Experimental evidence of the role of compound counting processes in random walk approaches to fractional dynamics

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We present dielectric spectroscopy data obtained for gallium-doped $Cd_{0.99}Mn_{0.01}Te$:Ga mixed crystals, which exhibit a very special case of the two-power-law relaxation pattern with the high-frequency power-law exponent equal to 1. We explain this behavior, which cannot be fitted by any of the well-known empirical relaxation functions, in a subordinated diffusive framework. We propose a diffusion scenario based on a renormalized clustering of a random number of spatio-temporal steps in the continuous-time random walk. Such a construction substitutes the renewal counting process, which is used in the classical continuous time random walk methodology, with a compound counting one. As a result, we obtain an appropriate relaxation function governing the observed nonstandard pattern, and we show the importance of the compound counting processes in studying fractional dynamics of complex systems.

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I. INTRODUCTION

The linear response theory [1] clearly plays a great role in physics, being the theoretical background for fundamental accomplishments in statistical physics. Its conventional formulation is based on two assumptions [2]: (i) The time evolution of the system variables is governed by Hamiltonian operators, and (ii) the external perturbation arising from an initial disturbance makes the system depart from canonical equilibrium weakly, and the linear response function is expressed in terms of the derivative of a stationary correlation function (of dipole orientation in the theory of relaxation, for example). This gives a rigorous and efficient approach to the temporal description of some Hamiltonian systems [3]. However, these foundations are refuted for many-body kernels in the dynamics of complex systems. The main cause for this is that the macroscopic evolution of such systems cannot be attributed to any particular object chosen from those forming the systems. Any form of many-body interactions in the complex systems should be introduced through cross-correlations between their different objects, yielding hence insurmountable mathematical problems. Therefore, it is very difficult, if not generally impossible, to describe the time evolution of many-body systems by Hamiltonian operators, and any stationary correlation function is not available at all. Since, in general, the dynamical processes in complex systems are strictly stochastic in nature, they should be analyzed in a corresponding manner.

For a description of relaxation and transport properties in such systems as glasses, liquid crystals, polymers, etc., the continuous-time random walk (CTRW) processes are one of the most useful mathematical tools. Despite their long history, starting with the brilliant Montroll-Weiss idea [4], the CTRWs are still far from being fully explored. Nevertheless, their connection with anomalous diffusion and fractional diffusion equations has been already recognized (see, e.g., [5]). Now the fractional diffusion equations find more and more applications in the different fields of physics, chemistry, and engineering [6,7]. Recently, progress in the understanding of this mathematical tool [8–12] has stimulated new developments in diffusive scenarios of the nonexponential relaxation phenomena and impedance effects [13–17].

In this paper, we present experimental data which confirm that the nonexponential relaxation behavior, in fact, is governed by compound counting processes strictly connected with clustered CTRWs [11]. In Sec. II, we study dielectric spectroscopy data measured for gallium-doped $Cd_{0.99}Mn_{0.01}Te$:Ga semiconducting mixed crystals possessing deep, metastable defects. We observe that this material exhibits such a relaxation pattern which cannot be fitted with any of the well-known empirical relaxation functions. Hence, we apply an alternative relaxation law, which is a modification of the result derived recently in [16]. In Sec. III we propose an anomalous diffusion scenario, based on the notion of a compound counting process, by means of which the observed relaxation behavior may be explained. We end with conclusions in Sec. IV.

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II. EXPERIMENT

Dielectric spectroscopy studies carried out on various physical systems reveal that a wide class of materials follows

the anomalous relaxation mechanism [18] represented by low- and high-frequency fractional power-law dependences of the imaginary part $\varepsilon''(\omega)$ of the complex dielectric permittivity $\varepsilon^*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$:

$$\begin{aligned} \varepsilon''(\omega) &\sim (\omega/\omega_p)^m, \omega \ll \omega_p, \\ \varepsilon''(\omega) &\sim (\omega/\omega_p)^{n-1}, \omega \gg \omega_p, \end{aligned} \tag{1}$$

where ω_p denotes the loss peak frequency and $0 < m,n < 1$. Depending on a mutual relation between the power-law exponents, two different types of relaxation responses can be distinguished. The relaxation response is called typical when the power-law exponents satisfy the relation $m \geqslant 1 - n$. In order to interpret this type of experimental data, the well-known Havriliak-Negami (HN) function [18,19]

$$\varphi^*_{HN}(\omega) = \frac{1}{[1 + (i\omega/\omega_p)^{\alpha}]^{\gamma}}, \quad 0 < \alpha,\gamma < 1, \tag{2}$$

is used, $\varepsilon^*(\omega) = (\varepsilon_0 - \varepsilon_\infty)\varphi_{HN}{}^*(\omega) + \varepsilon_\infty$, where ε_0 is the static permittivity and ε_∞ represents the asymptotic value of the dielectric permittivity at high frequencies. For the HN function (2), the two-power-law property (1) is fulfilled with the power-law exponents $m = \alpha$ and $1 - n = \alpha\gamma$. For a long time period, the HN function with extended parameters in the range $0 < \alpha,\alpha\gamma < 1$ was also used to fit the less typical relaxation data for which the power-law exponents yield the opposite inequality, $m < 1 - n$. Unfortunately, none of the known relaxation models [14,20] can justify the values $\gamma > 1$ appearing in the extended range of the power-law exponents. Recent progress in stochastic modeling of relaxation processes has resulted in the derivation of an alternative relaxation pattern [14,16] underlying the less typical responses:

$$\varphi^*(\omega) = 1 - \frac{1}{[1 + (i\omega/\omega_p)^{-\alpha}]^{\gamma}}, \quad 0 < \alpha,\gamma < 1. \tag{3}$$

This function exhibits the two-power-law property (1) with the power-law exponents $m = \alpha\gamma$ and $1 - n = \alpha$. It not only properly describes the less typical class of relaxation responses but also relates the experimentally observed power-law properties with random characteristics of the investigated system. Let us notice that both formulas (2) and (3) can be used as fitting functions also with parameters $\alpha = 1$ and/or $\gamma = 1$. In such cases, they result from slightly different diffusion scenarios and may exhibit only one fractional power law, or even none.

The less typical relaxation pattern is observed in semiconducting mixed crystals of $Cd_{0.99}Mn_{0.01}Te$:Ga possessing deep metastable defects–the so-called DX centers. It is widely accepted that, depending on the position of gallium (Ga) dopants in the CdMnTe lattice, shallow donor states or deep metastable traps may be formed [21]. As shown in Ref. [22], the relaxation response of $Cd_{0.99}Mn_{0.01}Te$:Ga is mainly influenced by the presence of deep, metastable traps within the band gap of the investigated Au-$Cd_{0.99}Mn_{0.01}Te$ Schottky junction. For purposes of the present study, we analyzed the frequency-domain response of two samples of the same $x = 0.01$ manganese (Mn) content, labeled as sample 1 and sample 2, respectively. Both of the samples possess the same net donor concentration of approximately $10^{15}cm^{-3}$ estimated from the capacitance-voltage measurements. A

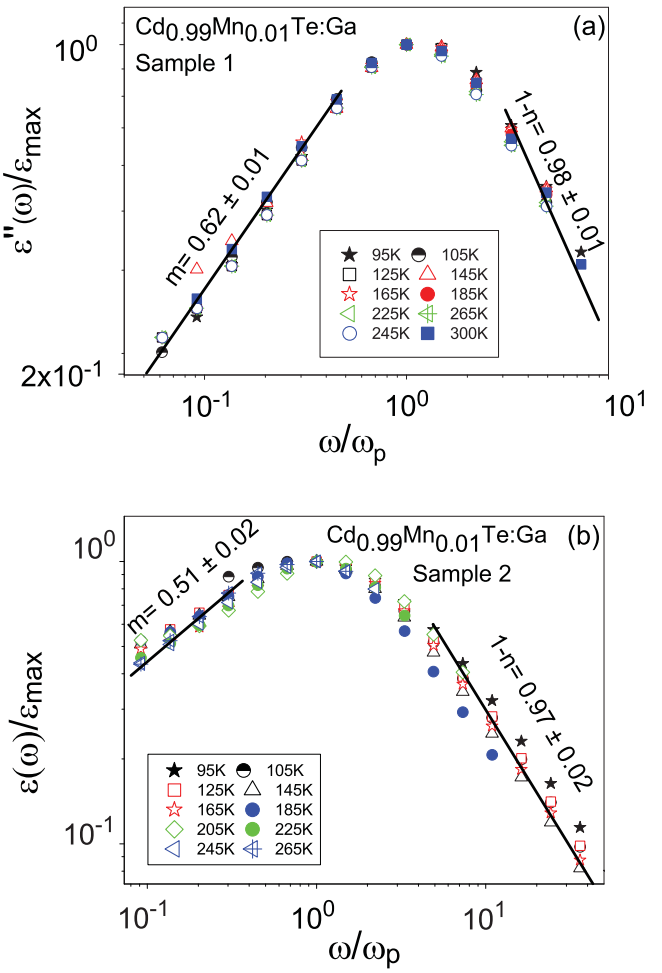

FIG. 1. (Color online) Normalized imaginary part of dielectric permittivity obtained for two samples of $Cd_{0.99}Mn_{0.01}Te$:Ga. Both samples exhibit two-power-law relaxation behavior with power-law exponents satisfying relaxation $m < 1 - n$.

description of the sample preparation can be found in detail elsewhere [23]. Gold Schottky contacts were thermally evaporated on the front side of the samples. Measurements were performed at zero bias using a Novocontrol impedance analyzer. An applied ac probe signal amplitude was equal to 10 mV.

In Fig. 1 the normalized imaginary part of the dielectric permittivity for two samples, investigated in a broad temperature range, is presented. It is clear from the plot that both samples exhibit the less typical, two-power-law relaxation pattern $m < 1 - n$, however, with different values of the low-frequency power-law exponent m. The change in values of this exponent in the samples of the same manganese content and gallium concentration may be associated with locally different surroundings of DX centers contributing to the effective relaxation response of the investigated sample.

In Fig. 2 the experimental data fitted by means of formula (3) are presented. It can be observed that this function perfectly covers the experimental data points. Values of the fitting parameters are collected in Fig. 3. Both α and γ values are temperature independent. It should be pointed out that for both analyzed samples, the α parameter remains the same,

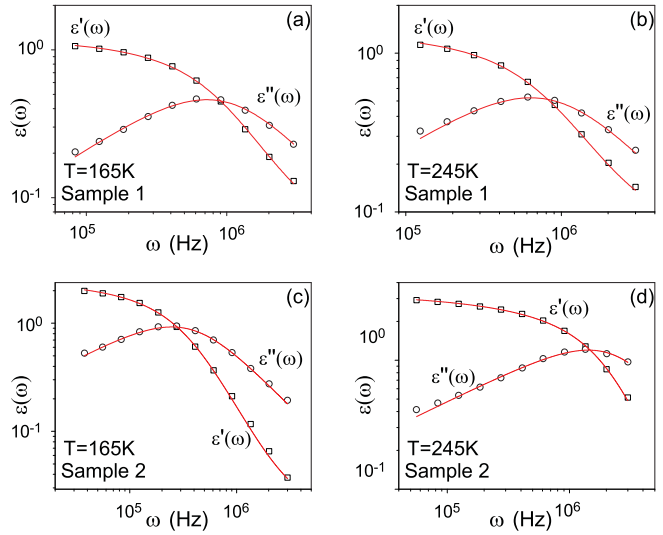

FIG. 2. (Color online) Sample real and imaginary part of the permittivity data as a function of frequency obtained for the two investigated samples of $Cd_{0.99}Mn_{0.01}Te$:Ga at various temperatures. Solid lines represent function (3).

whereas the value of γ is significantly different depending on the sample considered. Since α is approximately equal to 1, we observe the lack of the high-frequency fractional power-law behavior. Such a relaxation pattern, being a very special case of Eq. (1), suggests a nonstandard diffusion scenario underlying the experimental result.

III. DIFFUSION SCENARIO

The CTRW process $R(t)$ determines the total distance reached by a random walker until time t. It is characterized by a sequence of independent and identically distributed (i.i.d.) spatio-temporal random steps (R_i, T_i), $i \geqslant 1$. If we assume stochastic independence between jumps R_i and waiting times T_i, we get a decoupled random walk; otherwise we deal with

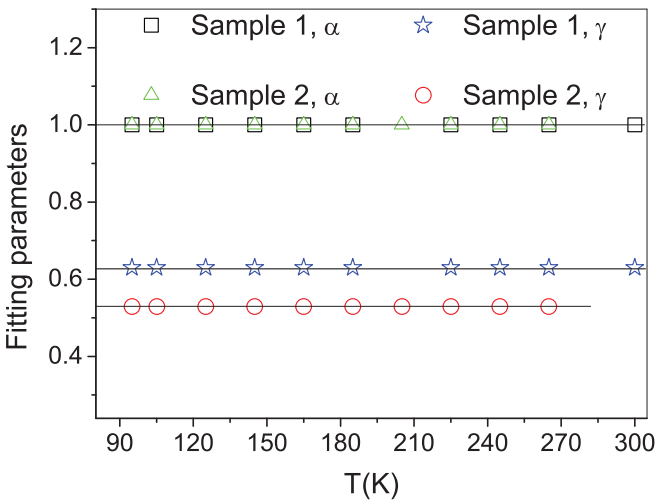

FIG. 3. (Color online) The values of fitting parameters obtained for two samples of $Cd_{0.99}Mn_{0.01}Te$:Ga obtained at various temperatures. Sample 1: $\alpha = 1.00 \pm 0.01$ and $\gamma = 0.63 \pm 0.01$; Sample 2: $\alpha = 1.00 \pm 0.01$ and $\gamma = 0.53 \pm 0.01$.

a coupled CTRW. The distance reached by the walker at time t is given by the following sum:

$$R(t) = \sum_{i=1}^{\nu(t)} R_i, \tag{4}$$

where $\nu(t) = \max\{n : \sum_{i=1}^{n} T_i \leqslant t\}$ counts the performed steps.

Theoretical studies of the relaxation phenomenon in the above framework are based on the idea of an excitation undergoing (generally anomalous) diffusion in the system under consideration [5]. The relaxation function $\phi(t)$ is then defined by the inverse Fourier or the Laplace transform of the diffusion front $\bar{R}(t) \approx R(t/\tau_0)/f(\tau_0)$, where the dimensionless rescaling parameter is $\tau_0 \approx 0$, and $f(\tau_0)$ is the appropriately chosen renormalization function. The diffusion front $\bar{R}(t)$ approximates a position at time t of the walker performing rescaled spatio-temporal steps $[R_i/f(\tau_0), \tau_0 T_i]$. The characteristics of the relaxation process are related to the properties of the diffusion front resulting from assumptions imposed on the spatio-temporal steps of the random walk. For example, the decoupled CTRW with power-law waiting-time distributions [i.e., with the random variables T_i satisfying $\text{Prob}(T_i \geqslant t) \sim (t/t_0)^{-a}$ as $t \to \infty$ with some $0 < a < 1$ and $t_0 > 0$] leads to the Cole-Cole relaxation [24]. However, the frequency-domain Cole-Cole relaxation with the corresponding time-domain Mittag-Leffler pattern is only one of the cases measured in various experiments with complex media, and the derivation of those more general patterns requires consideration of diffusion scenarios based on a compound coupled CTRW representation. It should be pointed out that the simple coupling of type $R_i \sim T_i^p$ (with positive power exponent p) does not lead behind the Cole-Cole relaxation [25]. In contrast, by introducing a dependence between the jumps and waiting times by a random clustering procedure, we can obtain another empirical relaxation law like the Cole-Davidson or Havriliak-Negami patterns [14,16,26]. Below, we present the diffusion scenario which leads directly to the results discussed in the preceding section.

Let M_j be a sequence of i.i.d. positive integer-valued random variables independent of the pairs (R_i, T_i). Next, assume that the jumps and waiting times are assembled into clusters of random sizes, $M_1, M_2, \ldots$. This assumption allows one to transform the sequence of spatio-temporal steps (R_i, T_i) into a new sequence $(\tilde{R}_j, \tilde{T}_j)$ of random sums,

$$\begin{aligned} (\tilde{R}_1, \tilde{T}_1) &= \sum_{i=1}^{M_1} (R_i, T_i), \\ (\tilde{R}_j, \tilde{T}_j) &= \sum_{i=M_1+\cdots+M_{j-1}+1}^{M_1+\cdots+M_j} (R_i, T_i), \quad j \geqslant 2. \end{aligned} \tag{5}$$

Then the position $R^M(t)$ of the walker is determined by $(\tilde{R}_j, \tilde{T}_j)$ and, in accordance with the general formula (4), it is given by

$$R^M(t) = \sum_{j=1}^{\tilde{\nu}(t)} \tilde{R}_j, \tag{6}$$

where $\tilde{\nu}(t) = \max\{n : \sum_{j=1}^{n} \tilde{T}_j \leqslant t\}$. The dependence between the jumps $\tilde{R}_j$ and the waiting times $\tilde{T}_j$ of the coupled CTRW process $R^M(t)$ is determined by the distribution of the cluster sizes M_j [11].

In the simple case when the waiting times are represented by equal intervals in time, i.e., $T_i = \Delta t$, we have

$$R(t) = \sum_{i=1}^{\lfloor t/\Delta t \rfloor} R_i, \tag{7}$$

where $\lfloor \cdot \rfloor$ denotes the integer part, while the clustering procedure (5) yields $\tilde{T}_j = M_j \Delta t$ for $j \geqslant 1$, and the coupled process $R^M(t)$ in (6) takes an equivalent form

$$R^M(t) = \sum_{i=1}^{U^M[\tilde{\nu}(t)]} R_i. \tag{8}$$

Here, $U^M[\tilde{\nu}(t)]$ is a compound counting process obtained from $U^M(n) = \sum_{j=1}^{n} \tilde{T}_j/\Delta t = \sum_{j=1}^{n} M_j$, and $\tilde{\nu}(t) = \max\{n : \sum_{j=1}^{n} \tilde{T}_j \leqslant t\} = \max\{n : U^M(n) \leqslant t/\Delta t\}$. Observe that formula (8) is an analog of (7), with the compound counting process $U^M[\tilde{\nu}(t)]$ substituting the deterministic number $\lfloor t/\Delta t \rfloor$ of performed jumps R_i. The counting process $U^M[\tilde{\nu}(t)]$ is always less than $\lfloor t/\Delta t \rfloor$, and it is hence a special case of the undershooting compound counting process [14]. It is also a clear signature of the spatio-temporal coupling provided by the clustering procedure (5).

The idea of compound counting processes in the CTRW approach is not new in physics. The resulting CTRW processes were examined in the context of the rareness hypothesis in the fractal-time random walk models (see, e.g., [27,28]). In general, the compound counting process cumulates a random number of random events. Physical situations where the relevance of this scheme holds are numerous. For instance, consider the energy release of individual earthquakes in geophysics, the random magnitude of claims sequence in insurance risk theory, or the random water inputs flowing into a dam in hydrology, where summing the individual contributions yields the total amount of the studied physical magnitude over certain time intervals.

The diffusion front $\bar{R}^M(t)$ related to (8) takes the subordinated form

$$\bar{R}^M(t) \stackrel{d}{=} X[Z(t)], \tag{9}$$

where the parent process $X(\tau)$ is just the diffusion front corresponding to the simple random walk $R(t)$ given by (7), while the undershooting subordinator $Z(t)$ corresponds to the limit of the rescaled counting process $U^M[\tilde{\nu}(t/\tau_0)]/g(\tau_0)$ as $\tau_0 \to 0$ [with renormalizing function $g(\tau_0)$ chosen appropriately]. Both the parent process and the undershooting subordinator are well defined if the distributions of the spatial steps R_i and cluster sizes M_j satisfy some conditions, referring to their asymptotic behaviors. In particular, if $\langle M_j \rangle < \infty$ (or equivalently $\langle \tilde{T}_j \rangle < \infty$), then we have $Z(t) = t/\Delta t$. However, by taking into account clustering with a heavy-tailed cluster-size distribution,

$$\text{Prob}(M_j \geqslant m) \underset{m\to\infty}{\sim} (m/c)^{-\gamma},$$

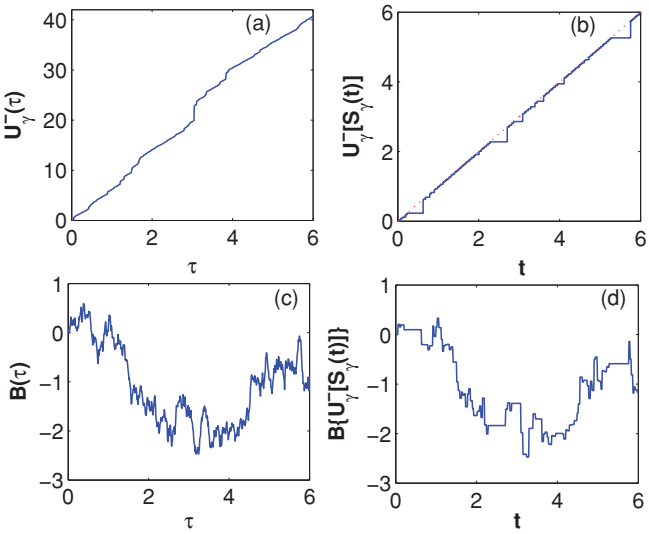

FIG. 4. (Color online) (a) Left-continuous process $U_\gamma^-(\tau)$, (b) Compound counting process $Z(t) = U_\gamma^-[S_\gamma(t)]$ as a new arrow of time, (c) 1D trajectory of standard Brownian motion (parent process), and (d) 1D trajectory of Brownian motion subordinated by $Z_U(t)$. For all plots, the index γ equals 0.9.

with the tail exponent $0 < \gamma < 1$ and some scaling constant $c > 0$, we obtain the compound form $Z(t) = U_\gamma^-[S_\gamma(t/\Delta t)]$, where $U_\gamma^-(\tau) = \lim_{x\to\tau_-} U_\gamma(x)$ is the left limit of the γ-stable subordinator $U_\gamma(\tau)$, and $S_\gamma(t) = \inf\{t \geqslant 0 : U(\tau) > t\}$ is its inverse process. For a fixed $t > 0$, the compound undershooting subordinator $U_\gamma^-[S_\gamma(t/\Delta t)]$ has the generalized arcsine distribution rescaled by $t/\Delta t$. The corresponding probability density function reads

$$p_\gamma(t,\tau) = \frac{\sin \pi\gamma}{\pi}\, \tau^{\gamma-1}(t/\Delta t - \tau)^{-\gamma}, \quad 0 < \tau < t/\Delta t\,. \tag{10}$$

It is easy to check that all moments of the random variable $U_\gamma^-[S_\gamma(t/\Delta t)]$ are finite.

The numerical approximation of the process $U_\gamma^-[S_\gamma(t/\Delta t)]$ is shown in Fig. 4. For simplicity, we take $\Delta t = 1$. To simulate the process $U_\gamma^-[S_\gamma(t)]$, one only needs to generate the values $U_\gamma(n\Delta\tau)$ [see Fig. 4(a)], where $n = 1,2,\ldots$, and $\Delta\tau$ is the step length. This can be carried out by the standard method of summing up the independent and stationary increments of the Lévy process [29]. Then the approximation of the process $U_\gamma^-[S_\gamma(n\Delta t)]$ is a simple continuous-time random walk in which each waiting time is exactly equal to the jump. In the result, Fig. 4(b) demonstrates the process $U_\gamma^-[S_\gamma(t)]$ nondecreasing in time.

As far as the parent process is concerned, for simplicity, but without loss of generality, we can consider the standard Brownian motion $B(t)$ that can easily be obtained if we take into account the spatial jumps satisfying conditions $\langle R_i \rangle = 0$ and $\langle R_i^2 \rangle < \infty$ [see Fig. 4(c)]. Then the subordinated process $B\{U_\gamma^-[S_\gamma(t)]\}$ represents the corresponding diffusion front. Its sample paths are shown in Fig. 4(d). The main feature of the subordinated process consists of random jumping along random trajectories of the parent process. For better visualization of this feature, we can consider a walk on a plane with

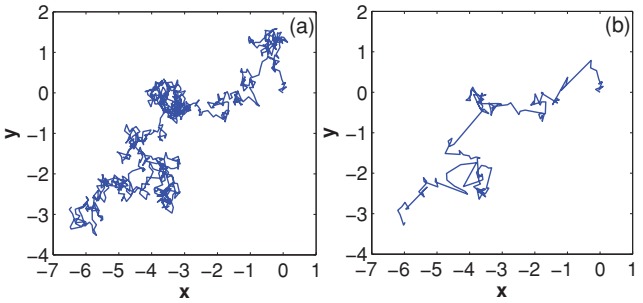

FIG. 5. (Color online) 2D trajectories of (a) standard Brownian motion and (b) Brownian motion under compound subordination with $\gamma = 0.7$.

two-dimensional (2D) vector jumps satisfying analogous conditions that lead to 2D Brownian motion as a parent process; see Fig. 5. The difference between Figs. 5(a) and 5(b) is that the walker, moving along a Brownian trajectory in the presence of the compound subordination, stops from time to time and overjumps through intermediate positions in the Brownian trajectory. In other words, the trajectory of Fig. 5(b) is something like a partition of the trajectory in Fig. 5(a) on random intervals. The appearance of the overjumps, as applied to the relaxation of semiconductors with metastable defects described in Sec. II, is connected with long-range interactions among DX centers.

The probability density function of $B\{U_\gamma^-[S_\gamma(t/\Delta t)]\}$ reads

$$p(x,t) = \int_0^\infty p^B(x,\tau)\, p_\gamma(t,\tau)\, d\tau, \tag{11}$$

where $p^B(x,\tau) = \frac{1}{\sqrt{2\pi\tau}} e^{-x^2/2\tau}$. Taking the inverse Fourier transform with respect to x for $p(x,t)$ in Eq. (11), we derive the relaxation function $\phi(t) = \langle e^{-ik\tilde{R}^M(t)}\rangle$ that characterizes the temporal decay of a given macroscopic mode k. We get

$$\phi(t) = \frac{\sin\pi\gamma}{\pi} \int_0^{t/\Delta t} e^{-(k^2/2)\tau}\, \tau^{\gamma-1} (t/\Delta t - \tau)^{-\gamma}\, d\tau, \tag{12}$$

where $k^2/2$ may be denoted as a characteristic frequency ω_p of the relaxing system. The integral representation of the relaxation function may be substituted by a series expansion observing that the relaxation function (12) can be expressed as $\phi(t) = E^\gamma_{1,1}(-\omega_p\, t)$, where

$$E^\gamma_{\alpha,\beta}(x) = \sum_{k=0}^\infty \frac{(\gamma,k)\, x^k}{\Gamma(k\alpha+\beta)k!}, \quad \alpha,\beta > 0,$$

is the generalized Mittag-Leffler function [30]. Here, $(\gamma,k) = \gamma(\gamma+1)(\gamma+2)\ldots(\gamma+k-1)$ is the Appell's symbol with $(\gamma,0) = 1$, $\gamma \neq 0$.

Based on the Laplace image of $\phi(t)$, with respect to t, we find the kinetic equation of relaxation in the pseudodifferential form

$$\left(\frac{d}{dt} + \omega_p\right)^\gamma \phi(t) = \frac{t^{-\gamma}}{\Gamma(1-\gamma)},$$

with the initial condition $\phi(0) = 1$. For the experimental studies of the dielectric spectroscopy data (dielectric permittivity or the corresponding susceptibility), the frequency-domain representation of the latter function,

$$\varphi^*(\omega) = \int_0^\infty e^{-i\omega t} \left(-\frac{d\phi(t)}{dt}\right) dt, \tag{13}$$

is of interest. By taking the Fourier transform, we get

$$\varphi^*(\omega) = 1 - \left(\frac{i\omega/\omega_p}{1+i\omega/\omega_p}\right)^\gamma, \quad 0 < \gamma < 1. \tag{14}$$

Let us note that formula (14) coincides with (3) for $\alpha = 1$. Moreover, for $\gamma = 1$, it takes the form of the Debye response, corresponding to the spatio-temporal clustering with $\langle M_j\rangle < \infty$, where the undershooting subordinator becomes deterministic: $Z(t) = t/\Delta t$.

Considering the dielectric susceptibility $\chi^*(\omega) = \chi'(\omega) - i\chi''(\omega) = \chi(0)\varphi^*(\omega)$, with $\varphi^*(\omega)$ as in (14), one can show that the real and imaginary parts of the susceptibility fulfill the following frequency-independent relations:

$$\begin{aligned} \lim_{\omega\to 0} \frac{\chi''(\omega)}{\chi'(0) - \chi'(\omega)} &= \tan\left(\frac{\pi\gamma}{2}\right), \\ \lim_{\omega\to\infty} \frac{\chi''(\omega)}{\chi'(\omega)} &= \infty. \end{aligned} \tag{15}$$

This means that for small ω, the gallium-doped $Cd_{0.99}Mn_{0.01}Te$ mixed crystals obey the energy criterion [19], while for large ω, the high-frequency energy lost per cycle does not have a constant relationship to the extra energy that can be stored by a static field. The similar feature takes place in the Cole-Davidson law where, however, the energy criterion is obeyed for large ω only. As is well known [19], the presence of the universal law in the relaxation in gallium-doped $Cd_{0.99}Mn_{0.01}Te$ mixed crystals is caused strictly by many-component interactions. This effect is just accounted for using the compound counting random processes.

IV. CONCLUSIONS

In the diffusive framework, we have brought to light the fractional dynamics of gallium-doped $Cd_{0.99}Mn_{0.01}Te$:Ga mixed crystals, for which the particular case of less typical relaxation behavior with the high-frequency power-law exponent $1 - n = 1$ has been detected. We have proposed the subordination scenario of anomalous diffusion underlying the observed pattern, based on the random, heavy-tailed, spatio-temporal clustering procedure applied to the random walk hidden behind the standard Brownian motion. Such a procedure has led to a substitution of the renewal counting process, used in the classical CTRW model, with the undershooting compound counting one. As a consequence, the explicit stochastic structure of the diffusion front as a subordinated Brownian motion has been obtained. The resulting frequency-domain relaxation function has been shown to fit the studied dielectric spectroscopy data.

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[1] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II* (Springer-Verlag, Berlin, 1985).
[2] P. Allegrini, M. Bologna, L. Fronzoni, P. Grigolini, and L. Silvestri, Phys. Rev. Lett. **103**, 030602 (2009).
[3] U. Balucani, M. H. Lee, and V. Tognetti, Phys. Rep. **373**, 409 (2003).
[4] E. W. Montroll and G. H. Weiss, J. Math. Phys. **6**, 167 (1965).
[5] R. Metzler and J. Klafter, Phys. Rep. **339**, 1 (2000).
[6] E. K. Lenzi, L. R. Evangelista, and G. Barbero, J. Phys. Chem. B **113**, 11371 (2009).
[7] J. Bisquert and A. Compte, J. Electroanal. Chem. **499**, 112 (2001).
[8] M. M. Meerschaert, D. A. Benson, H.-P. Scheffler, and P. Becker-Kern, Phys. Rev. E **66**, 060102(R) (2002).
[9] M. M. Meerschaert and H.-P. Scheffler, J. Appl. Prob. **41**, 623 (2004).
[10] A. Piryatinska, A. I. Saichev, and W. A. Woyczynski, Physica A **349**, 375 (2005).
[11] A. Jurlewicz, Diss. Math. **431**, 1 (2005).
[12] M. Magdziarz and K. Weron, Physica A **367**, 1 (2006).
[13] A. Weron and M. Magdziarz, Europhys. Lett. **86**, 60010 (2009).
[14] K. Weron, A. Jurlewicz, M. Magdziarz, A. Weron, and J. Trzmiel, Phys. Rev. E **81**, 041123 (2010).
[15] D. L. Sidebottom, Rev. Mod. Phys. **81**, 999 (2009).
[16] A. A. Stanislavsky, K. Weron, and J. Trzmiel, Europhys. Lett. **91**, 40003 (2010).
[17] J. R. Macdonald, J. Phys. Condens. Matter **22**, 495101 (2010).
[18] A. K. Jonscher, *Dielectric Relaxation in Solids* (Chelsea, London, 1983).
[19] A. K. Jonscher, *Universal Relaxation Law* (Chelsea, London, 1996).
[20] Y. P. Kalmykov, W. T. Coffey, D. S. F. Crothers, and S. V. Titov, Phys. Rev. E **70**, 041103 (2004).
[21] C. H. Park and D. J. Chadi, Phys. Rev. B **52**, 11884 (1995).
[22] J. Trzmiel, E. Placzek-Popko, E. Zielony, and Z. Gumienny, Acta Phys. Pol. A **116**, 956 (2009).
[23] J. Trzmiel, K. Weron, and E. Placzek-Popko, J. Appl. Phys. **103**, 114902 (2008).
[24] K. Weron and M. Kotulski, Physica A **232**, 180 (1996).
[25] M. Kotulski, in *Chaos The Interplay Between Stochastic and Deterministic Behaviour*, edited by P. Garbaczewski, M. Wolf, and A. Weron, Lecture Notes in Physics Vol. 457 (Springer, Berlin, 1995), p. 471.
[26] A. Jurlewicz and K. Weron, Acta Phys. Pol. **39**, 1055 (2008).
[27] H. Weissmann, G. H. Weiss, and S. Havlin, J. Stat. Phys. **57**, 301 (1989).
[28] J. Klafter and M. F. Schlesinger, J. Phys. Chem. **98**, 7366 (1994).
[29] A. Janicki and A. Weron, *Simulation and Chaotic Behavior of α-Stable Stochastic Processes* (Marcel Dekker, New York, 1994).
[30] A. M. Mathai, R. K. Saxena, and H. J. Haubold, *The H-Function: Theory and Applications* (Springer, Amsterdam, 2009).