

## Indirect control of transport and interaction-induced negative mobility in an overdamped system of two coupled particles

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One-dimensional transport of an overdamped Brownian particle biased by an external constant force does not exhibit negative mobility. However, when the particle is coupled to another particle, negative mobility can arise. We present a *minimal* model and propose a scenario in which only one (say, the first) particle is dc biased by a constant force and ac driven by an unbiased harmonic signal. In this way we intend to achieve two aims at once: (i) negative mobility of the first particle, which is exclusively induced by coupling to the second particle and (ii) indirect control of the transport properties of the second particle by manipulating the first particle only. For instance, the sign and amplitude of the averaged stationary velocity of the second particle can be steered by the driving applied to the first particle. As an experimentally realizable system, we propose two coupled resistively shunted Josephson junctions.

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### I. INTRODUCTION

Systems under nonequilibrium conditions can display new features as well as unexpected phenomena and processes which in equilibrium systems are forbidden by fundamental laws. One of the most prominent examples include the phenomenon of negative mobility (conductance, resistance): When a constant force is applied to a mobile particle, it moves in the direction opposite to that of the force. It is impossible in equilibrium states because it would violate the second law of thermodynamics.

The phenomenon of negative mobility can occur in  $p$ -modulation-doped multiple quantum-well structures [1], in semiconductor superlattices [2], in charge-density-wave conductors [3], in a three-terminal configuration in a two-dimensional (2D) electron gas [4], in transport of vortices in superconductors with inhomogeneous pinning under a driving force [5], and in Josephson junctions [6]. Some other examples that come to mind are the nonlinear response in ac-dc-driven tunneling transport [7], in the dynamics of cooperative Brownian motors [8], in Brownian transport containing a complex topology [9,10], and in some stylized multistate models with state-dependent noise [11]. The effect of negative mobility can occur also in driven systems such as in nonlinear inertial Brownian dynamics [12–15], in overdamped nonlinear Brownian motion in the presence of time-delayed feedback [16], and in transport of asymmetric particles in a periodically segmented 2D channel [17].

For a one-particle system, the simplest, one-dimensional (1D) model is formulated in terms of the Newton equation for a particle moving in a symmetric spatially periodic potential, biased by a static force and driven by an unbiased harmonic force  $A \cos(\omega t)$  [12–15]. This system is out of equilibrium and displays both absolute negative mobility (ANM) around zero static applied force (the linear response regime) and negative mobility in the nonlinear response regime (NNM). It is known that the corresponding *overdamped* system does not exhibit negative mobility and the inertial term in the Newton equation is absolutely necessary for the negative mobility to occur. However, phenomena which are absent in a single element perhaps can occur in a system of coupled elements.

The physics of many-body systems provides hundreds of examples. Therefore, we are going to check whether it is possible to model a system of two coupled overdamped particles that exhibits negative mobility. We can speculate that the combined effect of the time-dependent driving and the interaction between two particles can radically modify a one-particle overdamped dynamics, yielding transport anomalies as ANM or NNM. We want to construct a model composed of minimal essential ingredients and therefore we apply the static and time-periodic forces to only one (say, the first) particle. This scenario is intended to achieve two aims at once: (i) both positive and negative mobility of the first particle and (ii) control and steering of motion of the second particle by manipulation of the first particle. In particular, the second particle can be transported in the same direction as the static force or in the opposite direction. As an example of a real and experimentally accessible system, we propose to investigate two coupled Josephson junctions which play the role of two interacting particles in the mechanical framework. Similar effects (however, caused by radically different mechanisms) have been studied in Ref. [18], where it was proposed to use an active species of particles to control the passive species of particles. The authors of Ref. [18] have considered a mixture of two species of Brownian particles diffusing on 1D periodic substrates and showed that in the mean-field approximation the particles can move either together or in opposite directions, depending upon the strength of the interaction, and whether the interaction is attractive or repulsive.

The paper is organized as follows. We introduce in Sec. II the theoretical model in terms of a set of two coupled Langevin equations which describe the dynamics of two resistively shunted Josephson junctions. In Sec. III we analyze the transport properties of the system driven by the ac periodic force and dc bias, both applied to the first junction. Finally, we summarize the paper by conclusions in Sec. IV.

### II. THE MODEL

In a more general case, we can consider a system consisting of two coupled resistively shunted Josephson junctions with

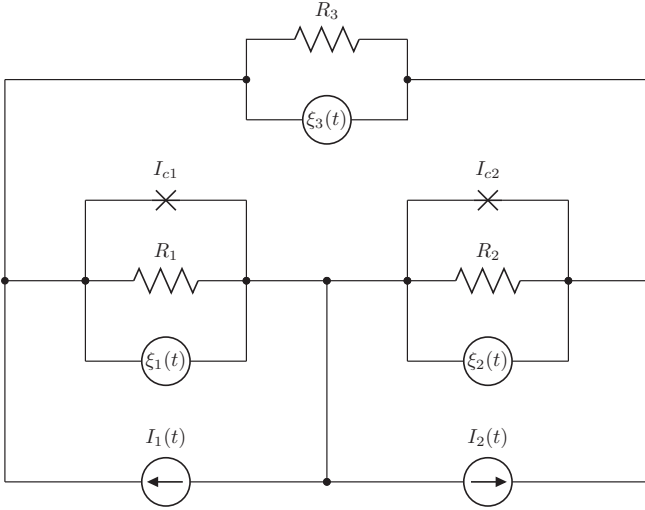


FIG. 1. The system of two resistively shunted Josephson junctions coupled by an external shunt resistance  $R_3$  and driven by the currents  $I_1(t) = I_1 + a_1 \cos(\omega t)$  and  $I_2(t) = I_2 + a_2 \cos(\omega t)$ .

critical currents ( $I_{c1}, I_{c2}$ ), resistances ( $R_1, R_2$ ), and phases ( $\phi_1, \phi_2$ ), respectively. The system is externally shunted by the resistance  $R_3$  as shown in Fig. 1. Moreover, the junctions can be dc biased by the currents  $I_1$  and  $I_2$ , and ac driven by the currents  $a_1 \cos(\omega t)$  and  $a_2 \cos(\omega t)$ , respectively. We consider the small junction area limit and a regime where photon-assisted tunneling phenomena do not contribute to the dynamics of the system. The phase dynamics of the junctions is determined by two equations, the dimensionless form of which read [19]

$$\dot{\phi}_1 = I_1 - I_{c1} \sin \phi_1 + b_1 \cos(\omega t) + \alpha (I_2 - I_{c2} \sin \phi_2) + \sqrt{D} \xi'(t), \quad (1a)$$

$$\dot{\phi}_2 = \alpha \beta (I_2 - I_{c2} \sin \phi_2) + b_2 \cos(\omega t) + \alpha (I_1 - I_{c1} \sin \phi_1) + \sqrt{\alpha \beta D} \xi''(t), \quad (1b)$$

where the dot denotes a time derivative, and the parameters  $\beta = 1 + R_3/R_1$ ,  $b_1 = a_1 + \alpha a_2$ , and  $b_2 = \alpha(a_1 + \beta a_2)$ . The coupling parameter  $\alpha = R_2/(R_2 + R_3) \in [0, 1]$  can be changed by adjusting the external resistance  $R_3$ . We use the same dimensionless units as Nerenberg *et al.* [19]: The current unit is the averaged critical supercurrent  $\bar{I}_c = (I_{c1} + I_{c2})/2$ , and the time unit is  $\hbar/2eV_0$ , where  $V_0 = \bar{I}_c R_1 (R_2 + R_3)/(R_1 + R_2 + R_3)$  is the characteristic voltage. We assume that all resistors are at the same temperature  $T$ , and that the noise sources (the Johnson thermal noise) in Fig. 1 are represented by zero-mean white noises  $\xi_i(t)$  ( $i = 1, 2, 3$ ) which are delta correlated, i.e.,  $\langle \xi_i(t) \xi_j(s) \rangle = \delta_{ij} \delta(t - s)$  for  $i, j \in \{1, 2, 3\}$ . The noises  $\xi'(t)$  and  $\xi''(t)$ , which appear in Eqs. (1), are linear combinations of the noises  $\xi_i(t)$ ,  $i = 1, 2, 3$ . The dimensionless noise strength is  $D = 4ek_B T/\hbar \bar{I}_c$ . Equations (1) are the extended version of the system studied by Nerenberg *et al.* [19,20], which now includes ac-driving and noise terms. The considered system assumes a series-opposing configuration of the bias currents—that is,  $I_1$  and  $I_2$  flow in opposite directions. It is easy to verify that the series-aiding

case (i.e., when  $I_1$  and  $I_2$  flow in the same direction) can be obtained by the substitution  $\alpha \rightarrow -\alpha$  and  $\beta \rightarrow -\beta$ .

Equations (1) describe the overdamped dynamics of a hypothetical mechanical system of two interacting particles of coordinates  $x_1 = \phi_1$  and  $x_2 = \phi_2$ , respectively. Our model is *minimal* in the sense that the phase space of the deterministic system (1) is 3D, namely,  $\{x_1 = \phi_1, x_2 = \phi_2, x_3 = \omega t\}$  and three is the minimal phase dimension necessary for it to display chaotic evolution, which is an important feature for anomalous transport to occur [12–15]. At a nonzero temperature,  $D > 0$ , the Johnson thermal fluctuations activate a diffusive dynamics where stochastic escape events among existing attractors become possible. Moreover, the system can now visit any part of the phase space and evolve within some finite time interval by closely following any existing orbits, either stable or unstable.

### III. DRIVING APPLIED TO ONE JUNCTION

We consider a simplified system of *two identical* junctions ( $R_1 = R_2$ ,  $I_{c1} = I_{c2} = 1$ ,  $\alpha \beta = 1$ ) and the case when  $I_2 = 0$  and  $a_2 = 0$ —that is, when only the first junction is dc biased and ac driven. In this way, we want to manipulate the first junction only and observe the response of both the first and second junctions. We address the question of under what conditions we can control transport properties of the second junction. To answer this question, we numerically study the dimensionless long-time averaged voltage  $v_1 = \langle \dot{\phi}_1 \rangle$  across the first junction and the voltage  $v_2 = \langle \dot{\phi}_2 \rangle$  across the second junction. The long-time physical voltage is then expressed as  $V_i = V_0 v_i$  ( $i = 1, 2$ ). If the problem is formulated in terms of overdamped motion of classical Brownian particles, the voltage  $v_i$  translates into the averaged velocity of the first or second particles, respectively, and the dc current  $I_1$  translates into an external constant force acting only on the first particle. The junction resistance (or equivalently conductance) translates then into the particle mobility. One can use this analogy to simplify the visualization of transport processes in junctions. The voltage  $v_i = v_i(I_1)$ ,  $i = 1, 2$ , is typically a nonlinear and nonmonotonic function of the dc current  $I_1$ . In the “normal” transport regime, the voltage  $v_i$  is positive for positive bias  $I_1$ , i.e., the “nonlinear resistance” or the static resistance  $r_i = v_i/I_1$  at a fixed bias current  $I_1$  is positive. The case of  $r_1 < 0$  (i.e., when the response of the first junction is opposite to the external load applied to it) is usually referred to as the anomalous transport regime with ANM or NNM. With the necessary changes, we will call the case with  $r_2 = v_2/I_1 < 0$  as ANM or NNM.

We begin the analysis of the system (1) by some general remarks about its long-time behavior. As expected from symmetry, in the zero bias case, i.e., when  $I_1 = 0$ , there is no net transport. If the dc bias  $I_1$  is sufficiently large in comparison to the amplitude  $a_1$  of the ac driving, one can detect the normal transport regime where  $v_1 > 0$  and  $v_2 > 0$  for  $I_1 > 0$ , i.e., the voltage across both junctions has the same sign as the dc current. This is rather obvious because the driving is not important in such a regime. More interesting effects can take place in the regime of small  $I_1$ . However, the parameter space  $\{\alpha, I_1, a_1, \omega, D\}$  is 5D and thus too large for an extensive numerical scan. Therefore, a number of low-resolution scans

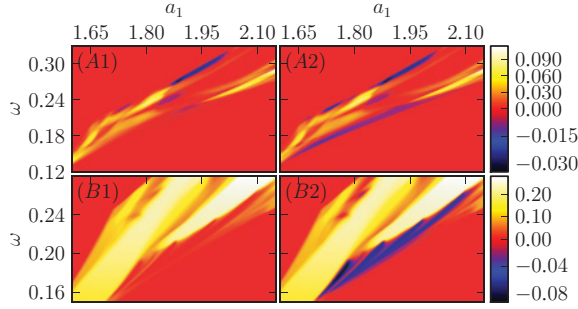


FIG. 2. (Color online) The long-time transport properties of the driven system of two Josephson junctions in the parameter space  $\{a_1, \omega\}$  of the ac driving acting on the first junction at a representative coupling strength  $\alpha = 0.77$ ,  $a_2 = I_2 = 0$ , and temperature  $D = 2 \times 10^{-5}$ . The transport regime (A) is illustrated at  $I_1 = 0.008$  in (A1) for the averaged voltage  $v_1$  and (A2) for the averaged voltage  $v_2$ . The transport regime (B) is illustrated at  $I_1 = 0.05$  in (B1) for  $v_1$  and (B2) for  $v_2$ .

over the parameters  $\{\alpha, a_1, \omega\}$  at fixed values of  $I_1$  and  $D = 0$  were first performed. Next, the interesting regions of the parameter space were analyzed in more detail and at a higher resolution. These initial scans were done for initial values of the phases  $\phi_1$  and  $\phi_2$  randomly chosen from the interval  $[0; 2\pi]$ . They were then supplemented by scans at nonzero temperatures. We have found that for  $I_1 < 0.1$  the modulus of the average voltage across both junctions takes its highest values for  $\omega \in (0, 1)$  and is gradually diminished for higher frequencies. At large ac-driving frequencies ( $\omega > 5$ ) there is no noticeable net transport regardless of the values of other parameters.

We have found that the normal transport regime dominates in the parameter space. However, we can also identify two remarkable and distinct regimes of anomalous transport, namely, (A)  $v_1 < 0$  and  $v_2 < 0$  for  $I_1 > 0$ , and (B)  $v_1 > 0$  and  $v_2 < 0$  for  $I_1 > 0$ . We have not found regimes where  $v_1 < 0$  and  $v_2 > 0$  for  $I_1 > 0$ .

For fixed but small values of the dc bias  $I_1 < 0.1$ , strips of nonzero average voltage are clearly visible in the parameter space  $\{a_1, \omega, \alpha\}$ . For weak coupling of two junctions (small  $\alpha$ ), there is no net transport in the second junction. For stronger coupling, strips of nonzero average voltage start to appear at progressively lower values of the amplitude  $a_1$  of the ac driving. The strips are also visible in the plots of the average voltage of the first junction, which means that they represent regimes of the parameter space where both junctions operate in synchrony. Figure 2 illustrates this behavior: the asymptotic, averaged voltages across the first and second junction in the two regimes (A) and (B) defined above are depicted. In doing so here we do not discriminate between ANM and NNM. Both these transport behaviors are jointly presented.

In Fig. 3 we present the influence of temperature on transport properties in regime (B), where we show how the parameter plane  $\{a_1, \omega\}$  is divided into regions of normal ( $v_1 > 0$  and  $v_2 > 0$  for  $I_1 > 0$ ) and anomalous ( $v_1 > 0$  and  $v_2 < 0$  for  $I_1 > 0$ ) transport. For small temperature, the structures visible in the plots become more complex [as in Fig. 3(e)] and regions of negative conductance of the second junction can be

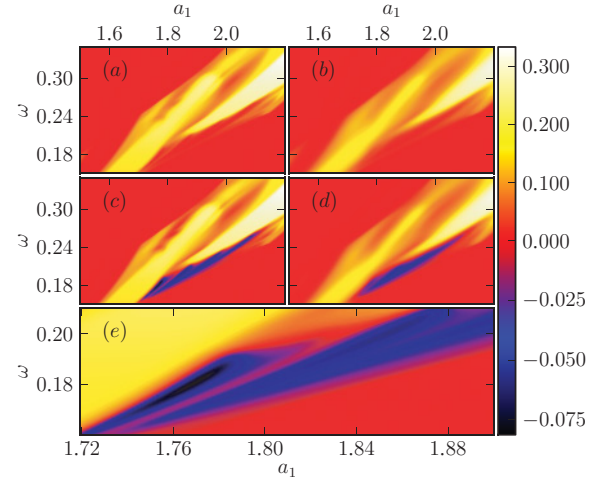


FIG. 3. (Color online) The influence of temperature on transport properties of the driven system of two Josephson junctions in the parameter space  $\{a_1, \omega\}$  of the ac driving acting on the first junction at a representative coupling value of  $\alpha = 0.77$ , the dc bias  $I_1 = 0.05$ , and  $a_2 = I_2 = 0$ . In (a) and (b) the averaged voltage  $v_1$  across the first junction is shown at two dimensionless temperatures  $D = 5 \times 10^{-5}$  and  $2.5 \times 10^{-4}$ , respectively. (c) and (d) The averaged voltage  $v_2$  across the second junction is depicted at the same temperatures as in (a) and (b), respectively. (e) presents an enlarged part of (c): the plot reveals the island structure of regions of negative resistance of the second junction.

identified. For higher temperature, a rich structure is gradually smeared out by thermal equilibrium fluctuations.

In Fig. 4 we depict the typical dependence of the voltages  $v_1$  and  $v_2$  on the bias  $I_1$  in the (A) regime with ANM. In the inset, we present the temperature dependence of the voltages. It follows that ANM is solely induced by thermal fluctuations, and this mechanism is explained in Ref. [12]. For small and large thermal fluctuations, ANM does not occur. Notably, there is a window of temperatures in which ANM can be detected. Moreover, there exists an optimal temperature at which ANM

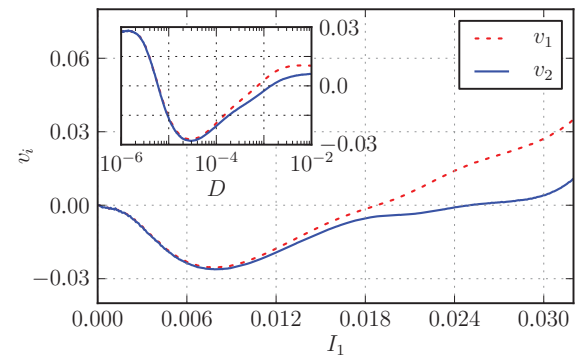


FIG. 4. (Color online) The ANM regime of long-time averaged voltages  $v_1$  and  $v_2$  of the first and second junctions, respectively, at a dimensionless temperature of  $D = 5 \times 10^{-5}$ . The inset depicts the influence of temperature: It reveals the noise-induced mechanism of ANM. The parameters are the coupling strength  $\alpha = 0.77$ , amplitude  $a_1 = 1.8912$ , and frequency  $\omega = 0.2708$  of the ac driving,  $a_2 = I_2 = 0$ .

is most pronounced, i.e.,  $v_1$  and  $v_2$  take their absolute minimum (negative) values.

We now discuss the dependence of the voltages  $v_1$  and  $v_2$  on the dc bias  $I_1 > 0$ , in the anomalous transport regime (B) illustrated in panels (B1) and (B2) of Fig. 2. The result is presented in Fig. 5(a). The characteristic feature is the emergence of intervals of  $I_1$ , where the voltage  $v_2$  is negative. Two anomalous effects are detected: ANM for  $I_1 \rightarrow 0$  (the linear response regime) and NNM when  $I_1$  has a value remote from zero. In the case presented in Fig. 5(a), there are two intervals of the bias  $I_1$  where NNM occurs. This is to be contrasted with the averaged voltage  $v_1$  of the first junction, which is never negative. However, one can note a rough synchronization in the dependency of  $v_1$  and  $v_2$  on the bias  $I_1$ : simultaneous increases and decreases of both voltages in some intervals of the bias. Similar synchronization is also observed in the temperature dependence—see Figs. 5(b) and 5(c). A closer inspection of Fig. 5(c) reveals another interesting result: There are two fundamentally different mechanisms generating anomalous transport. In the case  $I_1 = 0.025$ , the negative resistance is induced by thermal fluctuations [12].

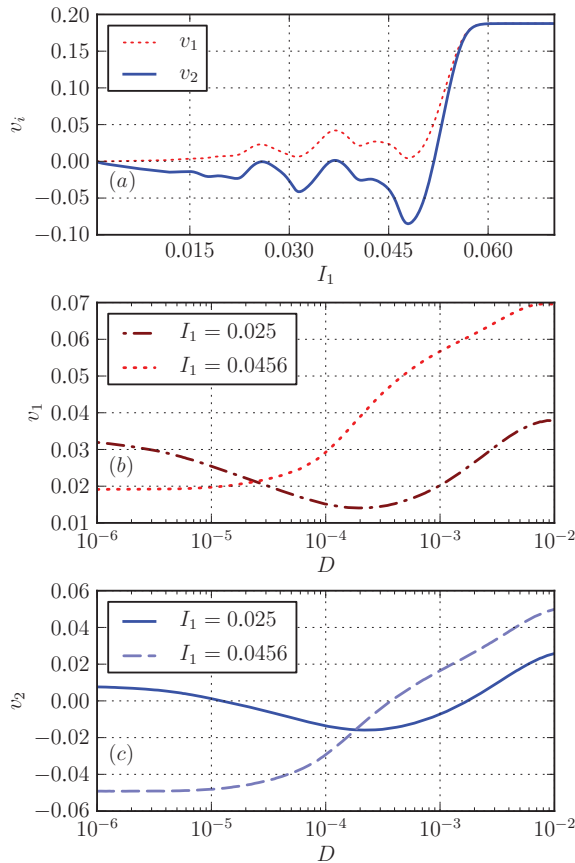


FIG. 5. (Color online) The long-time averaged voltages  $v_1$  and  $v_2$  of the first and second junctions, respectively. In (a) the dependence on the dc bias is depicted at a fixed temperature  $D = 2 \times 10^{-5}$ . (b) and (c) depict the temperature dependence of  $v_1$  and  $v_2$ , respectively. In (c), two distinct mechanisms for the negative voltage of the second junction can be observed: noise induced for  $I_1 = 0.025$  and deterministic, chaotic assisted for  $I_1 = 0.456$ . The parameters are the coupling strength  $\alpha = 0.77$ , amplitude  $a_1 = 1.7754$ , and frequency  $\omega = 0.1876$  of the ac driving,  $a_2 = I_2 = 0$ .

There is a finite interval of temperature where this effect occurs. For very low temperatures (lower than  $1.2 \times 10^{-5}$ ) and high temperatures (greater than  $2 \times 10^{-3}$ ) the voltage  $v_2$  takes positive values. Between these two temperatures, the voltage is negative. In contrast, in the case  $I_1 = 0.0456$ , the negative resistance is generated purely by deterministic dynamics, and this mechanism is described in detail in Ref. [15]. Even in zero temperature  $D = 0$ , the resistance is negative. For this chaos-assisted mechanism, temperature plays a destructive role: Increasing temperature monotonically diminishes the negative voltage  $v_2$ , and after crossing zero at some critical temperature, the voltage assumes positive values.

From the above analysis it follows how we can control the transport properties of the second junction. We should choose an optimal regime by fixing the temperature and the parameters of the ac driving, and next change the dc current  $I_1$  applied to the first junction. In this way we can change the sign and amplitude of the averaged stationary voltage of the second junction. It should be emphasized that the anomalous transport effects are all caused by the coupling between two junctions—without coupling, the negative voltage vanishes. Figure 6 shows how the average voltage of the second junction depends on the coupling constant  $\alpha$ . We note that there are always finite windows of  $\alpha$  for which this effect can be observed. The location and size of these windows depend on values of other system parameters. For the regimes depicted in Fig. 5, we illustrate it in Fig. 6. In the case when the bias is  $I_1 = 0.025$ , there are four distinct intervals of the coupling constant for which the voltage  $v_2$  is negative. In the case  $I_1 = 0.0456$ , in turn, there are two distinct intervals of the coupling constant for which the voltage  $v_2$  is negative.

We want to point out that adding a constant bias to the second particle has a destructive impact on the negative mobility—the higher the value of  $I_2$ , the smaller the parameter area where negative mobility can be observed. For instance, we find 0.1 to be the limiting value of  $I_2$  at which areas of negative mobility cease to exist everywhere in the analyzed area when  $I_1 = 0.05$ . Increasing  $I_2$  has a general smoothing effect on the transport properties of the second particle—at higher values

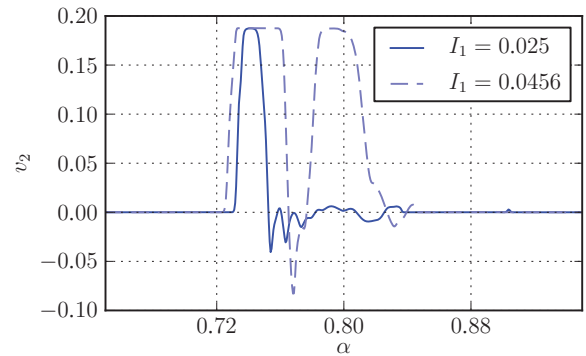


FIG. 6. (Color online) The influence of the coupling parameter  $\alpha$  on transport properties of the second junction. The long-time averaged voltage  $v_2$  across the junction is depicted for two fixed values of the externally applied dc current  $I_1$  and  $D = 2 \times 10^{-5}$ . For  $I_1 = 0.025$  there are four intervals of  $\alpha$  where the voltage  $v_2$  is negative, while for  $I_1 = 0.0456$  there are two intervals of such  $\alpha$ . Other parameters are the same as in Fig. 5.



of  $I_2$ , the fine details visible for  $I_2 = 0$  gradually disappear. Increasing  $I_2$  also causes all features to move slightly toward higher values of  $a_1$  and  $\omega$ . It is therefore possible to find areas of the parameter space where  $I_2$  induces a negative mobility effect, as well as areas where it increases the strength of an existing negative mobility effect.

#### IV. SUMMARY

It is of great importance to construct simple models which make it possible to explain and clarify the understanding of unusual transport properties not only in physical but also in biological systems such as, e.g., the bidirectionality of the net cargo transport inside living cells [21]. Such models can be used for fundamental studies of transport control and can also serve as a basis for the construction of more realistic and quantitative models for transport of interacting carriers in collective systems. In this paper, we constructed a minimal model of two coupled elements, described in terms of overdamped dynamics, which exhibits negative mobility both in the linear and nonlinear regimes. There are regimes where both particles can be transported in the same direction as the external static force acting on the first particle. There are also regimes where only the second particle can be transported

in the opposite direction to the bias. We propose to exploit these properties of the system for indirect control of transport properties of the second particle by manipulating the first particle only. The interaction between two particles constitutes a crucial ingredient of the model: Without coupling, the second particle is not transported at all, while the first particle can be transported only in the direction of the dc bias. The system can be manipulated in other ways. The source of energy, the driving  $I_1(t) = I_1 + a_1 \cos(\omega t)$ , can be replaced by another source of energy such as, e.g., an unbiased multiharmonic force or, in biological systems, chemical reactions. The model can be generalized to more than just two particles or by taking into account inertial effects. Finally, it is a promising topic which can stimulate experimentalists to perform measurements testing our findings in systems of two coupled Josephson junctions, where the coupling can be precisely controlled by an external resistance and, moreover, to plan experiments for other systems.

#### ACKNOWLEDGMENT

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