Dynamics of the Blume-Capel model with quenched diluted single-ion anisotropy in the neighborhood of equilibrium states

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The relaxation dynamics of a Blume-Capel model with a quenched diluted crystal field is formulated by a method combining the statistical equilibrium theory and the thermodynamics of linear irreversible processes. Using a mean-field approximation for the magnetic Gibbs free-energy production, a generalized force and a current are defined within the irreversible thermodynamics. Next the kinetic equation for the magnetization is obtained within linear response theory. Finally, the temperature dependence of the relaxation time in the neighborhood of the phase-transition points is obtained by solving the kinetic equation of the magnetization. We find that the relaxation time of the order parameter diverges near the critical and multicritical points, which corresponds to the familiar critical slowing down. On the other hand, it displays different behavior at the first-order phase transitions. It has a jump discontinuity at the first-order phase-transition temperatures. Moreover, the z dynamic critical exponent is calculated and compared with the z values obtained for a diverse class of systems, and good agreement is found with our results.

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I. INTRODUCTION

Investigation of the effect of disorder on various kinds of condensed matter orderings remains an open field of inquiry [1]. Random field effects on magnetic systems have been systematically studied, not only for their theoretical interests but also for their experimental realization implications [2]. Long-range order under random fields has been addressed within a variety of theoretical studies: the domain energy argument of Imry and Ma, renormalization-group treatment of domain interfaces [3], supersymmetry identifications [4], and perturbative expansions about the upper critical dimension [5]. In addition, it has been shown that the long-range order can be deduced from strong-coupling rescaling behavior [6]. It is a well-known fact that the degree to which quenched randomness affects the nature of a phase transition varies. For instance, in low dimensions random fields can altogether eliminate the phase transitions [6–8] and otherwise affects the numerical values of the critical exponents [5]. Especially for the case of a spin-1 Blume Emery Griffiths model [9] that exhibits multicritical behavior, it has been shown by renormalization group arguments that first-order transitions are replaced by continuous transition, and consequently tricritical points and critical endpoints are depressed in temperature, and a finite amount of disorder will suppress them [10]. We notice that the simplest model exhibiting a tricritical phase diagram in the absence of randomness is the Blume-Capel model [11]. It is a regular spin-1 Ising model, which was first used to model ^{3}He–^{4}He mixtures, and it plays a fundamental role in the multicritical phenomena associated with various physical systems, such as metalic alloys [12], liquid crystals [13], polymeric systems [14], and proteins [15].

The most interesting feature of the model is the existence of a tricritical point in the phase diagram represented in the plane of temperature versus crystal field. This phase diagram was first calculated within mean-field theory [11] and confirmed by Monte Carlo simulations [16]. The model has inspired a range methodologies to be applied to investigate the effect of crystal field disorder on the multicritical phase diagram of the Blume-Capel model, e.g., effective field theory [17] and the mean-field approach [18,19], as well as by introducing an external random field [20]. Recently Salmon and Tapia have studied the multicritical behavior of the Blume-Capel model with infinite-range interactions by introducing quenched disorder in the crystal field Δ_i, which is represented by a superposition of two Gaussian distributions [21]. On the other hand, the *nonequilibrium* behavior of the system has not been as thoroughly investigated because dynamical models of critical phenomena are of more speculative nature.

To the best of our knowledge, dynamics of the spin-1 Blume-Capel model with quenched diluted single ion anisotropy in the neighborhood of equilibrium states has not been studied by the methods of irreversible thermodynamics. In this paper, a method combining the statistical equilibrium theory of cooperative phenomena and the thermodynamics of linear irreversible processes is carried out for the Blume-Capel model under a two-valued random crystal field. This type of calculation was first performed for an AB-type alloy by Tanaka *et al*. [22] and AB-type ferromagnetic and antiferromagnetic Ising model by Barry [23] and Barry and Harrington [24]. Erdem and Keskin and co-workers investigated the dynamical behavior of the Blume-Emery-Griffiths model extensively [25,26]. Whereas Keskin and Canko has worked on the relaxation phenomena in spin-3/2 Ising system with bilinear and biquadratic interactions [27]. Gulpinar *et al*. investigated the relaxations dynamics of iron group dihalides with the same method [28].

Recently, the application of relaxation theory to a diverse class of systems has attracted much attention: Dziarmaga reviewed the dynamics of a quantum phase transition and relaxation to a steady state [29]; Etzkorn *et al*. performed in-field

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relaxation measurements of the spin glasses [30]; Atsarkin *et al*. investigated the spin relaxation in $La_{1-x}Ca_xMnO_3$ [31]; a theoretical study of the relaxation processes of the two-dimensional *XY* model has been performed by Lei and Zheng [32]; and a relaxation analysis of a quasi-one-dimensional frustrated *XY* model was made by Nogawa and Nemoto [33]. Additional applications of relaxation theory includes the relaxation theory of the antiferromagnetic Ising model on a triangular lattice [34], the relaxation process in the photoinduced neutral-ionic paraelectric-ferroelectric phase transition of tetrathiafulvalene-pchloranil [35], and the magnetic relaxation in antiferromagnetic nanoparticles [36]; and the relaxation processes in PbSe quantum dots of different sizes have been also investigated [37].

The paper is organized as follows: In Sec. II, we give a brief description of the model and its equilibrium properties within the framework of a mean-field approximation. In Sec. III, we obtain the magnetic *Gibbs free-energy production* for the Blume-Capel model with quenched diluted crystal field. Next we analyze, in Sec. IV, the relaxation behavior of the system near the critical and multicritical points. Finally, a summary and discussion of the results are given in the last section.

II. THE MODEL AND THE STATIC PROPERTIES IN THE MEAN-FIELD APPROXIMATION

The Hamiltonian of the system is given by

$$\hat{H} = -J \sum_{\langle i,j \rangle} S_{iz} S_{jz} + \sum_i D_i S_{iz}^2, \tag{1}$$

where the local spin variables S_i are restricted to taking the values $\pm 1,0$. The first term describes the ferromagnetic coupling ($J > 0$) between neighboring spins i and j. The second term describes the single-ion anisotropy. Finally D_i is a two-valued random crystal field distributed according to the probability distribution

$$F(D_i) = p\delta(D_i - D) + (1 - p)\delta(D_i). \tag{2}$$

In order to obtain the equilibrium properties of the model, we first have to calculate the magnetic Gibbs free energy following the definition given in Refs. [38,39]. Letting U, T, S, H, m, and N be spin-system configurational interaction energy, temperature, entropy, external magnetic field, magnetic moment, and number of spins, respectively, the magnetic Gibbs potential, $G_{\text{magnetic}} = U - TS - HNm$, can be written in the mean-field approximation for a given value of the magnetization m as

$$G_{\text{magnetic}} = \frac{NzJ}{2} m^2 + Nk_BT \int \ln\left[1 - \frac{\cosh\left(\frac{Jzm}{T}\right)}{2\cosh\left(\frac{Jzm}{T}\right) + \exp\left(\frac{D_i}{T}\right)}\right] \times F(D_i) dD_i - NHm. \tag{3}$$

Here k_B is Boltzmann constant [18]. In addition, it is convenient to introduce the reduced quantities

$$\theta = \frac{T}{zJ}, \qquad d = \frac{D}{zJ}, \qquad h = \frac{H}{zJ}. \tag{4}$$

Finally, by use of Eq. (2), the magnetic Gibbs potential can be written as

$$G_{\text{magnetic}} = NzJ\left(\frac{m^2}{2} + pd - hm - \theta\left\{p\ln\left[2\cosh\left(\frac{m}{\theta}\right) + \exp\left(\frac{d}{\theta}\right)\right] + (1-p)\ln\left[2\cosh\left(\frac{m}{\theta}\right) + 1\right]\right\}\right). \tag{5}$$

The equilibrium condition, $\delta G_{\text{magnetic}} = 0$, results in the following transcendental equation for the $h = 0$ case:

$$m_0 = 2\sinh\left(\frac{m_0}{\theta}\right)\left\{\frac{p}{2\cosh\left(\frac{m_0}{\theta}\right) + \exp\left(\frac{d}{\theta}\right)} + \frac{(1-p)}{2\cosh\left(\frac{m_0}{\theta}\right) + 1}\right\}. \tag{6}$$

The equilibrium phase diagrams of this system were studied extensively by Benyoussef *et al*. within the mean-field theory, and it was found that the spin-1 Blume-Capel model, with quenched diluted single-ion anisotropy, exhibits three kinds of phase diagrams in the (θ,d) plane depending on the value of p [18]. For the interval $(0.945 \leqslant p \leqslant 1)$, the phase diagram is identical to that of Blume-Capel model with homogenous crystal field; see Fig. 1(a). The ordered phase is separated from the disordered phase by a critical line. The transition along this line is of second order up to a tricritical point. At the tricritical point the transition becomes first order. This first-order part reaches the $\theta = 0$ axis at $d = 1/2p$. For $(0.926 \leqslant p \leqslant 0.945)$, the system still exhibits a tricritical behavior, but in this case the second-order transition line has a re-entrant part. Whereas the first-order line separates the ferromagnetic and the paramagnetic phases and reaches the $\theta = 0$ axis at $d = 1/2p$. In fact, this line also exhibits a re-entrant part. The two types of transitions are connected by a tricritical point; see Fig. 1(b). As one increases the concentration of vacancies in the crystal field, the phase diagram changes in nature: For $(\frac{8}{9} \leqslant p < 0.926)$, the transitions between the ferromagnetic and paramagnetic phases are of first order at low temperatures and strong crystal fields, while it is of second order at higher temperatures. In addition the second-order transition line has a re-entrant part. A portion of the second-order transition line is masked by a first-order transition line. This situation gives rise to a critical endpoint (CEP) where the second-order line meets two first-order lines; one of them, immersed in the ordered phase, ends at a bicritical endpoint (BCP); see Fig. 1(c). Finally, for $(p < \frac{8}{9})$, which is the threshold value of the nonzero crystal field concentration, the first-order phase transitions are totally eliminated, and all phase transitions are second order. Moreover, the ferromagnetic phase is stable for an infinitely large value of d at temperatures lower than $\theta_0 = \frac{2}{3}(1-p)$. The phase diagram given in Fig. 1(d) represents the case for $p = 0.4$.

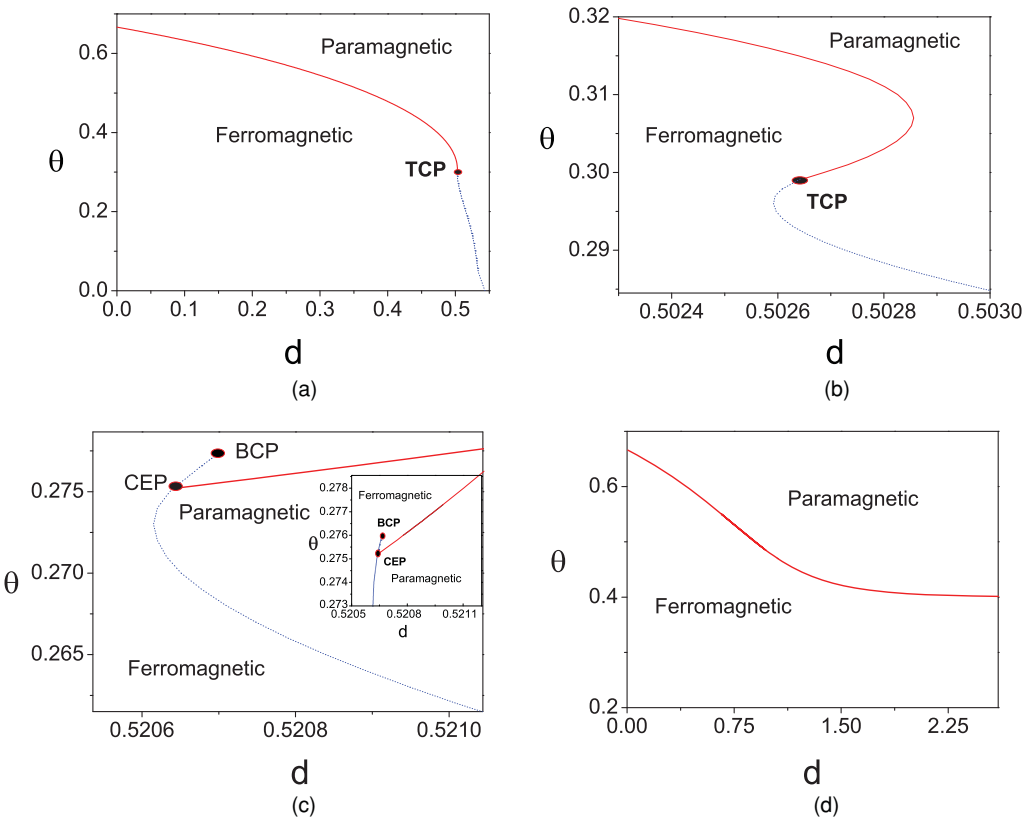

FIG. 1. (Color online) Mean-field phase diagram of the Blume-Capel model with quenched diluted single-ion anisotropy for (a) $p = 0.98$, (b) $p = 0.93$, (c) $p = 0.90$, and (d) $p = 0.4$. The dotted lines denote the coexistence curve and solid lines denote the λ line.

III. GIBBS POTENTIAL PRODUCTION FOR THE SPIN-1 BLUME-CAPEL MODEL WITH A RANDOM CRYSTAL FIELD IN A MAGNETIC FIELD

In order to study the relaxation of the magnetization to equilibrium in the spin-1 Blume-Capel model with quenched diluted single-ion anisotropy we assume a small uniform, external field is applied along the z axis only for a short while, which removes the system slightly from equilibrium, and we study the how rapidly the system relaxes back to thermal equilibrium. Throughout this study the external field will always be assumed to be sufficiently small to allow the spin system to be in the neighborhood of equilibrium. Consequently here we study only the final stage of the approach to equilibrium. In the case of the existence of a small deviation of the magnetic field from its equilibrium value ($\delta H = H - H_0$), the system will be removed slightly from equilibrium and a finite magnetic Gibbs free-energy production (ΔG) will arise:

$$\begin{aligned}\Delta \mathbf{G} &= \mathbf{G}(m,T,H) - \mathbf{G}^{(0)}(m_0,T,H_0)\\ &= \tfrac{1}{2}[a(m-m_0)^2 - 2b(H-H_0)(m-m_0)\\ &\quad + c(H-H_0)^2 + 2d(H-H_0)]. \end{aligned} \tag{7}$$

Here $\mathbf{G}^{(0)}$ is the equilibrium value of the free energy for the case $m = m_0, H = H_0 = 0$, and $\mathbf{G}(m,T,H)$ is the free energy in the neighborhood of equilibrium; the coefficients a–d are given as follows:

$$\begin{aligned} a &= \left(\frac{\partial^2 \mathbf{G}}{\partial m^2}\right)_{\text{eq}},\\ a &= zJ\left(1 - \theta\left\{ - \frac{4p\left[\sinh\left(\frac{m}{\theta}\right)\right]^2}{\theta^2\left[2\cosh\left(\frac{m}{\theta}\right) + e^{\frac{d}{\theta}}\right]^2}\right.\right.\\ &\quad + \frac{2p\cosh\left(\frac{m}{\theta}\right)}{\theta^2\left[2\cosh\left(\frac{m}{\theta}\right) + e^{\frac{d}{\theta}}\right]} + \frac{2(1-p)\cosh\left(\frac{m}{\theta}\right)}{\theta^2\left[2\cosh\left(\frac{m}{\theta}\right) + 1\right]}\\ &\quad \left.\left. - \frac{4(1-p)\left[\sinh\left(\frac{m}{\theta}\right)\right]^2}{\theta^2\left[2\cosh\left(\frac{m}{\theta}\right) + 1\right]^2}\right\}\right),\\ b &= -\left(\frac{\partial^2 \mathbf{G}}{\partial H \partial m}\right)_{\text{eq}} = NzJ,\\ c &= \left(\frac{\partial^2 \mathbf{G}}{\partial H^2}\right)_{\text{eq}} = 0,\\ d &= \left(\frac{\partial \mathbf{G}}{\partial H}\right)_{\text{eq}} = NzJm_0. \end{aligned} \tag{8}$$

IV. DERIVATION OF THE KINETIC EQUATION AND THE RELAXATION TIME FOR THE MAGNETIZATION

In the theory of irreversible thermodynamics, the time derivative of the ferromagnetic order parameter

(magnetization) is treated as the generalized flux (current) conjugate to its appropriate generalized force. One obtains the generalized force X_m conjugate to the current $\dot{m}$ by differentiating $\Delta\mathbf{G}$ with respect to $(m - m_0)$:

$$X_m = \frac{\partial \Delta \mathbf{G}}{\partial (m - m_0)} = a(m - m_0) - b(H - H_0). \tag{9}$$

In addition, the linear relations between the currents and forces may be expressed in terms of the phenomenological rate coefficient (L_{mm}):

$$\dot{m} = L_{mm} X_m. \tag{10}$$

Before starting the investigation of the longitudinal relaxation induced by the magnetic field, we emphasize the following preliminary remarks concerning our method. The Hamiltonian of the Blume-Capel model with a quenched diluted single-ion anisotropy is given by Eq. (1), where $\mathbf{S}_{lz}$ is the longitudinal component of the spin-1 Ising ($\mathbf{S} = 1$) spin angular momentum operator $\mathbf{S}_l$. This Hamiltonian does not admit longitudinal relaxation since the equation of motion for $\langle \mathbf{S}_{lz} \rangle$ vanishes identically:

$$\frac{d\langle \mathbf{S}_{lz} \rangle}{dt} = \int \left(\frac{J}{i\hbar} \sum_{\langle i,j \rangle} \langle [\mathbf{S}_{iz}\mathbf{S}_{jz}, \mathbf{S}_{lz}] \rangle + \sum_i D_i \langle [\mathbf{S}_{iz}^2, \mathbf{S}_{lz}] \rangle \right) \times F(D_i) dD_i = 0, \tag{11}$$

where $\langle ... \rangle$ signifies the appropriate ensemble average, $2\pi\hbar$ is Planck's constant, and $[\mathbf{S}_{iz}\mathbf{S}_{jz}, \mathbf{S}_{lz}]$ and $[\mathbf{S}_{iz}^2, \mathbf{S}_{lz}]$ denote the vanishing commutators $\mathbf{S}_{iz}\mathbf{S}_{jz}\mathbf{S}_{lz} - \mathbf{S}_{lz}\mathbf{S}_{iz}\mathbf{S}_{jz}$ and $\mathbf{S}_{iz}^2\mathbf{S}_{lz} - \mathbf{S}_{lz}\mathbf{S}_{iz}^2$, respectively. Therefore other operator quantities should be added to the Blume-Capel Hamiltonian, which does not commute with $\mathbf{S}_{lz}$, which will ensure transitions within the spin system and thereby allow longitudinal relaxation. It is well known that longitudinal relaxation is closely related to the spin-lattice relaxation time so that these added operator quantities should contain some kind of spin-lattice coupling, e.g., the frequently designated T_1 appearing in the longitudinal Bloch equation representation of spin-lattice relaxation in solids [23]. As discussed above, the method in the present paper contains the theory of irreversible processes, which is a phenomenological theory: The explicit form of the required operators will not be represented, but instead their effect will be embedded in the phenomenological rate coefficient L_{mm}, which itself must be obtained either in principle by a more powerful method or in practice by fit with the experimental data.

In order to obtain the relaxation time of the magnetization, one should consider the corresponding homogenous equation corresponding to the equilibrium value of the external field, i.e., $H = H_0 = 0$, where Eq. (10) becomes

$$\dot{m} = a L_{mm}(m - m_0). \tag{12}$$

The linearized equations of motions are obtained by using relations (9) in Eq. (10) and solved by assuming the following form for the solution ($m - m_0 \simeq e^{-\frac{t}{\tau}}$). Thus, one obtains the relaxation time as $\tau = -\frac{1}{aL_{mm}}$. If one inserts the first equation of Eq. (7) into Eq. (12), the relaxation time can be written as follows:

$$\tau(\theta) = -\frac{\frac{1}{zJL_{mm}}}{\left(1 - \theta\left\{ -\frac{4p\left[\sinh\left(\frac{m_0}{\theta}\right)\right]^2}{\theta^2\left[2\cosh\left(\frac{m_0}{\theta}\right) + e^{\frac{d}{\theta}}\right]^2} + \frac{2p\,\cosh\left(\frac{m_0}{\theta}\right)}{\theta^2\left[2\cosh\left(\frac{m_0}{\theta}\right) + e^{\frac{d}{\theta}}\right]} + \frac{2(1-p)\,\cosh\left(\frac{m_0}{\theta}\right)}{\theta^2\left[2\cosh\left(\frac{m_0}{\theta}\right) + 1\right]} - \frac{4(1-p)\left[\sinh\left(\frac{m_0}{\theta}\right)\right]^2}{\theta^2\left[2\cosh\left(\frac{m_0}{\theta}\right) + 1\right]^2} \right\}\right)}. \tag{13}$$

The behavior of the relaxation time near the phase-transition points can be obtained analytically from the critical exponents. It is a well-known fact that various thermodynamic functions represents singular behavior as one approaches the critical point. Therefore it is convenient to introduce an expansion parameter, which is a measure of the distance from the critical point:

$$\epsilon = \theta - \theta_c, \tag{14}$$

where θ_c is the reduced critical temperature. In the neighborhood of the critical point the relaxation time of the Blume-Capel Model with quench diluted crystal field, namely, Eq. (13), can be written in the form

$$\tau(\epsilon) = -\frac{1}{zJL_{mm}}\left(1 - (\theta_c - \epsilon)\left\{ -\frac{4p\left[\sinh\left(\frac{m_0}{\theta_c - \epsilon}\right)\right]^2}{(\theta_c - \epsilon)^2\left[2\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right) + e^{\frac{d}{\theta_c - \epsilon}}\right]^2} + \frac{2p\,\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right)}{(\theta_c - \epsilon)^2\left[2\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right) + e^{\frac{d}{\theta_c - \epsilon}}\right]} + \frac{2(1-p)\,\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right)}{(\theta_c - \epsilon)^2\left[2\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right) + 1\right]} - \frac{4(1-p)\left[\sinh\left(\frac{m_0}{\theta_c - \epsilon}\right)\right]^2}{(\theta_c - \epsilon)^2\left[2\cosh\left(\frac{m_0}{\theta_c - \epsilon}\right) + 1\right]^2} \right\}\right). \tag{15}$$

The equilibrium values of the order parameter appearing in Eq. (15) can be expressed in the vicinity of θ_c by

$$m_0(\epsilon) \sim (\epsilon)^{1/2}, \tag{16}$$

where the system undergoes a second-order phase transition, and here the $\sim$ sign indicates that $m_0(\epsilon)$ represents only the asymptotic behavior of the function $m_0(\epsilon)$ as $\epsilon \to 0$. More generally one might expect $m(\epsilon) = A|\epsilon|^{\lambda}(1 + b\epsilon^{\lambda_1} + \cdots)$, where $\lambda_1 > 0$. On the other hand, for $p > \frac{8}{9}$ the system undergoes a first-order phase transition at low temperatures, and in the vicinity of this transition the magnetization vanishes at θ_c as

$$m_0(\epsilon) \sim (-\epsilon)^{1/2}. \tag{17}$$

Only in the range $(0.926 \leqslant p \leqslant 1)$ is the ordered phase separated from the disordered phase by a critical line, part of which is of second order up to a tricritical point where the junction with the first-order part takes place:

$$m_0(\epsilon) \sim (-\epsilon)^{1/4}. \tag{18}$$

On the other hand, in the case of $(\frac{8}{9} \leqslant p < 0.926)$, the phase diagram is rather different. The tricritical point decomposes into a CEP and a BCP. At both points the magnetization scales as

$$m_0(\epsilon) \sim (\epsilon)^{1/2}. \tag{19}$$

The critical exponent $\tau(\epsilon)$ for the function is defined as

$$\lambda = \lim_{\epsilon \to 0} \frac{\ln|\tau(\epsilon)|}{\ln|\epsilon|}. \tag{20}$$

This definition is valid for all values of λ, where the negative values correspond to the divergence of the relaxation times $\tau(\epsilon)$ as ϵ goes to zero, and the positive values correspond to logarithmic divergence, cusps, or jump singularities [40]. On the other hand, in order to distinguish a cusp from a logarithmic divergence, another type of critical exponent, λ', is introduced. To find the exponent λ' that describes the singular parts of τ with a cusplike singularity, we first find the smallest integer k for which the derivative $\frac{\partial^k \tau}{\partial(\epsilon)^k} = \tau^{(k)}$ diverge as $\epsilon \to 0$:

$$\lambda' = k + \lim_{\epsilon \to 0} \frac{\ln|\tau(\epsilon)|}{\ln|\epsilon|}. \tag{21}$$

The behavior of the relaxation time τ as a function of the reduced temperature is shown in Figs. 2(a)–2(f) for several values of the amplitude of the reduced crystal field. Figs. 2(a)–2(c) correspond to temperature dependence of τ near the second-order phase-transition temperature, first-order phase-transition temperature, and tricritical point, respectively, for $p = 0.98$. Figure 2(d) represents the reduced temperature dependence of the relaxation time at the tricritical point for $p = 0.93$. Finally, Figs. 2(e) and 2(f) show the behavior of the magnetization relaxation time at double CEP and CEP, respectively, for $p = 0.90$. In these figures, the curves are labeled with the various values of the kinetic rate coefficient (L_{mm}). The vertical dotted lines refer to the phase-transition temperatures for each value of the reduced crystal field.

One can see from Fig. 2(a) that τ grows rapidly with increasing temperature and diverges as the temperature approaches the second-order phase-transition point. In accordance with this behavior, the critical exponent of τ is found to be $\lambda_c = -1.0$ for all values of the parameters. We should note that the analysis used in this paper is identical to Landau-Ginzburg kinetic theory of phase transitions of a spatially homogenous system. As is discussed extensively in Ref. [41], the case of spatially inhomogeneous medium where $m - m_0 = \delta m(t, \mathbf{r})$, the Landau-Ginzburg kinetic theory of phase transitions reveals the fact that the relaxation time becomes finite for $\theta = \theta_c$ for components with $\mathbf{k} \neq 0$. Here $\mathbf{k}$ is the Fourier transform of the spatial variable $\mathbf{r}$. On the other hand, the renormalization-group formalism has proved to be very useful in calculating not only the static behavior but also the dynamic scaling. By making use of this method, Halperin *et al.* found the critical-point singularity of the linear dynamic response of various models [42]. The linear response theory, however, describes the reaction of a system to an infinitesimal external disturbance, while in experiments and computer simulations it is often much easier to deal with nonlinear-response situations, since it is much easier to investigate the response of the system to finite changes in the thermodynamic variables. A natural question is whether the critical-point singularity of the linear and nonlinear responses is the same. The answer is yes for ergodic systems, which reach equilibrium independently of the initial conditions [43]. The assumption that the initial and intermediate stages of the relaxation do not affect the divergence of the relaxation time (motivated by the observation that the critical fluctuations appear only very close to equilibrium) led to the expectation that in ergodic systems τ^{nl} and τ^{l} diverge with same critical exponent. This view seemed to be supported by Monte Carlo calculations [44] and high-temperature series expansion of the two-dimensional one-spin flip kinetic Ising model [45]. Later Koch *et al.* presented field-theoretic arguments by making use of the Langevin equation for the one-component field $\varphi(\mathbf{r}, t)$ as well as numerical studies of finite-size effects on the exponential relaxation times τ_1 and τ_2 of the order parameter and the square of the order parameter near the critical point of three-dimensional Ising-like systems [46]. The diverging behavior of the relaxation time and corresponding slowing down of the dynamics of the system in the neighborhood of phase transitions has been a subject of experimental research for quite a long time. In 1958, Chase *et al.* reported that liquid helium exhibits a temperature dependence of the relaxation time consistent with the scaling relation $(\theta - \theta_c)^{-1}$ [47]. Later Naya and Sakai presented an analysis of the critical dynamics of the polyorientational phase transition, which is an extension of the statistical equilibrium theory in random-phase approximation [48]. In addition, Schuller *et al.* have shown that the relaxation time of the superconducting order parameter diverges close to the transition temperature [49], in accordance with the theoretical prediction of several authors [50]. Recently, Sperkach *et al.* measured the temperature dependence of acoustical relaxation times in the vicinity of a nematic-isotropic phase-transition point in 5CB liquid crystal [51]. Comparing Fig. 2(a) and Fig. 5 of Ref. [51] one can observe the similarity between the temperature-dependent behavior of the low-frequency relaxation time of the 5CB liquid crystal and the Blume-Capel model with random single-ion anisotropy. Moreover, very recently, Ahart *et al.* reported that a critical slowing down of the central peak,

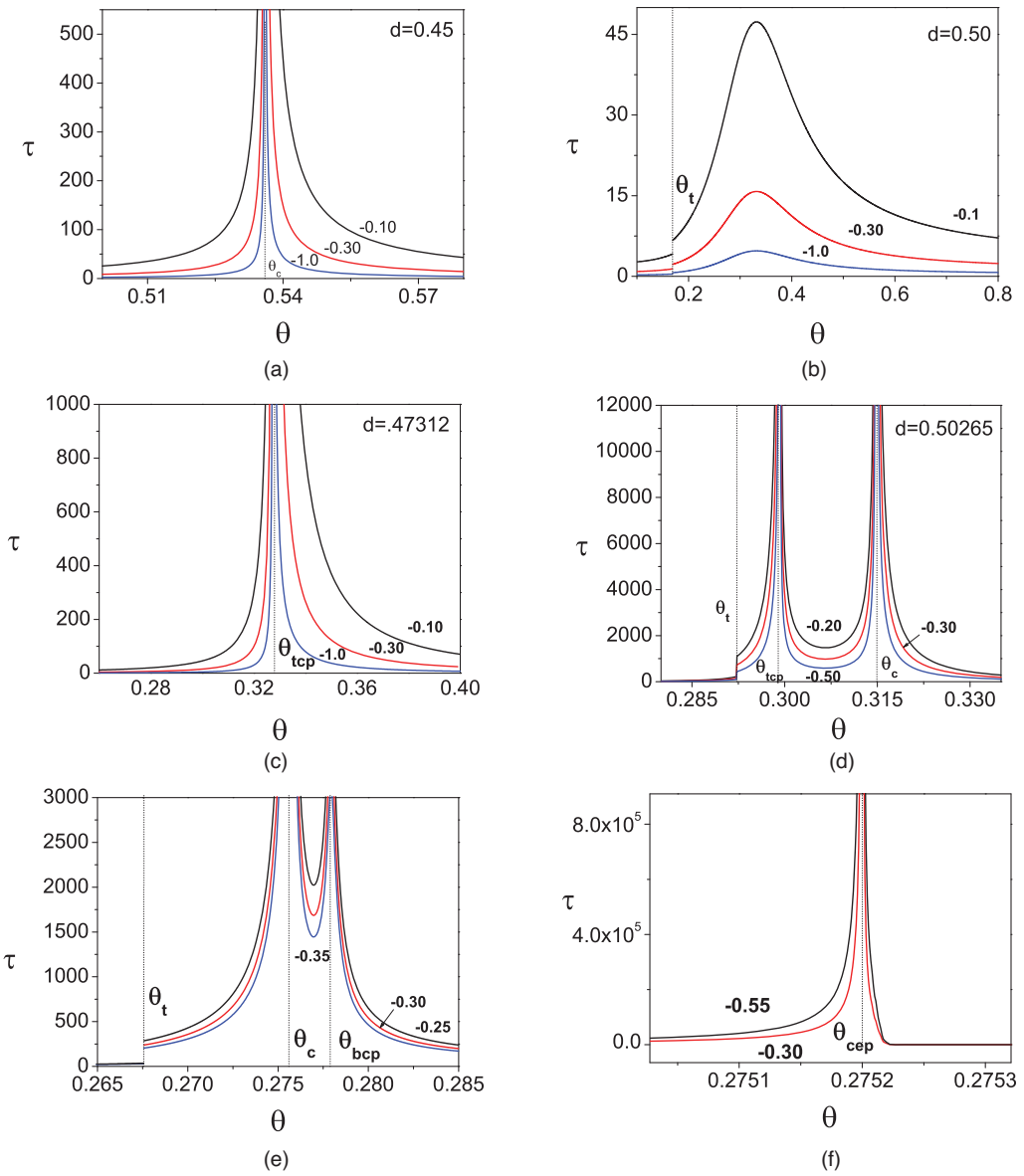

FIG. 2. (Color online) (a) Relaxation time τ as a function of the temperature in the neighborhood of a critical point for $p = 0.98$, (b) a first-order phase transition for $p = 0.98$, (c) the tricritical point for $p = 0.98$, (d) the tricritical point for $p = 0.93$, (e) the BCP for $p = 0.90$, and (f) the CEP for $p = 0.90$. The number accompanying each curve denotes the value of the kinetic rate coefficient (L_{mm}), and the vertical dotted lines represent the phase-transition temperatures.

a feature of order-disorder ferroelectric phase transitions of $Pb(Sc_{0.5}Nb_{0.5})O_{0.5}$ nano-crystals, is observed near θ_c [52].

The behavior of τ near the tricritical point for $p = 0.98$, where the phase diagram is identical to the pure Blume-Capel model, is plotted in Fig. 2(d). One can see from this figure that the relaxation time increases rapidly with increasing temperature and diverges at the tricritical point as in the second-order case. Our analysis in the mean-field approximation has given the tricritical exponent as $\lambda_{\rm tcp} = -1.0$. As is discussed above, the system changes its behavior near the tricritical point for the interval ($0.926 \leqslant p \leqslant 0.945$), where the second-order transition line has a re-entrant part. One can explicitly see this phenomena in Fig. 2(d). Here τ diverges at the tricritical point as well as at the critical point, which is located at higher temperature with same critical indices $\lambda_{\rm tcp} = \lambda_c = -1.0$. In Figs. 2(e) and 2(f), variation of τ with temperature for the reduced crystal field amplitude values corresponding to CEP and DCP has been illustrated for $p = 0.90$. The important feature of the system for $d = d_{\rm dcp}$ is the existence of a first-order phase transition at lower temperatures and a second-order phase transition at higher temperature regime. This situation could easily be seen in the phase diagrams given in Fig. 1(c) and Fig. 3 in Ref [18]. We should stress that τ has a jump discontinuity at the first-order transition point just below the $\theta_{\rm dcp}$, but there is no noncritical peak in this case. For $d = d_{\rm cep}$ CEP takes place in the low-temperature region in the ordered phase, and there is a second-order phase transition taking place in the high-temperature regime. In order to reveal the behavior at CEP, we have given the temperature dependence of the

relaxation time only in a narrow temperature range in the neighborhood of θ_{cep} in Fig. 2(f). In these multicritical points τ diverges with the following exponents: $\lambda_{dcp} = \lambda_{cep} = -1.0$. The Blume-Emery-Griffiths model is investigated for CEP in Ref. [53]. Later, Gulpinar *et al*. studied the metamagnetic Ising model for both CEP and DCP [28]. In agreement with the results of the present work, it is observed in Refs. [28,53] that one of the relaxation times exhibits a divergence in the vicinity of CEP and DCP. Consequently, the Blume-Capel model with random single-ion isotropy exhibits critical slowing down at the critical, tricritical, critical end, and double critical endpoints.

Unlike the typical critical slowing down behavior of the second-order phase transitions, systems exhibit different relaxation characteristics in the neighborhood of first-order transitions. For instance, it has been reported by Collins and Teh that the rate of disordering of a nickel-manganese alloy showed "critical slowing down" as if the transition were of higher order than first [54]. While various crystallographically identical transformations have been shown to be first order [55]. Beers and Guttman conjectured that a system that has a first-order phase transition can show critical slowing down, if at or near a virtual transition point where the order parameter vanishes in the metastable order phase. Apparently, the temperature of this virtual transition is only a little higher than the actual limit of stability of the ordered phase relative to the disordered state [56]. Meanwhile, a detailed investigation of the phase transitions in proton-conducting $Cs_3(HSO_4)_2$ single crystals by observation of ^1H and ^{133}Cs spin-lattice relaxations reveals the fact that abrupt changes in relaxation times are associated with structural phase transitions [57]. It can be seen from Fig. 7 of Ref. [57] that the temperature dependence of the spin-lattice relaxation time for ^{133}Cs in this single crystal has jump discontinuities at the first-order phase-transition temperatures. In accordance with this result we have observed a jump discontinuity at all of the first-order phase-transition points with ($\lambda_t = \lambda'_t = 0.0$) and an anomalous peak at higher temperatures [see Fig. 2(b) as well as Fig. 2(d) and 2(e)].

On the other hand, by making use of the scaling form of the relaxation time we can determine the dynamic critical exponent z. Since $\tau \sim \xi^z$, where ξ and z are the correlation length and dynamical exponent, respectively, hence $\tau \sim |\theta_c - \theta|^{-z\nu}$ [58]. According to our mean-field calculations, the dynamic critical exponent of the Blume-Capel model with random single-ion isotropy is $z = 2$ at the critical, critical endpoint, and double critical endpoints as well as tricritical point, whereas $z = 0$ for first-order critical transition points. If we compare our z values with the ones obtained for various systems by experimental and theoretical means, one can see that they are in good agreement with each other. Goswami has calculated the critical and tricritical dynamical critical exponents as $z_c = z_{tcp} = 2$ of a weakly interacting Fermi-Bose mixture in an approximation scheme that is based on an approximate form of the effective, low-momentum boson Hamiltonian derived in a previous paper and the Landau theory equation of state [59]. In addition, the dynamical exponent has been calculated by using damage spreading and heat bath dynamics and found to be 2.172 for two-dimensional and 2.0324 for three-dimensional spin-1/2 Ising models [60]; by using Monte Carlo simulations z was found to be 2.62(7) for a random-field Ising model; by using the Pade-Borel summation technique z was found to be 2.017 for two- and three-dimensional spin systems [61]; by using the Chisholm-Borel resummation technique z was found to be 2.012 for three-dimensional magnets with quenched extended defects [62]. In addition, z was found to be 2.18(1) for $Fe_{0.9}Zn_{0.1}F_2$, and 2.1(1) for $FeFl_2$ [63].

V. SUMMARY AND DISCUSSIONS

In this paper, we have investigated the relaxation phenomena of the Blume-Capel model with quenched diluted single-ion anisotropy via a theory combining equilibrium statistical mechanics and irreversible thermodynamics. For this aim, we derived the mean-field Gibbs free energy of the system and represented the phase diagrams for various values of the nonzero crystal field concentration (p). Next we have calculated the Gibbs free-energy production produced in the irreversible process. Then the time derivative of the magnetization is treated as flux conjugate to its appropriate forces in the sense of the theory of irreversible thermodynamics [64]. The kinetic equation is obtained by introducing a phenomenological kinetic coefficient in which the effect of the operators that cause longitudinal relaxation is included. From the solution of the kinetic equation near the equilibrium states, a relaxation time, which describes the nonequilibrium behavior in the cooperative system, is obtained. In order to investigate the critical behavior of the relaxation phenomena, the temperature dependence of the relaxation time is determined in the neighborhood of the phase-transition points. The results can be summarized as follows: (τ) grows drastically with increasing temperature and tends to infinity near the second- and the higher-order phase-transition points as $(\theta_c - \theta)^{-1}$, but a jump discontinuity ($\lambda = 0.0$) is observed for the first-order behavior of τ. The findings are in accordance with the results obtained by Erdem and Keskin for the relaxation dynamics of the Blume-Emery-Griffiths model [26] and with the z values obtained in different systems, experimentally [60,63] and theoretically [59,61].

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