Generalized Eden model with a screening effect

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We generalize the Eden model to take into account the screening effect, i.e., the end point grows much faster than the interior points do. Highly anisotropic clusters are obtained in our generalized Eden model. It is found that the length in the long direction scales differently than that in the short direction does as the number of sites increases.

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I. INTRODUCTION

The diffusion-limited aggregation (DLA) model has attracted a lot of attention since proposed by Witten and Sander [1–9]. It has been recognized that the screening effect [1,5], i.e., the end points have much more chance of growing in DLA, plays an important role in giving rising to the fractal dimensionality. On the other hand, the Eden model [10,11], in which an adjacent site to the occupied site is randomly selected among all possible adjacent sites in each step, gives only a compact structure in the large size limit. This may be easily understood (see Sec. III), because no screening effect is present in the Eden model. However, the algorithm to simulate the Eden model can be very efficient and proportional to the number of particles in the system. Therefore, it may be desirable to introduce the screening effect into the Eden model, and see how the compact structure varies as the screening parameter varies. In this paper, we shall study a generalized Eden model in which the screening effect is taken into account. At the same time, the algorithmic efficiency, depending on the screening parameter, is still much higher than the algorithmic efficiency for the DLA model. Although our generalized Eden model cannot be used to substitute for the DLA model, its study allows us to understand the screening effect more clearly. As we shall see, introducing the screening effect into the Eden model changes the two-dimensional compact object into the highly anisotropic object which actually resembles a one-dimensional object in the large size limit. This paper is organized as follows: In Sec. II, the model is defined and the numerical simulation results are presented. In Sec. III, some analytic results, which become exact in the large size limit, are presented. Finally in Sec. IV, we shall briefly discuss the relation between the DLA model and our generalized Eden model.

II. MODEL AND THE NUMERICAL RESULTS

Our model is defined as follows. First, we have a square lattice. At time step 1, a seed particle occupies the lattice point (0,0). At time step N, there are N occupied sites which are connected. Those N occupied sites have Q adjacent empty sites. Calculate the distance between those Q adjacent sites and the (0,0) site. Find out the maximum value and denote it as $r_{Q\max}$. Then, select one of the Q adjacent sites according to the probability proportional to $\frac{1}{(r_{Q\max}+1-r_q)^\beta}$, where r_q is the distance between the selected adjacent site and the (0,0) site, and $0<\beta<1$ is the screening parameter which is adjustable in our model. Then, it goes to the time step $N+1$. Repeat until a large cluster grows. It may be helpful to mention that the Eden model just corresponds to the case $\beta=0$ [10]. At first sight, one sees that the far-most site, usually just the end point, grows much faster than the interior sites. Therefore, one may expect a different scaling behavior for the $\beta>0$ case than the Eden model ($\beta=0$ case) in which the growing cluster is a nearly homogeneously compact structure. Figure 1 plots a typical cluster having $N=30\,000$ sites with $\beta=0.5$. One sees that the cluster is an anisotropic object with the long axis along the radial direction and the short axis along the azimuthal angle direction. Although the cluster appears to be a two-dimensional object when $N=30\,000$, the long axis and the short axis scale differently as the number of particles N increases. Table I shows the dependence of $r_{\max}$, the distance between the far-most occupied site and the (0,0) site, and Q, the number of adjacent sites, on the number of occupied sites N. The exponents are defined as $N\sim r_{\max}^\alpha$ and $Q\sim r_{\max}^\eta$. From Table I, one may find that $\eta\approx 1$ and $\alpha\approx 2-\beta$. The width of our anisotropic object $W\sim r_{\max}^\delta$ with the scaling relation $\delta=\alpha-1=1-\beta$. These results will be explained in the subsequent section.

III. ANALYTIC RESULTS

Our generalized Eden model can be solved analytically in the large size limit. For our generalized Eden model in

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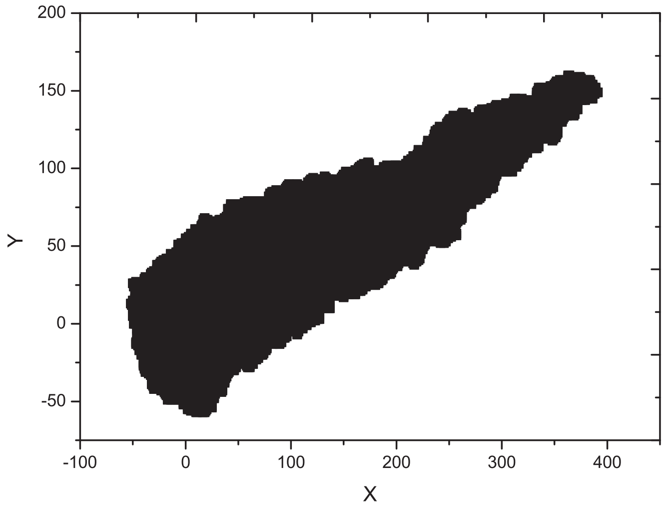

FIG. 1. Simulation pattern of our generalized Eden model. $N = 30000$, $\beta = 0.5$.

D dimensions, similar results can be also obtained and will be published in another paper. Denote $R(N)$ to be the average distance between the far-most occupied sites and the (0,0) sites, and $Q(N)$ to be the average number of adjacent sites as function of N. Since the increase in the rate of $R(N)$ is directly proportional to the probability of the far-most site being selected, which is equal to $\frac{1}{\sum_{i=1}^{Q(N)}\frac{1}{[R(N)+1-r_i]^\beta}}$, we have, up to a numerical factor,

$$\frac{dR(N)}{dN} \approx \frac{O(1)}{\sum_{i=1}^{Q(N)}\frac{1}{[R(N)+1-r_i]^\beta}}, \tag{1}$$

where r_i is the distance between the ith adjacent site and the (0,0) site. For almost all adjacent sites, we have $R + 1 - r_i$ as the order of R because our cluster is highly anisotropic and almost all $r_i \sim \lambda_i R$ for $\lambda_i < 1$, and we obtain

$$\frac{dR(N)}{dN} \sim \frac{R(N)^\beta}{Q(N)}. \tag{2}$$

Assume that our giant cluster only has the anisotropic property, but does not have the fractal property; we then have the approximate relation

$$Q(N) \sim R(N). \tag{3}$$

Therefore, we have

$$\frac{dR(N)}{dN} \sim R(N)^{\beta-1}, \tag{4}$$

TABLE I. Some exponents in our generalized Eden model.

β	0.25	0.50	0.75
N	10^6	10^6	3×10^5
$\alpha + \beta - 2$[a]	-0.085 ± 0.018[b]	-0.017 ± 0.010	0.067 ± 0.009
η	0.914 ± 0.010	0.962 ± 0.007	1.001 ± 0.007

[a]The exponents α and η are obtained by the linear fit in the log-log plot for N from $N/2$ to N.
[b]This table lists the average values ($\pm$statistical error only) of α and η over 10 clusters for $\beta = 0.25, 0.50$, and 0.75, respectively.

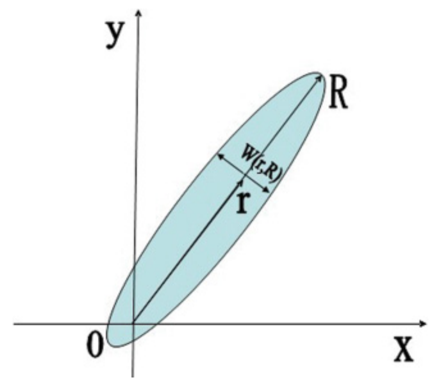

FIG. 2. (Color online) Illustration of various variables in the text.

which implies

$$N \sim R^{2-\beta}, \tag{5}$$

or

$$\alpha = 2 - \beta. \tag{6}$$

In the large size limit as $N \to \infty$, the cluster approaches a definite shape (the stochastic fluctuation becomes insignificant) which scales as $N^{\frac{1}{(2-\beta)}}$ in the long direction because $R \sim N^{1/(2-\beta)}$. The width W in the perpendicular direction is much less than R and is expected to be proportional to $N^{(1-\beta)/(2-\beta)}$ so that the number of total points can be correctly obtained, i.e., $N \sim RW$. To find out the shape function, denote $W(r,R)$ as the width of the long bar at the distance r from the (0,0) site when the maximum distance is R (see Fig. 2). Since a site with the distance r is selected with the probability

$$\frac{1/(R-r)^\beta}{2\int_0^R du 1/(R-u)^\beta} = \frac{1-\beta}{2}\frac{R^{\beta-1}}{(R-r)^\beta}, \tag{7}$$

$W(r,R)$ satisfies the differential equation

$$\frac{dW(r,R)}{dN} = 2\frac{(1-\beta)R^{\beta-1}}{2(R-r)^\beta}, \tag{8}$$

with the initial condition

$$W(r,r) = 0. \tag{9}$$

Note that in the above equation, we have used the fact that the normal direction of the surface of the long bar is almost perpendicular to the long axis in the large size limit when $\beta > 0$, because the width $W(r,R) \ll R$ when $R \to \infty$. [$\beta = 0$ is a special case, and the above equation is invalid in the large size limit because $W(r,R) \sim R$.] Notice that

$$\frac{dR}{dN} = CR^{\beta-1}. \tag{10}$$

So we obtain

$$\frac{dW}{dR} = \frac{1-\beta}{C(R-r)^\beta}. \tag{11}$$

Direct integration yields

$$W(r,R) = \frac{1}{C}(R-r)^{1-\beta} = \frac{R^{1-\beta}}{C}\left(1-\frac{r}{R}\right)^{1-\beta}, \tag{12}$$

which indeed scales as $R^{1-\beta}$. It may be interesting to point out that the long bar grows not necessarily along a straight line but slightly fluctuates around it. Since the fluctuation is very small, Eq. (12) still holds in the large size limit. To conclude this section, we would like to discuss the finite size effect in the numerical simulation results. From Table I, one sees that when $\beta = 0.25$, $\alpha < 2 - \beta$ up to $N = 10^6$. The deviation of α from $2 - \beta$ must be due to the finite size effect. The reason is that $\eta < 1$ for $\beta = 0.25$ and $N = 10^6$; this is definitely impossible in the large size $N \to \infty$ limit because of the inequality $Q(N) > R(N)$. Our results strongly suggest that the finite size effect can be still very significant to the correction of α even for $N = 10^6$.

IV. DISCUSSIONS

From the above two sections, we see that the Eden model ($\beta = 0$) can only produce a two-dimensional homogeneous cluster. On the other hand, when $\beta > 0$, our model produces a quite anisotropic cluster which scales differently than the simple two-dimensional object. Also, in the DLA model, the end point should grow much faster than other points because of the screening effect. For example, consider the Possion equation $\nabla^2 u = 0$ with $u\mid_\Gamma = 0$, where the boundary Γ is an ellipse with the long axis R and the short axis R^δ, with $R \gg 1$ and $\delta < 1$. Then, $\frac{\partial u}{\partial n}\mid_\Gamma$ is much larger at the long end than on the short side. This ratio is $R^{1-\delta} \gg 1$. This is an illustration of the screening effect in the DLA model and is effectively taken into account in our generalized model (i.e., the end points grow much faster).

We would like to point out since the growth rate of only one end point is greatly enhanced in our model, we only obtain a highly anisotropic cluster which scales differently along different directions as the number of occupied sites N increases. The fractal structure is absent in our model. Only when the growth rate of many end points of the big trunks is greatly enhanced, can one observe the fractal structure as seen in the DLA model.

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