may be of practical importance even if they are exhibited in only part of the spectrum. To increase the sensitivity of a GMR sensor, the bias field on the free layer is almost made perpendicular to the magnetization of the pinned layer. Unfortunately, it is this kind of biasing that gives rise to the "$1/f$"-type noise studied here, since it permits the system to hop between two states. The results reported here are valid even if the pinning of the bottom layer is not perfect. In fact, large bias fields tend to "distort" the magnetization

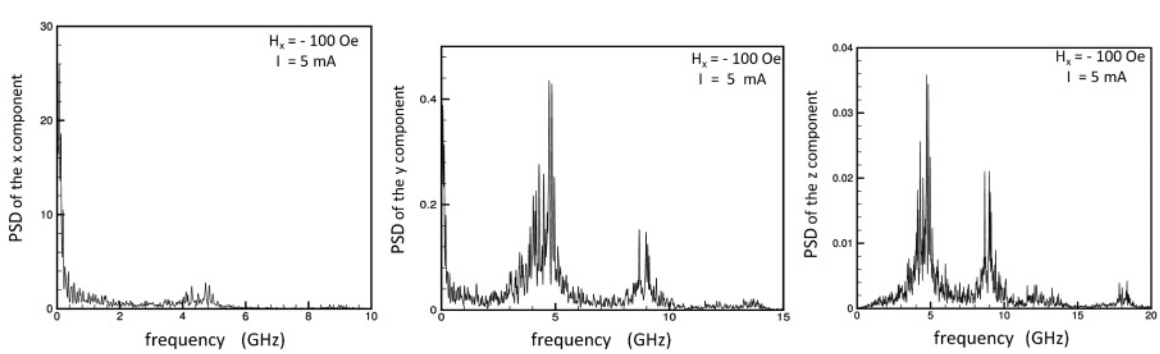

FIG. 6. Noise spectrum in the components of the magnetization of the FL in the absence of ac current: (1) x component; (2) y component; (3) z component. The x component of the magnetization shows significant low-frequency noise compared to the other components. The major two peaks in the y and z components are due to the inhomogeneous two states in the system.

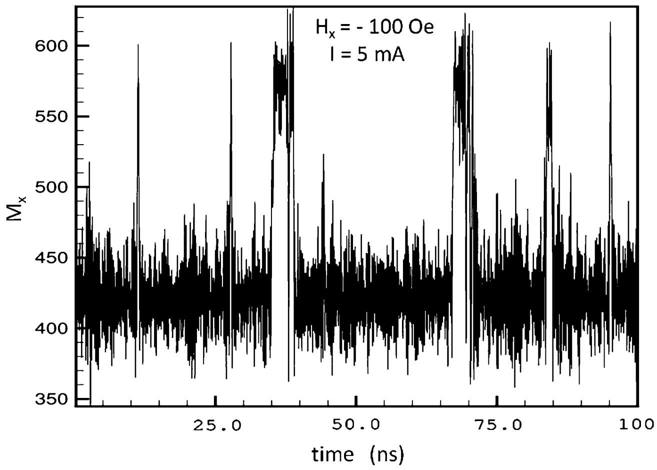

FIG. 7. x component of the magnetization as a function of time. The behavior of the average x component indicates that the magnetization is switching between two states—one stable and the other unstable due to thermal fluctuations.

in the pinned layer, but only slightly and do not contribute to the "1/f"-type noise observed here. Hence a solution where we can keep the bias field and get rid of the interwell transitions is needed.

One possible solution to this problem is to use ac spin torques, as in the previous section. Therefore, we expect the ac spin torque to affect the noise spectrum in a nontrivial way. In the presence of a spin torque with constant current $I =$ 5 mA, the FMR frequency for this system is shifted to around 4.5 GHz. In magnetic recording, e.g., such devices operate at frequencies below the FMR frequency and hence this system is considered too noisy to be used as a sensor. One possible way to address this problem is to seek a way to suppress the noise for frequencies less than 1.0 GHz and maybe shift the noise to higher frequencies that are outside the operational range of the device. In this sense, we are selectively suppressing the noise below the FMR frequency only. Previous applications of stochastic resonance were interested in suppressing the noise around the frequency of the driving force, which is usually the signal to be measured, as we did in the previous section.

From the results in Fig. 6, we observe that it is the x component of the magnetization of the FL that is the noisiest and hence would interfere with a possible GMR signal. The out-of-plane z component of the magnetization is very quiet due to the demagnetization field. The large peak close to $f = 0$ is due to the switching between the two C and S states of this system, Fig. 5, and is the major source for the noise in this structure. It also falls within the operating bandwidth of any possible sensor based on this structure. Our aim is therefore to suppress this component of the noise. Figure 7 confirms that the source of the noise is from the switching of the magnetization between two configurations and it is of telegraphic nature—one with an average x component of about 430 emu/cc and the other less stable one with an average x component around 580 emu/cc. The spectral noise in the y and z components shows more than the usual FMR peak, since the system is not in a ground state. The higher-order harmonics are due to nonhomogeneity of the magnetization, i.e., spin-wave excitations.

In Fig. 8, we change the sign of the current to show the effect of the spin torque on the magnetization and noise. In this case, the magnetization spends almost equal time in both states. This suggests the use of the time-dependent spin torque as a regulator of the transition rate between the two bistable states, as was done in our simpler system in Sec. II. Unfortunately, in this case, a low-frequency spin torque does not alter the shape of the power spectrum in a significant way. Instead, we want to explore the idea of adding a strongly periodic current in addition to the dc bias current in order to suppress the telegraph noise associated with the $f = 0$ peak.

Figure 9 shows the result of applying an ac current with amplitude equal to the bias current, $i = I$, and a frequency $f = 8$ GHz for the same system of Fig. 6. The noise spectrum for frequencies below 1 GHz is greatly suppressed by this additional current source in comparison to the FMR peak of

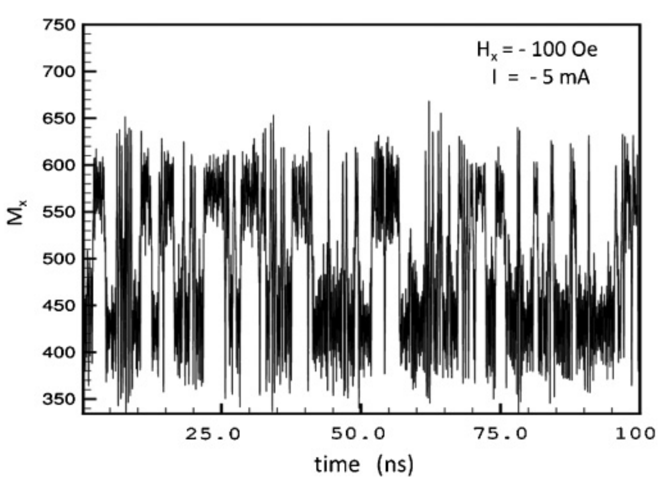

FIG. 8. x component of the magnetization as a function of time for opposite sign of the dc current in Fig. 7. The spin torque appears in this case to change the topology of the energy surface to one where both states are equally visited by the magnetization.

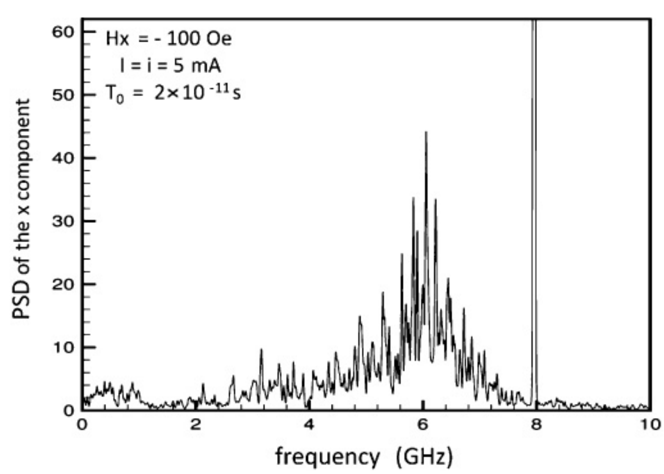

FIG. 9. Noise spectrum in the x component of the magnetization of the FL in the presence of an ac component to the current with same the amplitude as the dc part. The low-frequency part of the spectrum has been completely suppressed by the addition of the ac current. The peak at $f = 8$ GHz is that of the ac current and $T_0 = 1/2\pi f$. I and i are the magnitudes of the dc and ac part of the current, respectively.

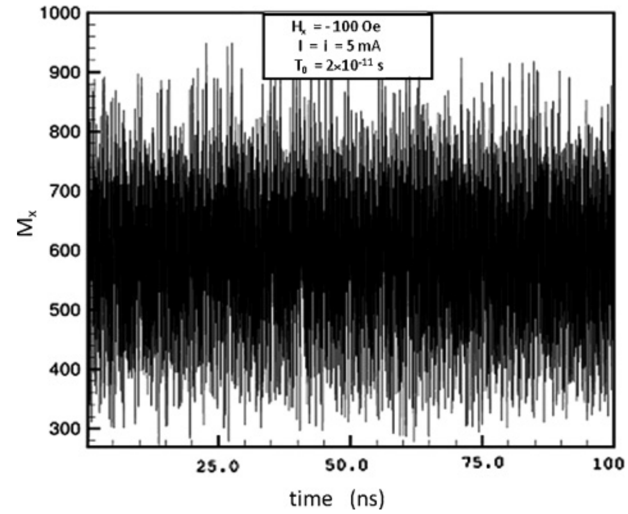

FIG. 10. Noise spectrum in the x component as a function of time in the presence of the ac current. The energy surface in this case appears to have only one stable minimum and no switching is observed. The noise in the x component has been pushed to high frequencies by the ac current with $T_0 = 1/\Omega$. I and i are the magnitudes of the dc part and ac part of the current, respectively.

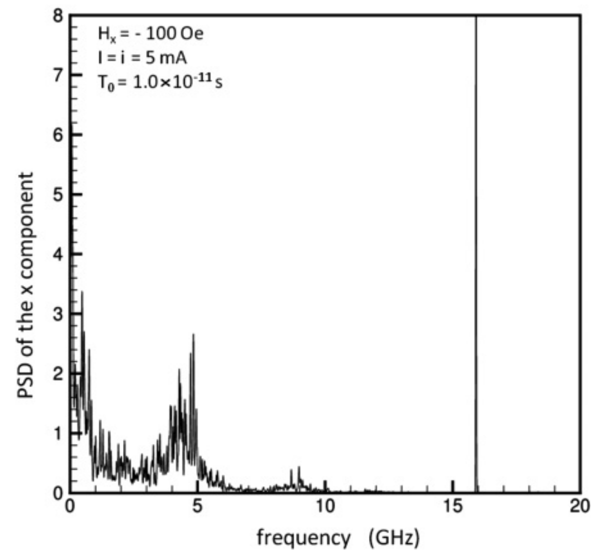

FIG. 11. Noise spectrum in the x component of the magnetization of the FL. The frequency of the ac current has been doubled compared to Fig. 9.

the system. The peak around the FMR frequency is now much more pronounced and so is its width. Hence, effectively, the dissipation in the system has been increased by the periodic spin torque. The second narrow peak is that of the ac current or signal. It is also interesting to observe that the FMR frequency of the system is now closer to the original FMR at zero currents. A closer look at the real-time behavior of the x component of the magnetization, Fig. 10, shows that the strong periodic current is making the system less susceptible to random switching induced by thermal fluctuations, which is the source of the telegraph noise. To be an effective suppressant, the frequency of the torque has to be outside the bandwidth of the sensor device, which is what we observed in the simulations. Since it is not feasible in our case to have enough simulations to plot a figure as in Fig. 2 to identify any SR behavior in the system, we can instead use the residence time behavior of a system to detect SR-type behavior [2]. It is therefore very clear from our results that the suppression of the $1/f$ behavior implies less random transitions between the initial C and S states.

The last figure, Fig. 11, shows that an increase in the frequency of the ac current degrades the effectiveness of the ac current component to suppress the low-frequency noise. This appears to be the case also in systems that show SR behavior [2]. At high frequencies, the SNR curve ceases to have a peak at nonzero temperature. In our system, too, we found that, for current amplitudes at 5 mA, the maximum frequency the current should have is about twice the FMR frequency of the system. This latter criterion seems to depend strongly on the energy surface and the strength of the driving torque, but more study is needed to determine the frequency dependence.

Finally, we make a few comments regarding the strength of the current. From our calculation in Eq. (35), it is clear that the stronger the ac component of the spin torque, the bigger the SNR and hence the more effective the ac component to suppress the noise in relation to the signal. For example, for $i = 1$ mA, we did not find a substantial influence on the relative size of the $1/f$-type noise component to that of the natural response of the system. This is partly confirmed by the work of Pankratov [6], who also studied the effect of using strong periodic forces in a simpler system. He showed that, in the highly nonlinear regime, SR will not disappear and may in fact be enhanced, but becomes frequency dependent.

IV. CONCLUSION

In summary, we have shown that the ideas of SR are also useful in magnetic systems using nonconservative torques. We have studied two systems of current practical interest due to their potential applications in information storage. In Sec. II, we have shown analytically that a system of a uniaxial magnetic nanoparticle driven by a periodic spin torque exhibits stochastic resonance. To derive this result, we have used the Fokker-Planck equation in the adiabatic limit. Our result points to a possible solution to suppress the telegraph noise observed lately in similar systems [10]. In Sec. III, we studied a more complex energy surface with telegraph noise and also demonstrated numerically that strong periodic torques are able to suppress the noise associated with the telegraph noise. Here we had to resort to a numerical solution of the stochastic Landau-Lifshitz equation. This study also suggests that telegraph noise due to pinning and unpinning of domain walls can be addressed by periodic spin torques.

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[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981).
[2] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[3] I. Dayan, M. Gitterman, and G. H. Weiss, Phys. Rev. A **46**, 757 (1992).
[4] M. Ishihara, e-print arXiv:cond-mat/0508758.
[5] A. Rebei, M. Simionato, and G. J. Parker, Phys. Rev. B **69**, 134412 (2004).
[6] A. L. Pankratov, Phys. Rev. E **65**, 022101 (2002).
[7] V. Berdichevsky and M. Gitterman, Europhys. Lett. **36**, 161 (1996).
[8] L. Gammaitoni, F. Marchesoni, E. Menichella Saetta, and S. Santucci, Phys. Rev. E **49**, 4878 (1994).
[9] Y. Jia, S.-N. Yu, and J.-R. Li, Phys. Rev. E **62**, 1869 (2000).
[10] J. Cucchiara *et al.*, Appl. Phys. Lett. **94**, 102503 (2009).
[11] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
[12] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[13] W. F. Brown, Jr., Phys. Rev. **130**, 1677 (1963).
[14] C. W. Gardiner, J. Stat. Phys. **30**, 157 (1983).
[15] B. McNamara and K. Wiesenfeld, Phys. Rev. A **39**, 4854 (1989).
[16] R. Mantegna, B. Spagnolo, L. Testa, and M. Trapanese, J. Appl. Phys. **97**, 10E519 (2005).
[17] R. Zhu and P. B. Visscher, J. Appl. Phys. **103**, 07A722 (2008).
[18] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).
[19] H. Risken, *The Fokker-Planck Equation: Methods of Solutions and Applications* (Springer-Verlag, Berlin, Heidelberg, 1996).
[20] H. A. Kramers, Physica **7**, 284 (1940).
[21] B. Caroli, C. Caroli, and B. Roulet, J. Stat. Phys. **28**, 757 (1982).
[22] B. Caroli, C. Caroli, B. Roulet, and D. Saint-James, Physica A **108**, 233 (1981).
[23] Z. Li and S. Zhang, Phys. Rev. B **69**, 134416 (2004).
[24] J. Foros, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Phys. Rev. Lett. **95**, 016601 (2005).
[25] Z.-G. Zhu, G. Su, Q.-R. Zheng, and B. Jin, Phys. Rev. B **68**, 224413 (2003).
[26] S. Urazhdin, N. O. Birge, W. P. Pratt, Jr., and J. Bass, Phys. Rev. Lett. **91**, 146803 (2003).
[27] W. F. Brown, Jr., *Magnetostatic Principles in Ferromagnetism* (North-Holland, New York, 1962).
[28] A. Rebei, L. Berger, R. Chantrell, and M. Covington, J. Appl. Phys. **97**, 10E306 (2005).
[29] A. Rebei and O. Mryasov, Phys. Rev. B **74**, 014412 (2006).