Externally driven transmission and collisions of domain walls in ferromagnetic wires

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Analytical multidomain solutions to the dynamical (Landau-Lifshitz-Gilbert) equation of a one-dimensional ferromagnet including an external magnetic field and spin-polarized electric current are found using the Hirota bilinearization method. A standard approach to solve the Landau-Lifshitz equation (without the Gilbert term) is modified in order to treat the dissipative dynamics. I establish the relations between the spin interaction parameters (the constants of exchange, anisotropy, dissipation, external-field intensity, and electric-current intensity) and the domain-wall parameters (width and velocity) and compare them to the results of the Walker approximation and micromagnetic simulations. The domain-wall motion driven by a longitudinal external field is analyzed with especial relevance to the field-induced collision of two domain walls. I determine the result of such a collision (which is found to be an elastic one) on the domain-wall parameters below and above the Walker breakdown (in weak- and strong-field regimes). Single-domain-wall dynamics in the presence of an external transverse field is studied with relevance to the challenge of increasing the domain-wall velocity below the breakdown.

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I. INTRODUCTION

Description of the magnetic-field- and electric-current-induced motions of domain walls (DWs) in nanowires has became a hot topic because of novel methods of storing and switching the (magnetically encoded) binary information. These proposals offer progress in the miniaturization of memory and logic elements, utilizing crucial advantages of magnetic information encoding (when a bit is identified with a single magnetic domain). Such information is insensitive to the voltage fluctuations while its maintenance does not cost any energy, which enables data processing with the production of a small amount of heat. Currently investigated random-access memories are built of metallic nanowires, formed into a parallel-column structure, which store magnetic domains separated by DWs. Such a three-dimensional (3D) magnetic system has the potential of storing more information than devices based on 2D systems, like hard-disk drives or electronic memories, in a given volume [1,2]. Also, an interesting concept of logical operation via transmitting magnetic DWs through nanowires of specific geometries is being developed [3,4]. I mention that ferroelectric nanosystems offer similar capabilities while their basic properties are studied with the same dynamical (Landau-Lifshitz-Gilbert) equation even though the effects of electroelastic coupling are strong [5,6].

In order to write and switch information, one can move the DWs via the application of an external magnetic field (parallel to the easy axis) or via the application of a voltage which induces a spin-polarized electric current through the DW. The directions of the field-driven motion are different for the tail-to-tail and head-to-head DWs; thus the magnetic field induces DW collisions, while the direction of the voltage-driven motion uniquely corresponds to the current direction. Field-driven motion and current-driven (below the Walker breakdown) motion are possible due to the magnetic dissipation, and their description demands inclusion of the Gilbert term into the Landau-Lifshitz (LL) equation. However, existing many-domain analytical solutions to the LL equation do not include the dissipation [7,8], while approximate solutions using the Walker ansatz describe a single DW only [9]. Since the parameters of the DW solutions to the Landau-Lifshitz-Gilbert (LLG) equation determine accessible values of technological characteristics (e.g., the minimal domain length, the bit-switching time, etc.), it is of interest to know the analytic DW solution to the LLG equation. Knowledge of the many-domain solution is of importance for preventing unwanted DW collisions, which can result in an instability of the record, and it enables verification of DW-collision simulations regardless of the internal structure or the geometry of the simulated system.

In the present paper, I perform an analytical study of the dynamics of multidomain systems including the dissipation. The dynamical LLG system is bilinearized following the Hirota method of solving nonlinear equations and it is extended, via doubling the number of freedom degrees and the number of equations, into a time-reversal-invariant form. The field solving the extended system contains proper and virtual (unphysical) dynamical variables and the physical components of the solution are shown to satisfy the pimary LLG system in a relevant time regime. In particular, aiming to analytically describe the field-induced collision, I establish asymptotic three-domain magnetization profiles (relevant in the time limits $t \to \pm\infty$). With connection to the phenomenon of the Walker breakdown, (a cusp in the dependence of the DW velocity on the external field and in its dependence on the current intensity), I modify the LLG model in a way to make it applicable below the breakdown. I reduce it to a model of plane rotators. In this weak-field regime, the spin alignment in the DW area is saturated in a plane while, above the breakdown (the strong-field regime), the spins rotate about the magnetic-field axis (the easy axis). These considerations supplement micromagnetic simulations of the DW collisions in terms of the studied DW-parameter regimes [10]. Within the present method, I verify the Walker-ansatz predictions on the current-driven DW motion above and below the breakdown (including adiabatic and nonadiabatic parts of the spin-transfer torque) [11–15], thus showing the applicability of the present formalism to the field-driven motion of multi-domain systems

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with an additional voltage applied. With relevance to the challenge of controlling the maximum DW velocity below the Walker breakdown, we analyze the longitudinal-field- and current-driven motion of the DWs in the presence of an additional perpendicular (with respect to the easy axis) field.

A complementary study of the DW collision of magnetic DWs in the subcritical regime is performed in a separate paper [16].

In Sec. II, I extend the dissipative equations of motion of the ferromagnet in such a way as to make equations applicable to the unlimited range of time $t \in (-\infty,\infty)$. Section III is devoted to the analysis of its single-and double-DW solutions in the presence of a magnetic field and electric current. I study the field-induced collision in detail. The plane-rotator approach to the DW dynamics is described in Sec. IV. In Sec. V, consequences of the application of an external field perpendicular to the easy axis for the DW statics and dynamics are considered. Conclusions are given in Sec. VI.

II. DYNAMICAL EQUATIONS

The dynamics of the magnetization vector $\mathbf{m}$ ($|\mathbf{m}| = M$) in the one-dimensional ferromagnet is described with the LLG equation

$$\begin{aligned}\frac{\partial \mathbf{m}}{\partial t} &= \frac{J}{M}\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2} + \gamma \mathbf{m} \times \mathbf{H} + \frac{\beta_1}{M}(\mathbf{m}\cdot\hat{\imath})\mathbf{m}\times\hat{\imath} \\ &\quad -\frac{\beta_2}{M}(\mathbf{m}\cdot\hat{\jmath})\mathbf{m}\times\hat{\jmath} - \delta\frac{\partial \mathbf{m}}{\partial x} - \frac{\delta\beta}{M}\mathbf{m}\times\frac{\partial \mathbf{m}}{\partial x} \\ &\quad -\frac{\alpha}{M}\mathbf{m}\times\frac{\partial \mathbf{m}}{\partial t}. \end{aligned} \tag{1}$$

The first term of the right-hand side (RHS) of (1) relates to the exchange spin interaction while the second term depends on the external magnetic field $\mathbf{H}$; thus, γ denotes the giromagnetic factor (up to its sign). The constant $\beta_{1(2)}$ determines the strength of the easy axis (plane) anisotropy and $\hat{\imath} \equiv (1,0,0)$, $\hat{\jmath} \equiv (0,1,0)$. Note that the long axis of the magnetic nanowire is an easy axis for the majority of real systems; however, another choice of the anisotropy axes does not influence the magnetization dynamics. The constant δ is proportional to the intensity of the electric current through the wire and δ changes its sign under time-arrow reversal (the inversion of the electron flow) [17,18]. The nonadiabatic part of the current-induced torque (which depends on β) is of dissipative origin; thus, $\beta \to 0$ with decreasing Gilbert damping constant $\alpha \to 0$ (one takes $\alpha,\beta \ll 1$). Its inclusion is necessary if one describes an observed monotonic motion of the DW below the Walker breakdown [11–14]. Notice that including the magnetic-dissipation term following the original LL approach (changing the last term of (1) into $-\alpha\mathbf{m}\times[\mathbf{m}\times\mathbf{h}_{\text{eff}}]$, where $h_{\text{eff}\,j} = -\delta\mathcal{H}/\delta m_j$, and $\mathcal{H}$ denotes the Hamiltonian) would lead to changing the constant β into $\beta - \alpha$ [13]. Although the discussion of the relevance of both approaches to the magnetic dissipation remains open [13,19], I believe the clinching argument for the Gilbert approach is the expectation for the proper dissipative term to be dependent on the time derivative of the dynamical parameter $\mathbf{m}$. In the other case, the dissipative term could influence static solutions to the LL equation while one expects the magnetic friction to be kinetic.

Since (1) is valid only when the constraint $|\mathbf{m}| = M$ is satisfied, I intend to write equations of the unconstrained dynamics equivalent to (1). Introducing the complex dynamical parameters $m_\pm = m_y \pm i m_z$, I represent the magnetization components using a pair of complex functions $g(x,t), f(x,t)$. This way I reduce the number of independent degrees of freedom. The relation between the primary and secondary dynamical variables

$$m_+ = \frac{2M}{f^*/g + g^*/f}, \quad m_x = M\frac{f^*/g - g^*/f}{f^*/g + g^*/f} \tag{2}$$

[where $(\cdot)^*$ denotes the complex conjugate (c.c.)] ensures that $|\mathbf{m}| = M$ while there are no constraints on g and f. The transform (2) enables bilinearization ("trilinearization") of (1) following the Hirota method of solving nonlinear equations [7,8]. In the particular case $H_y = H_z = 0$, from (1) and (2), we arrive at the trilinear equations for f and g:

$$\begin{aligned} &f[-iD_t + JD_x^2 + \delta(\beta - i)D_x + \alpha D_t]f^*\cdot g + Jg^* D_x^2 g\cdot g - \left(\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)|f|^2 g - \frac{\beta_2}{2}f^{*2}g^* = 0, \\ &g^*[-iD_t - JD_x^2 + \delta(\beta - i)D_x + \alpha D_t]f^*\cdot g - JfD_x^2 f^*\cdot f^* + \left(-\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)|g|^2 f^* + \frac{\beta_2}{2}g^2 f = 0, \end{aligned} \tag{3}$$

where D_t, D_x denote Hirota operators of differentiation which are defined by

$$D_t^m D_x^n b(x,t)\cdot c(x,t) \equiv (\partial/\partial t - \partial/\partial t')^m(\partial/\partial x - \partial/\partial x')^n b(x,t)c(x',t')|_{x=x',t=t'}.$$

The inclusion of the dissipation into the LLG equation is connected to breaking the symmetry with respect to time reversal. Therefore, neither (1) nor (3) can describe the magnetization evolution on the whole time axis. In particular, the application of an external magnetic field or current to the DW system (or creation of a domain in the presence of an external field) initiates a nonequilibrium process of the DW motion. Such a motion cannot be present in the distant past ($t \to -\infty$) since a nonzero value of the dissipative function [relevant to the Gilbert term in (1)] would indicate unlimited growth of the energy as $t \to -\infty$. Thus, *solitary-wave solutions to* (1) *are relevant only in the limit of large positive values of time* [11,13]. *This fact makes impossible an exact analysis of the DW collisions using* (3) and motivates extension of the dynamical system within a formalism applicable to the whole length of the time axis.

I write modified equations of motion using a similar trick to the one proposed by Bateman with application to the Lagrangian description of the damped harmonic oscillator [20]. It is connected to the concept by Lakshmanan and Nakamura of removing the dissipative term from the evolution equation of ferromagnets via multiplying the time variable by a complex constant [21]; however, it demands an improvement in the spirit of Bateman's idea [22]. The concept is to extend the dynamical system, doubling the number of degrees of freedom and adding equations which differ from the original ones by the sign of the dissipation constants. The resulting extended system is symmetric with respect to the time-arrow reversal; however, its solution consists of physical and virtual fields. Let me mention that different quantum dissipative formalisms (nonequilibrium Green functions, thermofield dynamics, rigged Hilbert space) are based on Bateman's trick [23].

I extend the system of secondary dynamical equations (3) since it describes unconstrained dynamics unlike the primary LLG equation (1). We replace g, g^*, f, and f^* in (3) with fields of the corresponding set g_1, g_2^*, f_2, and f_1^* and of the set of their c.c.s. For $\alpha,\beta = 0$, in the absence of dissipation, $g_1 = g_2 = g$, $f_1 = f_2 = f$. For the case $H_y = H_z = 0$, the secondary dynamical equations transform into

$$\begin{aligned} &f_2\left[-iD_t + JD_x^2 + \delta(\beta - i)D_x + \alpha D_t\right]f_1^*\cdot g_1 + Jg_2^* D_x^2 g_1\cdot g_1 - \left(\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)f_2 f_1^* g_1 - \frac{\beta_2}{2} f_2^* f_1^* g_1^* = 0,\\ &g_2^*\left[-iD_t - JD_x^2 + \delta(\beta - i)D_x + \alpha D_t\right]f_1^*\cdot g_1 - Jf_2 D_x^2 f_1^*\cdot f_1^* + \left(-\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)g_2^* g_1 f_1^* + \frac{\beta_2}{2} g_2 g_1 f_1 = 0. \end{aligned} \tag{4}$$

In writing (4), I have replaced the last terms on the LHS of (3) in such a way as to be linear in $g_2^{(*)}, f_2^{(*)}$, which ensures that they vanish (diverge) with time in the presence of $H_x \neq 0$ with similar damping (exploding) rates as all other terms of these equations (in particular, their damping does not modify the anisotropy). The additional equations of the dynamical system differ from (4) by the sign of the dissipation constants α,β:

$$\begin{aligned} &f_1\left[-iD_t + JD_x^2 + \delta(-\beta - i)D_x - \alpha D_t\right]f_2^*\cdot g_2 + Jg_1^* D_x^2 g_2\cdot g_2 - \left(\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)f_1 f_2^* g_2 - \frac{\beta_2}{2} f_1^* f_2^* g_2^* = 0,\\ &g_1^*\left[-iD_t - JD_x^2 + \delta(-\beta - i)D_x - \alpha D_t\right]f_2^*\cdot g_2 - Jf_1 D_x^2 f_2^*\cdot f_2^* + \left(-\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right)g_1^* g_2 f_2^* + \frac{\beta_2}{2} g_1 g_2 f_2 = 0. \end{aligned} \tag{5}$$

Though the previous dynamical variables $g(f)$ and $g^*(f^*)$ were mutually independent, they had to be c.c.s. of each other in order that the system of equations (3) and their c.c.s. was closed. In the system of eight equations (4) and (5) and their c.c.s. $g_1(f_1)$ is not a c.c. of $g_2(f_2)$, while, comparing (4) and (5), one sees that $g_2(x,t)$ $[f_2(x,t)]$ can be obtained from $g_1(x,t)$ $[f_1(x,t)]$ via changing the sign of its parameters α,β.

Under the time-arrow inversion, the system of equations obtained transforms into itself if one accompanies this operation by the transform of the dynamical variables $g_{1(2)} \to f_{2(1)}$, $f_{1(2)} \to -g_{2(1)}$. The equations (4) and their c.c.s, which determine the magnetization dynamics for large positive values of time (in particular, for $t \to \infty$), contain the differentials of the functions g_1, g_1^*, f_1, and f_1^*. Therefore, the magnetization vector should be expressed with these functions in the relevant time regime. Writing the magnetization in the form

$$m_+ = \frac{2M}{f_1^*/g_1 + g_1^*/f_1},\quad m_x = M\frac{f_1^*/g_1 - g_1^*/f_1}{f_1^*/g_1 + g_1^*/f_1} \tag{6}$$

ensures that their components satisfy $|\mathbf{m}| = M, m_x = m_x^*$, and they reproduce (2) for $\alpha = \beta = 0$. In the regime of large negative values of time, in particular, for $t \to -\infty$, we can analyze the evolution of the magnetization with the reversed time arrow. It is described with the reversed magnetization vector

$$\tilde{m}_+ = -\frac{2M}{f_2^*/g_2 + g_2^*/f_2},\quad \tilde{m}_x = -M\frac{f_2^*/g_2 - g_2^*/f_2}{f_2^*/g_2 + g_2^*/f_2}. \tag{7}$$

III. DOMAIN-WALL MOTION

Let us analyze the multidomain solutions to (4) and (5) in the absence of any external magnetic field, $\mathbf{H} = \mathbf{0}$. We search for solutions which describe a single DW and two DWs in the forms $f_1^* = 1$, $g_1 = w_1 e^{k_1 x - l_1 t}$, and $f_1^* = 1 + v^* e^{k_1 x - l_1 t} e^{k_2 x - l_2 t}$, $g_1 = w_1 e^{k_1 x - l_1 t} + w_2 e^{k_2 x - l_2 t}$, respectively, where $k_j = \mathrm{Re}(k_j)$, and $\mathrm{sgn}(k_1) = -\mathrm{sgn}(k_2)$. Inserting these *Ansätze* into (4) and (5), one finds

$$\begin{aligned} l_j &= \frac{1}{1+i\alpha}\left\{-\sqrt{-\left(Jk_j^2 - \beta_1\right)\left[Jk_j^2 - (\beta_1 + \beta_2)\right]}\right.\\ &\quad \left. + \delta(1 + i\beta)k_j\right\},\\ k_j &= \sqrt{\frac{\beta_1 + \beta_2(w_j + w_j^*)^2/(4|w_j|^2)}{J}}. \end{aligned} \tag{8}$$

The single-wall (two-domain) solutions with $|k_1| \in (\sqrt{\beta_1/J}, \sqrt{(\beta_1+\beta_2)/J})$ describe moving solitary waves (topological solitons), [7,24]. One sees the correspondence between the wall width and the spin deviation from the easy plane in the dependence of k_j on w_j. When $\delta = 0$, static two-domain solutions represent the Bloch DW ($|k_1| = \sqrt{\beta_1/J}$, $w_1 = -w_1^*$) or Néel DW ($|k_1| = \sqrt{(\beta_1+\beta_2)/J}$, $w_1 = w_1^*$), respectively (the nomenclature of [24] which differs from Neel and Bloch wall definition of e.g. [26]). These two solutions correspond to the ones found in [25,27] within the XY model. The DW profiles can be described with the functions

$$m_+(x,t) = M\frac{w_1 e^{-i\mathrm{Im}l_1 t}}{|w_1|}\mathrm{sech}[k_1 x - \mathrm{Re}l_1 t + \ln|w_1|],$$
$$m_x(x,t) = -M\tanh[k_1 x - \mathrm{Re}l_1 t + \ln|w_1|] \tag{9}$$

($k_1 > 0$ relates to the head-to-head structure and $k_1 < 0$ to the tail-to-tail structure). At the time points of the discrete set $t = \pi n/\mathrm{Im}l_1$, $n = 0, \pm 1, \pm 2, \ldots$, one finds $g_1 = g_2 = g$, $f_1 = f_2 = f$ and (4) coincides with (3). Therefore, (9) is a solitary-wave solution to (1) (in particular, it coincides with the one representing a Bloch or a Néel DW for $\delta \neq 0$ [12]). Throughout the paper, I focus my attention on the externally driven dynamics of the Bloch and Néel DWs, since these initially static structures are the most important with relevance to magnetic data storage.

I establish that static double-wall (three-domain) solutions to the LLG equation cannot be written with the above Hirota expansion when $k_1 = -k_2$. In this case the coefficient v,

$$v = -\frac{\beta_2 J k_1^2 w_1^* w_2^*}{(Jk_1^2-\beta_1)(Jk_1^2-\beta_1-\beta_2)}, \tag{10}$$

diverges with $|k_1| \to \sqrt{\beta_1/J}$ or $|k_1| \to \sqrt{(\beta_1+\beta_2)/J}$. Analogously to the XY model, the Hirota expansion is inapplicable to static three-domain configurations of the Bloch or Néel walls, while there exists a static solution to (4) and (5) which describes a pair of different-type (Néel and Bloch) walls [28]. In particular, for $k_1 = \sqrt{\beta_1/J}$, $k_2 = -\sqrt{(\beta_1+\beta_2)/J}$, and $w_1 = -w_1^*$, $w_2 = w_2^*$, one finds

$$v = -\frac{\beta_2 w_1 w_2}{2\beta_1+\beta_2-2\sqrt{\beta_1(\beta_1+\beta_2)}}. \tag{11}$$

Let me emphasize that I have not excluded the coexistence of a pair of Néel or Bloch walls in a magnetic wire. However, the overlap of both the topological solitons induces their interaction which leads to an instability of their parameters and, unlike for nontopological solitons, is not a temporal one [29].

Solving (4) and (5) in the presence of a longitudinal magnetic field $H_x \neq 0$, we apply the *Ansatz*

$$f_1^* = (1 + v^* e^{k_1 x - l_1 t} e^{k_2 x - l_2 t}) e^{\gamma H_x t/(-2i+2\alpha)},$$
$$g_1 = (w_1 e^{k_1 x - l_1 t} + w_2 e^{k_2 x - l_2 t}) e^{-\gamma H_x t/(-2i+2\alpha)} \tag{12}$$

at the discrete time points $t = t_n \equiv 4\pi n(1+\alpha^2)/(\gamma H_x)$, where $n = 0, \pm 1, \pm 2 \ldots$ (let $\delta = 0$ for simplicity). Behind these time points, in the presence of the longitudinal field, the last terms on the LHS of (4) and (5) change faster (they oscillate with a three times higher frequency) than the other ones. Therefore, taking the above *Ansatz*, we apply an approach similar to the rotating wave approximation in quantum optics. Since this *Ansatz* describes the spin structure rotation about the x axis, it is applicable when the external field exceeds the Walker-breakdown critical value, $|H_x| > H_W$. From the single-wall solution (the case of $w_2 = 0$ or $w_1 = 0$) for $k_1 = \sqrt{\beta_1/J}$, $k_2 = -\sqrt{(\beta_1+\beta_2)/J}$, I establish that applying the magnetic field in the easy-axis direction drives the DW motion with the velocity

$$c_{1(2)} = \gamma |H_x| \alpha/[|k_{1(2)}|(1+\alpha^2)]. \tag{13}$$

Correspondingly, for $H_x = 0$, applying the electric current through the initially static wall drives it to move with the velocity

$$c = \frac{\delta(1+\alpha\beta)}{1+\alpha^2} \tag{14}$$

which is independent of the DW width.

The essential difference between the two kinds of driven motion emerges from the analysis of the three-domain solutions. Under the external field, the two consecutive DWs move in opposite directions. The walls which are closing up to each other collide and eventually they can annihilate or wander away from each other. The application of an electric current along the magnetic wire drives both the DWs to move in the same direction with the same velocity. Analyzing the long-time limits of the magnetization vector in different regions of the coordinate x, I establish the consequences of the field-induced collision of the complex of a Bloch DW interacting with a Néel DW. We use the *Ansatz* (12) and assume $\delta = 0$.

Let $\eta_j \equiv k_j(x - x_{0j}) - \gamma H_x \alpha t/(1+\alpha^2)$, $\tilde{\eta}_j \equiv k_j(x - x_{0j}) + \gamma H_x \alpha t/(1+\alpha^2)$. For $H_x > 0$, at $t = t_n$ (within the above rotating wave approximation), we find the distant-future limit of the magnetization (6):

$$m_+ \approx \begin{cases} m_+^{(1)}, & \eta_2 \ll \eta_1 \sim 0, \\ m_+^{(2)}, & \eta_1 \ll \eta_2 \sim 0, \end{cases} = \lim_{t\to\infty} m_+,$$
$$m_x \approx \begin{cases} m_x^{(1)}, & \eta_2 \ll \eta_1 \sim 0, \\ m_x^{(2)}, & \eta_1 \ll \eta_2 \sim 0, \end{cases} = \lim_{t\to\infty} m_x, \tag{15}$$

where

$$m_+^{(j)} = 2M\frac{v/w_j^* e^{\eta_k} e^{-i\gamma H_x t/(1+\alpha^2)}}{1+|v|^2/|w_j|^2 e^{2\eta_k}},$$
$$m_x^{(j)} = -M\frac{1-|v|^2/|w_j|^2 e^{2\eta_k}}{1+|v|^2/|w_j|^2 e^{2\eta_k}}, \tag{16}$$

and $j \neq k$. Identifying the parameters x_{0j} with the DW-center positions, I introduce the restriction on w_j, $|v|/|w_j| = 1$. Note that $m_+^{(1)}, m_x^{(1)}$ as well as $m_+^{(2)}, m_x^{(2)}$ are the Walker single-DW solutions to the primary LLG equation, which describe the motion of well-separated DWs [9,30]. Thus, our three-domain profiles of the fields (6) tend to satisfy (1) in the limit $t \to \infty$ according to the requirement formulated in the previous section.

In the distant-past limit, I describe the magnetization evolution with the reversed time arrow. Following (7),

$$\tilde{m}_+ \approx \begin{cases} \tilde{m}_+^{(1)}, & \tilde{\eta}_1 \ll \tilde{\eta}_2 \sim 0, \\ \tilde{m}_+^{(2)}, & \tilde{\eta}_2 \ll \tilde{\eta}_1 \sim 0, \end{cases} = \lim_{t\to-\infty} \tilde{m}_+,$$
$$\tilde{m}_x \approx \begin{cases} \tilde{m}_x^{(1)}, & \tilde{\eta}_1 \ll \tilde{\eta}_2 \sim 0, \\ \tilde{m}_x^{(2)}, & \tilde{\eta}_2 \ll \tilde{\eta}_1 \sim 0, \end{cases} = \lim_{t\to-\infty} \tilde{m}_x, \tag{17}$$

where

$$\tilde{m}_+^{(j)} = -2M\frac{v/w_k^* e^{\eta_j} e^{-i\gamma H_x t/(1+\alpha^2)}}{1+|v|^2/|w_k|^2 e^{2\eta_j}},\quad \tilde{m}_x^{(j)} = M\frac{1-|v|^2/|w_k|^2 e^{2\eta_j}}{1+|v|^2/|w_k|^2 e^{2\eta_j}}, \tag{18}$$

and $j \neq k$. In order to consider the collision of a pair of DWs which are infinitely distant from each other at the beginning of their evolution, I determine the magnetization dynamics in the limit $t \to -\infty$. For this aim, one has to invert the propagation direction of the kinks of $\tilde{\mathbf{m}}$ and to reverse the arrowhead of the field vector $\tilde{\mathbf{m}}$. Utilizing the properties $\tilde{m}_+^{(j)}(x+x_{0k},0) = \tilde{m}_+^{(j)}(-x+x_{0k},0)$, $\tilde{m}_x^{(j)}(x+x_{0k},0) = -\tilde{m}_x^{(j)}(-x+x_{0k},0)$, I arrive at

$$m_+(x,t) = \begin{cases} -\tilde{m}_+^{(1)}(-x+2x_{01},t), & \eta_1 \gg \eta_2 \sim 0,\\ -\tilde{m}_+^{(2)}(-x+2x_{02},t), & \eta_2 \gg \eta_1 \sim 0,\end{cases}$$
$$m_x(x,t) = \begin{cases} \tilde{m}_x^{(1)}(-x+2x_{01},t), & \eta_1 \gg \eta_2 \sim 0,\\ \tilde{m}_x^{(2)}(-x+2x_{02},t), & \eta_2 \gg \eta_1 \sim 0.\end{cases} \tag{19}$$

The applicability of the above procedure to study the asymptotic evolution of a single DW is easy to verify since any single-DW solution satisfies

$$m_+(x,t) = -\tilde{m}_+(-x+2x_{01},t),\quad m_x(x,t) = -\tilde{m}_x(-x+2x_{01},t). \tag{20}$$

Typically, one should consider the formulas (15) and (19) with relevance to the case $\beta_1 \gg \beta_2$, and thus $k_1 \approx -k_2$, which corresponds to commonly studied crystalline magnetic nanowires, e.g., for Fe and FePt, $\beta_2/\beta_1 \sim 10^{-1}$ [31]. For noncrystalline (Permalloy) nanowires $Fe_{1-x}Ni_x$ deposited on a crystalline substrate, the easy-axis anisotropy constant determined from uniform-resonance measurements was found to be, unexpectedly, as big as in the crystalline nanowires [32]. Therefore, even when neglecting structural effects in real systems, which lead to the saturation of the spin alignment in the DW area to the easy or hard plane (the Walker breakdown), thus suppressing their spontaneous motion, the spectrum of spontaneously propagating DWs in nanowires will be very narrow and their velocities very small.

According to (15) and (19), two initially closing up DWs have to diverge after the collision. If one of the colliding DWs was initially, for $t \to -\infty$, described with the field ingredient $\tilde{m}_+^{(j)}, \tilde{m}_x^{(j)}$, it is finally, for $t \to \infty$, described with the field ingredient $m_+^{(j)}, m_x^{(j)}$. Therefore, reflecting DWs exchange their parameters $x_{01} \leftrightarrow x_{02}$, $w_1 \leftrightarrow w_2$, $k_1 \leftrightarrow -k_2$. This is connected to exchanging the directions of the spin orientation in the yz plane in the wall areas (after the collision the Néel wall changes it into the Bloch wall and vice versa as shown in Fig. 1). My prediction corresponds to the result of the collision analysis performed for spontaneously propagating DWs (in the absence of external field, electric current, and dissipation). According to findings of [24,33], the DWs reflect during the collision in such a way that one can say they pass through each other without changing their widths and velocities; however, they do change their character from the head-to-head to the tail-to-tail one, and vice versa.

Up to now, I have considered systems of infinite domains whose energy cannot be defined. However, the smaller a domain is the bigger percentage of the Zeeman part of its energy is lost per time unit due to the DW motion. The condition of domain-energy minimization determines the direction of this motion. The domains aligned parallel to the external field grow while the domains aligned antiparallely to the field diminish. Any DW reflection induces a motion which contradicts this rule. Such a motion has to be decelerated and, eventually, it has to be suppressed when the decrease of the DW interaction energy equals the increase of the Zeeman energy. The outcome of a many-collision process in a finite-size system is the appearance of a 1D magnetic-bubble structure similar to the widely known 2D bubble structures [34]. The bubble size and concentration depend on the magnetic-field intensity. Each bubble is ended with a Néel DW at one of its sides

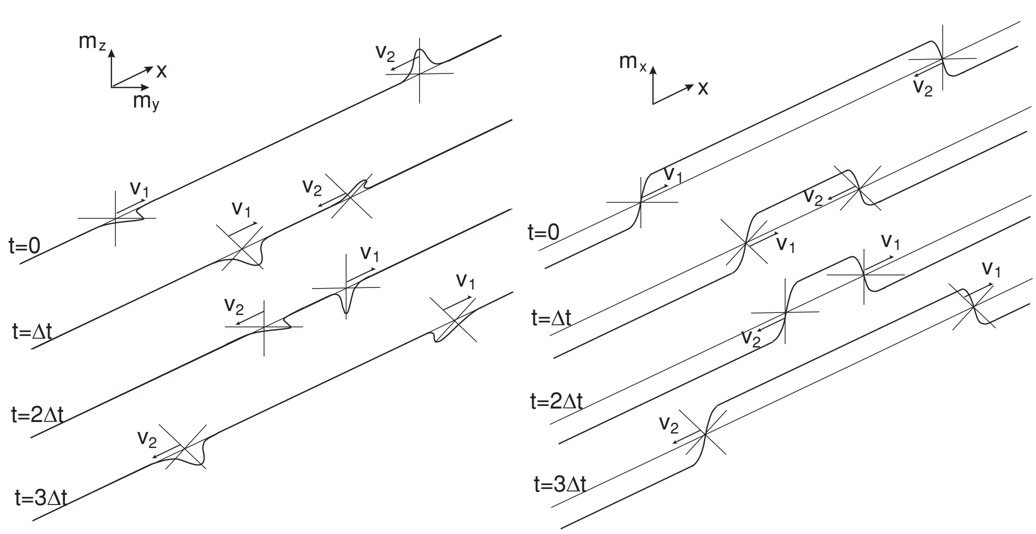

FIG. 1. The magnetization dynamics of a system of one Néel and one Bloch DW in a longitudinal field above the Walker breakdown. Their reflection takes place in the time region $(\Delta t, 2\Delta t)$ and it is accompanied by a change of the Bloch wall (of velocity v_2) into the Néel wall (of velocity v_1) and vice versa. Since $|H_x| > H_W$, the spin structure monotonically rotates about the x axis.

and with a Bloch DW at the other. Some analogy with a complex boundary of hard (quasi-2D) magnetic bubbles can be noticed since such a border contains alternating Néel and Bloch points in its structure [35]. Let me mention that interesting concepts of storing and transforming binary information were developed several decades ago with relevance to 2D magnetic bubble systems, although they were abandoned because of technological problems of the time [36].

Numerical analyses of the field-induced DW collision (micromagnetic studies) have been performed with relevance to flattened nanowires (quasi-1D nanostripes) below and just above the Walker breakdown using the dissipative LL equation [10]. The systems below threshold correspond to the plane-rotator model studied in Sec. IV, while the systems above threshold are qualitatively described with the present model. The mentioned simulations focus on the collisions of similar-type (Néel or Bloch) DWs, neglecting the anisotropy. They predicted mutual annihilation or reflection of the walls depending on the (parallel or antiparallel) spin alignments in both the DW centers. This result is partially supported by a perturbation analysis of Bloch-wall interactions within the *XY* model which showed that such DWs repel or attract each other depending on their chiralities [29]. The method of the present study cannot be applied to these collisions since one is unable to determine either the double-Néel- or double-Bloch-wall analytical solutions to the dynamical equations. However, except in the case of periodically distributed DWs, multi-Bloch or multi-Néel structures are unstable because of unbalanced DW interactions; thus, they seem to be less suitable for information-storing purposes than the Néel-Bloch DW structures. To the best of the author's knowledge, the field-induced collision of the Néel DW with the Bloch DW has not been simulated.

IV. PLANE-ROTATOR MODEL

In order to describe the DW dynamics below the Walker breakdown, I consider a system of plane rotators. Let me reduce the primary (LLG) dynamical system to its single component. Saturating the magnetization dynamics to the easy plane ($m_y = 0$), I neglect the spin rotation about the x and z axes since the relevant torque components are equal to zero. For $\mathbf{H} = (H_x,0,0)$, inserting

$$m_x = M\frac{1-a_1^2}{1+a_1^2}, \quad m_y = 0, \quad m_z = 2M\frac{a_1}{1+a_1^2} \tag{21}$$

(where a_1 takes real values) into the y component of (1), one arrives at a nonlinear diffusion equation,

$$\left(-\alpha\frac{\partial a_1}{\partial t} - \gamma H_x a_1 - \delta\beta\frac{\partial a_1}{\partial x} + J\frac{\partial^2 a_1}{\partial x^2}\right)\left(1+a_1^2\right) - 2Ja_1\left(\frac{\partial a_1}{\partial x}\right)^2 - \beta_1 a_1\left(1-a_1^2\right) = 0. \tag{22}$$

I use another *Ansatz* describing the dynamics constrained to the xy plane (a hard plane),

$$m_x = M\frac{1-a_2^2}{1+a_2^2}, \quad m_y = 2M\frac{a_2}{1+a_2^2}, \quad m_z = 0. \tag{23}$$

Then, I insert it into the z component of (1) and arrive at

$$\left(-\alpha\frac{\partial a_2}{\partial t} - \gamma H_x a_2 - \delta\beta\frac{\partial a_2}{\partial x} + J\frac{\partial^2 a_2}{\partial x^2}\right)\left(1+a_2^2\right) - 2Ja_2\left(\frac{\partial a_2}{\partial x}\right)^2 - (\beta_1+\beta_2)a_2\left(1-a_2^2\right) = 0, \tag{24}$$

which differs from (22) by a constant at the anisotropy term. With relevance to the case $\delta = 0$, one finds the two-domain solution

$$a_{1(2)} = we^{k_{1(2)}x-\gamma H_x t/\alpha}, \quad |k_1| = \sqrt{\frac{\beta_1}{J}}, \quad |k_2| = \sqrt{\frac{\beta_1+\beta_2}{J}}, \tag{25}$$

which correspond to the Bloch (Néel) DW. When $H_x \neq 0$, the DW propagates with the velocity

$$c_{1(2)} = \frac{\gamma|H_x|}{|k_{1(2)}|\alpha}. \tag{26}$$

The applicability of the plane-rotator model is limited by the Walker-breakdown condition. The magnetic field $|H_x|$ cannot exceed a critical value H_W [see Fig. 2(a)] which corresponds to the spin deviation from the basic magnetization plane (a canting) at the center of the DW about a limit angle equal to or smaller than $\pi/4$. I estimate an upper limit of the Walker critical field by considering the x component of the LLG equation at the DW center

$$\left.\frac{\partial m_x}{\partial t}\right|_{x=x_{01(2)}} \approx \left.\left[\frac{\beta_2}{M}m_y m_z - \delta\frac{\partial m_x}{\partial x}\right]\right|_{x=x_{01(2)}}, \tag{27}$$

where $x_{01(2)} \equiv -\ln(w)/k_{1(2)}$. Let φ denotes the angle of the spin deviation (canting) at the DW center. Inserting (21) and transforming $m_y m_z \to m_z^2\sin(\varphi)\cos(\varphi)$ in (27) or inserting (23) and transforming $m_z m_y \to m_y^2\sin(\varphi)\cos(\varphi)$ in (27), one arrives at

$$\frac{\partial a_{1(2)}}{\partial t} = \frac{\beta_2}{2}\sin(2\varphi)a_{1(2)} - \delta\frac{\partial a_{1(2)}}{\partial x} \tag{28}$$

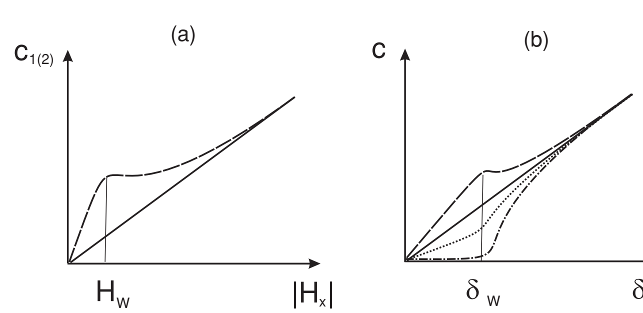

FIG. 2. (a) A scheme of the longitudinal-field dependence of the DW velocity for wires with single-axis anisotropy (solid line) and double-axis anisotropy (dashed line). (b) A scheme of the current-intensity dependence of the DW velocity for wires with single-axis anisotropy (solid line) and double-axis anisotropy; $\beta > \alpha$ (dashed line), $\beta < \alpha$ (dotted line), $\beta = 0$ (dash-dotted line).

and finally, assuming $|\varphi| \leqslant \pi/4$, at

$$|H_x| \leqslant H_W \leqslant \max H_W \equiv \frac{\alpha\beta_2}{2\gamma} + \frac{\alpha\delta}{\gamma}|k_2|. \tag{29}$$

This expression corresponds to the one given in [30,37,38]. However, notice that, for typical nanowires whose width-to-thickness ratio is bigger than 20 (double-atomic or triple-atomic layers of submicrometer width), in measuring H_W, one has estimated the canting angle to take a value of a few degrees at most [14,30,39].

For $\mathbf{H} = \mathbf{0}$, $\delta \neq 0$, the two-domain solution to (22)–(24) takes the form

$$a_{1(2)} = we^{k_{1(2)}(x-\delta\beta t/\alpha)},$$
$$|k_1| = \sqrt{\frac{\beta_1}{J}}, \quad |k_2| = \sqrt{\frac{\beta_1+\beta_2}{J}}. \tag{30}$$

It is seen that only the nonadiabatic part of the spin-transfer torque contributes to (22), (24) since the current-dependent term is proportional to β. From (28), the current-induced Walker breakdown corresponds to the critical current intensity

$$\delta_W \leqslant \beta_2/[2|k_1|(1+\beta/\alpha)] \tag{31}$$

if $H_x = 0$ [see Fig. 2(b)]. It has been observed that H_W and δ_W decrease with decreasing nanowire width-to-thickness ratio [39,40], because this ratio determines the strength of the easy-plane anisotropy while $H_W,\delta_W \to 0$ with $\beta_2 \to 0$ [41]. Notice that analytical calculations using the 2D *XY* model, experimental observations, and simulations of the spin ordering in nanostripes show this ordering to vary along the cross section width of the nanostripe in the DW area, thus revealing a complex topological structure [10,42]. Therefore, in terms of the application to nanostripes, my plane-rotator description is valid for a qualitative analysis of the DW dynamics at most.

Neither finding nonstationary double-Bloch nor double-Néel solutions in the form of the Hirota expansion (including its second order) does not manage. In particular, insertion of

$$a_{1(2)} = \frac{w_1 e^{k_{1(2)}x} + w_2 e^{k'_{1(2)}x}}{1 + v_{1(2)}e^{k_{1(2)}x + k'_{1(2)}x}} e^{-\gamma H_x t/\alpha} \tag{32}$$

into (22), (24), for $k_1 = -k_1' = \pm\sqrt{\beta_1/J}$, $k_2 = -k_2' = \pm\sqrt{(\beta_1+\beta_2)/J}$, leads to the divergence of $v_{1(2)}$ as follows from the approach of Sec. III. In order to describe the collision of Bloch and Néel walls below the Walker breakdown, I propose to apply an effective 1D model assuming the magnetization precession to be overdamped, and thus, taking the LHS of (1) to be equal to zero. Inclusion of the constraint $|\mathbf{m}| = M$ leads to the modified (by neglecting the first terms on the LHS) system (4) and (5). Solving it, I predict the field-induced DW collision below the Walker breakdown to result in their reflection similar to the one described in Sec. III. The reflection is accompanied by the change of the Bloch wall into the Néel wall and vice versa. Let us emphasize that there is no spontaneous DW motion below the Walker breakdown when magnetostatic effects are neglected [43].

The technological challenge of increasing the DW speed is especially important below the Walker breakdown, where the driving field is relatively weak. Referring to this purpose, I mention an attempt utilizing an increase of the nanostripe-edge roughness, and thus an increase of the damping constant α [44]. This approach fails since, according to simulations of [39], the maximum of the field-induced DW velocity is insensitive to α below the breakdown. It is because $c_{1(2)} \propto \alpha^{-1}|H_x|$ while $|H_x| \leqslant H_W \propto \alpha$. On the other hand, since β grows with α (the nonadiabatic part of the spin-transfer torque is of dissipative origin), the velocity of the current-induced DW motion,

$$c = \delta\beta/\alpha, \tag{33}$$

can be insensitive to the increase of the nanostripe-edge roughness as well. The reported DW-velocity increase due to the nanostripe-edge roughness has been attributed to a decrease of its effective cross-section width. Another attempt utilized an increase of H_W due to the increase of the anisotropy constant β_2. This was done via nanowire deposition on a specific crystalline substrate [45]. However, the most efficient method of influencing the maximum DW velocity below the Walker breakdown is the application of the transverse magnetic field, which is considered in the next section [46].

V. DOMAIN WALL IN PERPENDICULAR TO EASY-AXIS FIELD

Let me define $H_\pm \equiv H_y \pm iH_z$. For $H_x = 0$, we search for a two-domain solution to (1) using a different multilinearization than used in the previous sections:

$$f_2\left[-iD_t + JD_x^2 + \delta(\beta - i)D_x + \alpha D_t\right] f_1^* \cdot g_1 + \frac{\gamma H_+}{2} f_1^*\left(f_1^* f_2 + g_2^* g_1\right) - \left(\beta_1 + \frac{\beta_2}{2}\right) f_2 f_1^* g_1 - \frac{\beta_2}{2} f_1^{*2} g_2^* = 0,$$
$$g_2^*\left[-iD_t - JD_x^2 + \delta(\beta - i)D_x + \alpha D_t\right] f_1^* \cdot g_1 - \frac{\gamma H_-}{2} g_1\left(f_1^* f_2 + g_2^* g_1\right) + \left(\beta_1 + \frac{\beta_2}{2}\right) g_2^* g_1 f_1^* + \frac{\beta_2}{2} g_1^2 f_2 = 0, \tag{34}$$
$$f_2 g_1 D_x^2 f_1^* \cdot f_1^* - f_1^* g_2^* D_x^2 g_1 \cdot g_1 = 0.$$

Let us focus our attention on the case $\alpha,\beta,\delta=0$ for simplicity. Then one has $f_1=f_2=f$, $g_1=g_2=g$, while in the general case the relations (6), (7) apply. I analyze the two cases of the external-field direction; the one parallel to the easy plane $H_+=iH_z$, and the one perpendicular to the easy plane $H_+=H_y$.

In the case of $H_+=iH_z$, I apply the *Ansatz*

$$f_1^*=f_2=q_1+s_1e^{k_1x-l_1t},$$
$$g_1=-g_2^*=i(s_1+q_1e^{k_1x-l_1t}), \tag{35}$$

where k_1,q_1, and s_1 denote real constants, (the parameter l_1 can take complex values when $\alpha\neq 0$). The solution in the form (35) describes the wall between two domains whose spins are deviated from the easy axis onto the external-field direction about an angle which grows with $|H_+|$. Inserting this *Ansatz* into (34), one finds

$$s_1=\frac{\beta_1-\sqrt{\beta_1^2-\gamma^2H_z^2}}{\gamma H_z}q_1. \tag{36}$$

Considering the solutions which are static in the absence of the electric current, $l_1=0$ for $\delta=0$, one arrives at

$$|k_1|=\sqrt{\frac{\beta_1^2-\gamma^2H_z^2}{\beta_1 J}}. \tag{37}$$

In the case of $H_+=H_y$, the *Ansatz* relating to the deviation of the domain magnetization from the easy axis onto the external-field direction takes the form

$$f_1^*=f_2=q_2+s_2e^{k_2x-l_2t},$$
$$g_1=g_2^*=s_2+q_2e^{k_2x-l_2t} \tag{38}$$

with real k_2,q_2,s_2. From (34), I find

$$s_2=\frac{\beta_1+\beta_2-\sqrt{(\beta_1+\beta_2)^2-\gamma^2H_y^2}}{\gamma H_y}q_2. \tag{39}$$

The static solutions correspond to

$$|k_2|=\sqrt{\frac{(\beta_1+\beta_2)^2-\gamma^2H_y^2}{(\beta_1+\beta_2)J}}. \tag{40}$$

The transverse external field does not drive the DW motion even in the presence of the magnetic dissipation ($\alpha\neq 0$). When the current through the wire and the dissipation are applied, under the transverse magnetic field, the DW moves with the velocity c given by (14), which is independent of the value of this field. Then the solution to (34) satisfies the bilinearized LLG system [Eqs. (3) with additional H_+-dependent terms] at the time points of the discrete set $t=\pi n/\mathrm{Im}l_1$, where $n=0,\pm 1,\pm 2,\ldots,$ since $f_1=f_2=f$, $g_1=g_2=g$ at these points. Including a longitudinal component of the magnetic field H_x, additional to H_+, drives the DW motion. For the realistic case $H_W\sim|H_x|<|H_+|\ll|\beta_1/\gamma|$, neglecting small contributions to the H_x-dependent part of the torque, one finds the velocity of such a DW propagation (13) or (26) above and below the Walker breakdown, respectively, with $|k_{1(2)}|$ given by (37) and (40). This velocity nonlinearly increases with $|H_+|$ [47]. Searching for $c_{1(2)}$, in the case $|H_x|<H_W$, additionally, I have taken the LHS of (1) equal to zero as discussed in Sec. IV. The manipulation of $c_{1(2)}$ via the application of the transverse magnetic field is potentially useful for speeding up the processing with a magnetically encoded information. I also note that the transverse-field dependence of $|k_{1(2)}|$ enables influencing the magnitude of the critical current of the Walker breakdown δ_W, following (31).

VI. CONCLUSIONS

I have analytically studied the DW dynamics in the presence of an external magnetic field and an electric current along a magnetic wire within the LLG approach. It has required overcoming the difficulty arising from breaking the time-reversal symmetry by inclusion of the magnetic dissipation. I removed this asymmetry of the dynamical system by introducing additional (virtual) dynamical variables, which is a similar trick to the Lagrangian approach to the damped harmonic oscillator. Determining the connection of the additional dynamical variables to the evolution of the magnetization vector in specific ranges of time, I have analyzed the dynamics of a single DW and of a pair of DWs.

The magnetic-field-induced velocities of the DWs, the formulas (13) and (26), and the current-induced velocities (14) and (33) are found to correspond to those of the Walker approach above and below the breakdown, respectively. According to [28], static three-domain solutions to the 1D LLG equation describe pairs of Néel and Bloch walls. For the purposes of the qualitative dynamics analysis of a number of DWs below the Walker breakdown, especially of the Néel-Bloch pairs, I have proposed a dynamical equation which differs from the LLG one by neglecting the LHS in (1). Below and above the breakdown, the neighboring Néel and Bloch walls move in the presence of a longitudinal external field in opposite directions. Their collision results in the DW reflection accompanied by the reorientation of the Néel wall into the Bloch wall and vice versa. In other words, the DWs pass through each other without changing their widths and velocities; however, the head-to-head DW structure changes into the tail-to-tail one and vice versa.

The present method is useful for the analysis of two-domain systems under a transverse (with respect to the easy axis) external field, which enables a verification of numerical and experimental results [46–48]. A reorientation of the magnetic domains due to the transverse field induces a widening of the DW area whose width diverges when the field intensity approaches the coercivity value. The consequence of the transverse-field application is an increase of the DW mobility (the ratio $c_{1(2)}/|H_x|$) and an increase of the critical current (a shift of the current-driven Walker breakdown).

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