Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential

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(Received 10 June 2010; revised manuscript received 30 September 2010; published 30 March 2011)

We determine analytical extended traveling-wave and spatiotemporal solitary solutions to the generalized $(3+1)$-dimensional Gross-Pitaevskii equation with time-dependent coefficients, for the sinusoidally time-varying diffraction and quadratic potential strength. A number of periodic and localized solutions are obtained whose intensity does not decrease in time in the absence of externally induced gain or loss. Stability analysis of our solitary solutions is carried out, to display their modulational stability.

DOI: 10.1103/PhysRevE.83.036609 PACS number(s): 05.45.Yv, 42.65.Tg

I. INTRODUCTION

The Gross-Pitaevskii equation (GPE) is essential for the description of Bose-Einstein condensates (BECs), where it elucidates the behavior of condensate's macroscopic wave-function [1]. It has been introduced by Gross [2] and Pitaevskii [3] for unrelated problems, but was later found useful in different quantum systems [4]. Solutions to GPE are of great interest, because they can be applied to a variety of physical systems. Various solutions to GPE have been discovered [5], including localized (solitary) waves. However, proven stable soliton solutions to GPE exist only in $(1+1)$-dimensions [$(1+1)$D] [6,7]; there are no known exact stable solitons in higher dimensions.

In Ref. [8] we discovered a class of exact extended traveling-wave and spatiotemporal solutions, but only for constant values of the quadratic potential strength and the diffraction coefficient. Here we present analytical traveling-wave and solitary solutions to the GPE in $(3+1)$D when the potential and the diffraction are changing sinusoidally in time. Such a periodic choice for the coefficients in GPE is believed to improve the stability of its solutions [9]. Still, the stability of solutions to GPE is an involved question, to be addressed separately. Our preliminary results indicate that, being extended solitary waves, they are prone to modulational instabilities, however, often displaying prolonged quasistable propagation behavior, which extends over hundreds of diffraction lengths. For particular choices of coefficients and modulation parameters, we find our solitary solutions modulationally stable. These results agree with a recent analysis [10], in which it was shown that $(2+1)$D extended solitons of GPE with constant coefficients, similar to ours but called there the "line-solitons," are prone to collapse, whereas when they are rotated and a dissipative loss is included, they become more stable. It is known that in GPE with optical *spatially periodic* lattice potential there exist stable 2D and 3D solitons [9,11]. It should also be mentioned that in principle all solutions to the generalized GPE considered here are transient in nature, because the coefficients in the equation are time dependent.

The paper is divided into five sections. Section II describes the model and the solution procedure. Section III presents some of the solitary and traveling-wave solutions. Section IV discusses modulational stability of solitary waves, and Sec. V brings conclusions.

II. GROSS-PITAEVSKII EQUATION AND ITS SOLUTION

In [8] we considered the generalized GPE in $(3+1)$D with distributed time-dependent coefficients [1]

$$i\partial_t u + \frac{\beta(t)}{2}\Delta u + \chi(t)|u|^2 u + \alpha(t) r^2 u = i\gamma(t) u. \tag{1}$$

Here t is time, $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the 3D Laplacian, $r = \sqrt{x^2+y^2+z^2}$ is the radial position coordinate, and $\alpha(t)$ is the strength of the quadratic potential as a function of time. The functions β, χ, and γ stand for the diffraction, nonlinearity, and the gain or loss coefficient, respectively.

According to the F-expansion and the balance principle techniques [12–16], we search for the solution in the form [8]:

$$u(x,y,z,t) = A(x,y,z,t)\exp[iB(x,y,z,t)], \tag{2}$$

where

$$A = f(t)F(\theta) + g(t)F^{-1}(\theta), \tag{3}$$

$$\theta = k(t)x + l(t)y + m(t)z + \omega(t), \tag{4}$$

$$B = a(t)r^2 + b(t)(x+y+z) + e(t). \tag{5}$$

Here f, g, k, l, m, ω, a, b, e are parameter functions to be determined, and F is one of the Jacobi elliptic functions (JEFs) [17]. When expressions (3)–(5) are plugged into Eq. (1), the following set of differential equations for the parameter functions is obtained:

$$\frac{df}{dt} + 3a\beta f - \gamma f = 0, \tag{6}$$

$$\frac{dg}{dt} + 3a\beta g - \gamma g = 0, \tag{7}$$

$$\frac{dk}{dt} + 2ka\beta = 0, \tag{8}$$

$$\frac{dl}{dt} + 2la\beta = 0, \tag{9}$$

$$\frac{dm}{dt} + 2ma\beta = 0, \tag{10}$$

TABLE I. Jacobi elliptic functions.

	c_0	c_2	c_4	F	$M=0$	$M=1$
1	1	$-(1+M^2)$	M^2	sn	sin	tanh
2	$1-M^2$	$2M^2-1$	$-M^2$	cn	cos	sech
3	M^2-1	$2-M^2$	-1	dn	1	sech
4	M^2	$-(1+M^2)$	1	ns	cosec	coth
5	$-M^2$	$2M^2-1$	$1-M^2$	nc	sec	cosh
6	-1	$2-M^2$	M^2-1	nd	1	cosh
7	1	$2-M^2$	$1-M^2$	sc	tan	sinh
8	$1-M^2$	$2-M^2$	1	cs	cot	cosech
9	1	$-(1+M^2)$	M^2	cd	cos	1
10	M^2	$-(1+M^2)$	1	dc	sec	1

$$\frac{da}{dt}+2\beta a^2-\alpha=0, \tag{11}$$

$$\frac{db}{dt}+2\beta ab=0, \tag{12}$$

$$\frac{d\omega}{dt}+\beta(k+l+m)b=0, \tag{13}$$

$$\frac{de}{dt}+\frac{\beta}{2}[3b^2-(k^2+l^2+m^2)c_2]-3\chi fg=0, \tag{14}$$

plus two relations connecting some of the parameter functions with some of the coefficients:

$$f[\beta(k^2+l^2+m^2)c_4+\chi f^2]=0, \tag{15}$$

$$g[\beta(k^2+l^2+m^2)c_0+\chi g^2]=0. \tag{16}$$

The parameters c_0, c_2, and c_4 are related to the elliptic modulus M of JEFs (see Table I).

Note that in all these differential equations the parameter function $a(t)$ appears either explicitly or implicitly. On the other hand, the differential equation for a, Eq. (11), involves only the physical coefficients α and β. One has to solve Eq. (11) for a first and then find solutions to the rest of equations; this testifies to the importance of the parameter function a, which is known as the *chirp* function. Unfortunately, Eq. (11) is a Riccati equation, which does not admit *analytical* solutions for *arbitrary* $\alpha(t)$ and $\beta(t)$. It was relatively easy to solve this equation when α and β were constant, which straightforwardly yielded different time-dependent solutions [8]. Here we extend the analysis to the case when $\alpha(t)$ and $\beta(t)$ are not constant.

In principle, the general case when α and β are not constant greatly complicates the solution of the Riccati equation (11) for the chirp function. In [18] we have introduced a procedure for solving Eq. (11) analytically, when there exists a relation between $\alpha(t)$ and $\beta(t)$. In this paper we present and analyze the solutions to GPE when both the diffraction and the quadratic potential are sinusoidal functions of time. In other words, we solve Eq. (1) for two cases: $\alpha(t)=\alpha_0\sin(\Omega t)$, $\beta(t)=\beta_0\sin(\Omega t)$ and $\alpha(t)=\alpha_0\cos(\Omega t)$, $\beta(t)=\beta_0\cos(\Omega t)$. Such a choice of sign-changing periodic coefficients is of importance for the diffraction or dispersion management of systems described by Eq. (1) [9]. The solution procedure employed here is different from the procedure in Ref. [18]; the procedure there requires that $\alpha(t)$ and $\beta(t)$ be of the opposite signs, whereas here the requirement is that the ratio of these two functions is constant. In this case Eq. (11) separates the variables. There are two separate solutions: when the constant is positive and when the constant is negative. We present only the case when the ratio of $\alpha(t)$ and $\beta(t)$ is a positive real number, i.e., when the quadratic potential coefficient is proportional to the diffraction coefficient. While previously [8] two constant parameters p and C are found necessary to describe the solutions, now two parameter functions $p(t)$ and $q(t)$ are found necessary.

After a lengthy calculation, the solutions are obtained for both cases, in the form:

$$f=f_0p^{3/2}\exp\left(\int_0^t\gamma\,dt\right),\quad g=\epsilon\sqrt{\frac{c_0}{c_4}}f, \tag{17}$$

$$k=pk_0,\quad l=pl_0,\quad m=pm_0, \tag{18}$$

$$\omega=\omega_0-(k_0+l_0+m_0)b_0q,\quad b=pb_0, \tag{19}$$

$$e=e_0+\tfrac{1}{2}\left[\kappa_0^2(c_2-6\epsilon\sqrt{c_0c_4})-3b_0^2\right]q, \tag{20}$$

where $\kappa_0^2=k_0^2+l_0^2+m_0^2$ and the subscript "0" denotes the value of a given function at $t=0$. The parameter functions p and q, as well as the solution for a, can conveniently be expressed via an auxiliary function $\tau(t)$:

$$p=\sqrt{\frac{\alpha_0}{\alpha_0-2a_0^2\beta_0}}\operatorname{sech}[\tau(t)], \tag{21}$$

$$q=\frac{\sqrt{\alpha_0\beta_0}}{\sqrt{2}(\alpha_0-2a_0^2\beta_0)}\tanh[\tau(t)]-\frac{a_0\beta_0}{\alpha_0-2a_0^2\beta_0}, \tag{22}$$

$$a=\sqrt{\frac{\alpha_0}{2\beta_0}}\tanh[\tau(t)], \tag{23}$$

where

$$\tau(t)=\operatorname{arctanh}\left(a_0\sqrt{\frac{2\beta_0}{\alpha_0}}\right)+\sqrt{2\alpha_0\beta_0}\int_0^t\beta(t)\,dt. \tag{24}$$

The form of the auxiliary function $\tau(t)$ naturally differs in the two cases for $\alpha(t)$ and $\beta(t)$. Functions p and q place the following restriction on the parameters: $\alpha_0>2a_0^2\beta_0$, which for positive α_0 and β_0 implies $|a_0|<\sqrt{\frac{\alpha_0}{2\beta_0}}$.

The final solution for u is thus

$$u=f_0p^{3/2}\exp\left(\int_0^t\gamma\,dt\right)\left[F(\theta)+\epsilon\sqrt{\frac{c_0}{c_4}}\frac{1}{F(\theta)}\right],$$
$$\times\exp i[a(x^2+y^2+z^2)+b(x+y+z)+e], \tag{25}$$

where

$$\theta=\omega_0+(k_0x+l_0y+m_0z)p-(k_0+l_0+m_0)b_0q. \tag{26}$$

The parameter ϵ can take the values $0,\pm1$. As a consequence of Eqs. (15) and (16), our solution method imposes an integrability condition on the coefficients:

$$\chi(t)=-\beta(t)c_4\kappa_0^2f_0^{-2}p^{-1}\exp\left[-2\int_0^t\gamma(t)\,dt\right]. \tag{27}$$

This condition, naturally, places a restriction on the utility of the method of solution. In principle, such a condition is possible for a realistic physical system. However, it has not been contemplated before, for a simple reason: No exact solutions of the type described here were thought possible.

The form of solutions depends on what JEFs are utilized. Table I lists some of JEFs that may appear in the solutions. The elliptic modulus M varies between 0 and 1, and the functions to which F converges to for these two values are also indicated in the table. Depending on the value of M, one can obtain either traveling-wave or localized (solitary) solutions.

III. SOLITARY AND TRAVELING-WAVE SOLUTIONS

We present in this paper a few typical examples of solutions for both cases, $\alpha(t) = \alpha_0 \sin(\Omega t)$, $\beta(t) = \beta_0 \sin(\Omega t)$ and $\alpha(t) = \alpha_0 \cos(\Omega t)$, $\beta(t) = \beta_0 \cos(\Omega t)$. The initial conditions ($\sin 0 = 0, \cos 0 = 1$) produce a crucial difference in the chirp parameter a, which in turn affects both p and q. Another important point to note is that while the traveling-wave solutions are periodic in time, they are *not* periodic along the traveling variable $k_0x + l_0y + m_0z$, in contrast to the solutions found in Ref. [19]. Similar to [8], the initial value of the chirp is of not much importance; the solutions remain qualitatively the same. The major change however, is a shift of all the parameters a, p and q, which causes a shift in the graphs along the transverse variable, and a decrease in the magnitude (for positive a_0), which causes a narrowing of the peaks.

Figure 1 presents the sine case, and Fig. 2 the cosine case of the extended solitary waves; Fig. 3 presents the traveling-wave solutions for both the sine and the cosine case. For a better perspective, a small loss ($\gamma = -0.05$) is included in all the figures; without it, the waves keep the amplitude constant, but still breathe. Note the influence of the parameter b_0, which causes the solitons to wiggle.

Despite the apparent complexity of these solutions, they yield relatively straightforward and elegant spatiotemporal breathing solitary solutions for $F = sn$ (dark solitons) and $F = cn$ (bright solitons) without any need for positive γ. In other words, unlike the solutions in Ref. [8], which required a positive value of gain to form solitary waves, the signals here stay at the same peak intensity for $\gamma = 0$, but breathe. This is apparent from the fact that p and q, as well as a, the parameter functions on which all other variables in the final

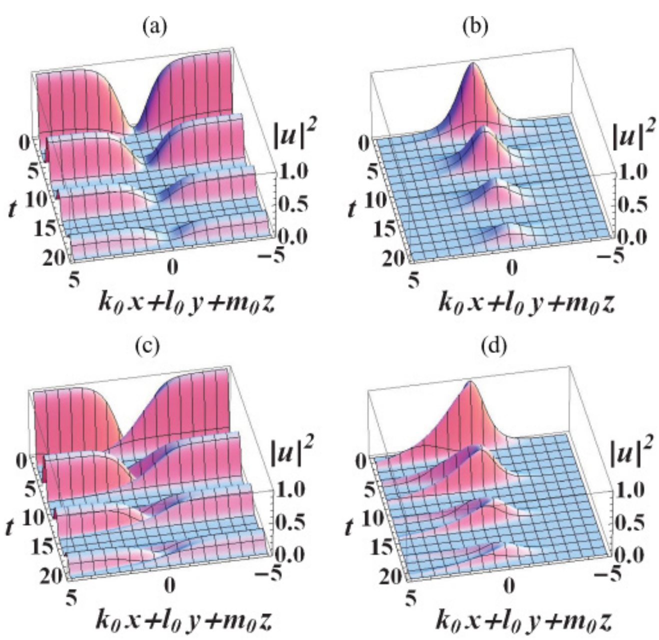

FIG. 1. (Color online) Soliton solutions to the Gross-Pitaevskii equation as functions of time, for the sine case: $\alpha(t) = \alpha_0 \sin(\Omega t)$, $\beta(t) = \beta_0 \sin(\Omega t)$. Intensity $|u|^2$ for (a) and (c) dark solitons ($F =$ tanh), and (b) and (d) bright solitons ($F =$ sech) presented as functions of $k_0x + l_0y + m_0z$ and t. For (a) and (b) $b_0 = 0$, for (c) and (d) $b_0 = 1$. Other parameters: $\alpha_0 = 1$, $\beta_0 = 1$, $\Omega = 1$, $\gamma(t) = -0.05$, $a_0 = 0$, $e_0 = 0$, $k_0 = l_0 = m_0 = 1$, $\omega_0 = 0$, $\epsilon = 0$.

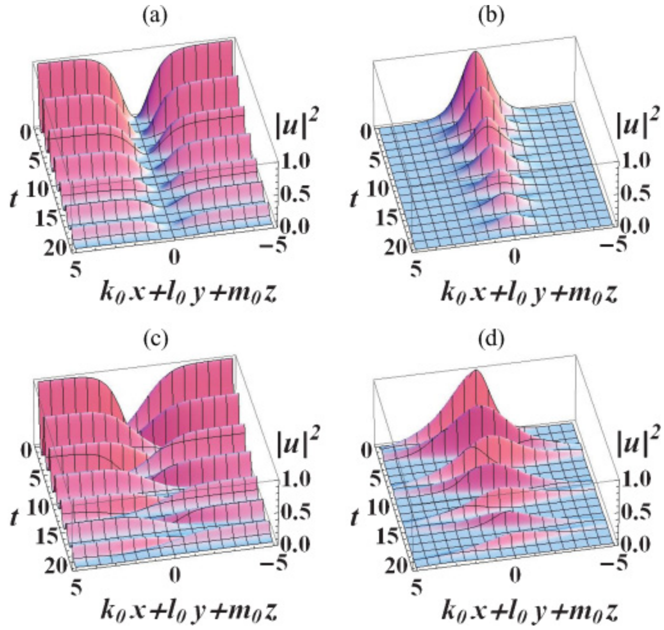

FIG. 2. (Color online) Soliton solutions to the Gross-Pitaevskii equation as functions of time for the cosine case: $\alpha(t) = \alpha_0 \cos(\Omega t)$, $\beta(t) = \beta_0 \cos(\Omega t)$. The other parameters are the same as in Fig. 1.

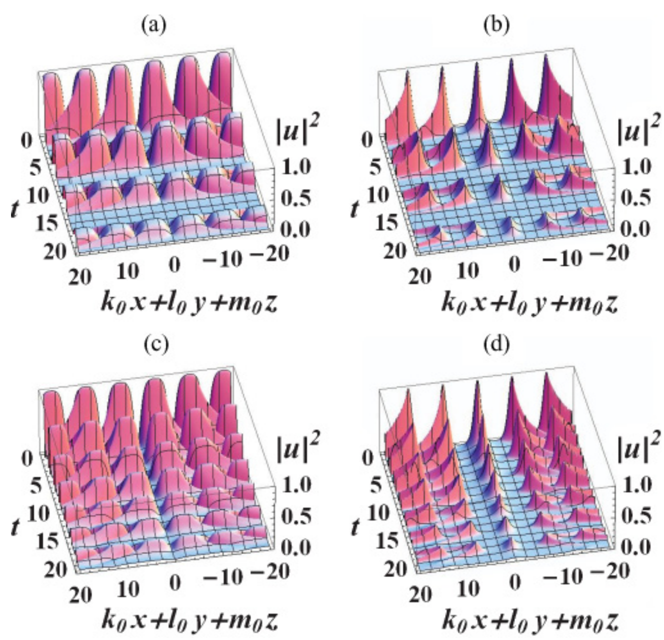

FIG. 3. (Color online) Traveling wave solutions to Gross-Pitaevskii solutions in terms of JEFs for the sine and the cosine case. The parameters for panels (a) and (b) are the same as in Figs. 1(a) and 1(b), and the parameters in panels (c) and (d) are the same as in Figs. 2(a) and 2(b), except for $M = 0.99$. In panels (a) and (c), $F =$ sn, and in panels (b) and (d), $F =$ cn.

solution depend, are periodic functions of time in both cases. Note also that the width of the solitary solutions in Ref. [8] was increasing in time, because of the positive value of γ. Energy was pumped into these solitary waves, which is why they were not the solitons in the usual sense of word.

It was curious to find that if arbitrary phase shifts are added to α and β, i.e., $\alpha(t) = \alpha_0 \sin(\Omega t + \phi_1)$ and $\beta(t) = \beta_0 \sin(\Omega t + \phi_2)$, very different solutions are found, often without a closed analytical form. Even a simple choice, such as $\phi_1 = 0$ and $\phi_2 = \pi$, which makes α and β oscillate out of phase relative to the cases presented here, causes the solutions to the Riccati equation for a to belong to a widely different class from the ones found here, where α and β oscillate in phase [18]. Furthermore, if one parameter is a sine function and the other is a constant, the solutions collapse rapidly. In the next section we present a short stability analysis of our solitary solutions.

IV. STABILITY ANALYSIS

We present an outline of the stability analysis concerning modulational instability of our extended solitary solutions. A more complete analysis, including numerical simulations of Eq. (1), will be presented elsewhere. The crux of our approach is to transform the starting Eq. (1) into a form more amenable to stability analysis [20]. This is accomplished as follows.

Owing to its linear nature, the gain or loss term cannot affect the stability of solutions. In fact, the gain or loss coefficient can be eliminated from Eq. (1) by a simple transformation of u and χ, $u = E \exp[\int_0^t \gamma(t)\,dt]$ and $\chi \exp[2\int_0^t \gamma(t)\,dt] \to \chi$, as is visible from Eqs. (25) and (27). Therefore, we concentrate on the part of the final solution u excluding the exponent:

$$E = f_0 p^{3/2} G \exp[ia(x^2+y^2+z^2) + ib(x+y+z) + ie], \tag{28}$$

where $G(x,y,z,t)$ stands for the expression in brackets in Eq. (25), but now considered as an explicit function of the independent variables. The transformation of variables $x \to x' = p\bar{x} = p\kappa_0(x-\varsigma)$, $y \to y' = p\bar{y} = p\kappa_0(y-\varsigma)$, $z \to z' = p\kappa_0(z-\varsigma)$, $t \to t' = \kappa_0^2 \int \beta p^2\,dt$, brings Eq. (1) into an equation for G in the new variables:

$$i\frac{\partial G}{\partial t'} + \left(\frac{\partial^2 G}{\partial x'^2} + \frac{\partial^2 G}{\partial y'^2} + \frac{\partial^2 G}{\partial z'^2}\right) - \frac{c}{2}G - c_4|G|^2 G = 0, \tag{29}$$

where $c = c_2 - 6\epsilon\sqrt{c_0 c_4}$ is a constant. The form of the function $\varsigma(t)$ in the transformation is of no immediate interest, other than it explicitly depends on $p(t)$ and $\tau(t)$. However, the new variable t', which depends only on t, involves an integral over β and can change sign. This is important in the analysis of Eq. (29).

Equation (29) is the usual (3 + 1)D nonlinear Schrödinger equation with constant coefficients, which is prone to instabilities and the wavefunction collapse. Instabilities in G translate into instabilities of the general solution u. This would bode disaster for the stability of exact traveling-wave and solitary solutions found, were it not for the possibility of diffraction and nonlinearity management [9] in Eq. (29), due to the form of the primed variables [21]. We find that, for the choice of coefficients $\alpha(t)$ and $\beta(t)$ made here, the typical extended soliton solutions of Eq. (29) do not collapse when perturbed, but keep oscillating in a typical breathing behavior.

Without loss of generality we put $f_0 = 1$, $\kappa_0 = 1$, $\beta_0 = 1$, and place the z' axis in the direction of inhomogeneity of our extended solitary solutions. In this manner the z' variable takes the role of the θ variable. The intensity $|E| \sim |G|$ is homogenous in two of the three spatial dimensions [i.e., in the plane perpendicular to the direction (k_0,l_0,m_0) of inhomogeneity.] It is in this plane that, owing to nonlinearity, the modulational instability can develop. For this reason, of particular interest is the analysis of modulational (in)stability of perturbations in the plane of homogeneity of $|G|$.

We consider the perturbation of G in this plane for the two fundamental solutions, the dark $G = \tanh(z')$ and the bright $G = \mathrm{sech}(z')$ solitons, in the form:

$$G \to G[1 + (U_r + iU_i)\cos(K\bar{x})], \tag{30}$$

where $U(t) = U_r(t) + iU_i(t)$ is the complex amplitude, and K is the wavenumber of the perturbation in the direction $\bar{x}$ perpendicular to z'. In a standard linear stability analysis, the perturbation is substituted into Eq. (29) and linear first-order differential equations for U_r and U_i obtained. These equations can be solved analytically, to yield the following solutions:

$$U_r(t) = \left\{C_1 P_\nu^\mu[\tanh(\tau)] + C_2 Q_\nu^\mu[\tanh(\tau)]\right\} p^{-1}, \tag{31}$$

$$U_i(t) = \frac{2\sqrt{2\alpha_0}}{pK^2}\frac{\partial(pU_r)}{\partial\tau}, \tag{32}$$

where P_ν^μ and Q_ν^μ are the associated Legendre functions, with $\nu = -1/2[1 - \sqrt{1 - dK^2/2(\alpha_0 - 2a_0^2)}]$, and $\mu = iK^2/2\sqrt{2\alpha_0}$. Here $d = -4$ for the dark soliton and $d = 8/3$ for the bright soliton. The constants C_1 and C_2 are determined by the initial conditions for U_r and U_i. [We take $U_r(t=0) = U_0$ and $U_i(t=0) = 0$.]

The solutions in Eqs. (31) and (32) determine the dynamics of the modulational instability. Figure 4 depicts a typical evolution of the modulus of the perturbation amplitude $|U|$ for different values of the parameters. In all cases, we have a periodic dependence in time, with the period $2\pi/\Omega$. For large K we see a superposition of two oscillations, one with the frequency Ω and the other with the frequency $\sim K\sqrt{K^2-d}/2$. For small Ω, independent of the value of K, the amplitude

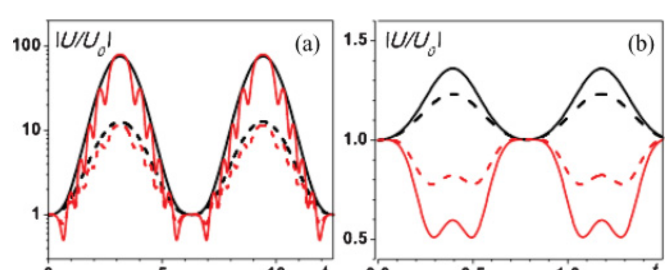

FIG. 4. (Color online) Evolution of the modulus of the perturbation amplitude $|U|$ of bright solitons in time for different values of Ω: (a) $\Omega = 1$, (b) $\Omega = 8$. In both figures $a_0 = 0.3$. Other parameters: $K = 1, \alpha_0 = 0.3$ [black, (upper) solid line]; $K = 1, \alpha_0 = 0.1$ [black, (upper) dashed line]; $K = 4$, $\alpha_0 = 0.3$ [red, (lower) solid line]; $K = 4, \alpha_0 = 0.1$ [red, (lower) dashed line].

of the perturbation may, for a time period equal to π/Ω, grow for several orders of magnitude compared to the initial value. This is expected: The generalized GPE should display sensitivity to low-frequency (long-period) perturbations. Such perturbations of the coefficients bring GPE closer to the limit of NLSE with constant coefficients, which naturally is prone to instability and collapse. In contrast to this case, for large Ω the variation of the perturbation amplitude is much smaller. In addition, a decrease in the initial chirp a_0 and in the strength of the potential α_0 causes a reduction in the variation of the perturbation amplitude. In fact, for large $\Omega \gg \sqrt{2\alpha_0}$ and small chirp $\alpha_0 \gg 2a_0^2$ the maximum variation of the perturbation amplitude $|U|$ in the lowest approximation can be expressed as

$$\max ||U/U_0| - 1| < 2|d^2 - dK^2 + 8\alpha_0|/\Omega^2,$$

and by increasing Ω can be made arbitrarily small. In all of this the crucial point is that the amplitude, while oscillating, remains finite. Hence, no collapse of the solitons occurs; they are modulationally stable against the perturbations considered.

V. CONCLUSION

In conclusion, we have solved analytically the generalized $(3+1)$D GPE with distributed coefficients, for a sinusoidal time-varying quadratic potential and diffraction. A number of exact traveling-wave solutions are found, and novel exact spatiotemporal soliton solutions are obtained. Modulational stability analysis is carried out, to display stability of our extended solitary solutions.

ACKNOWLEDGMENTS

Work at the Texas A&M University at Qatar is supported by the Qatar National Research Foundation project NPRP 25-6-7-2. Work at the Institute of Physics is supported by the Ministry of Science of the Republic of Serbia, under the project OI 171006.

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[21] For example, by transferring the factor $\kappa_0^2\beta$ from dt' to the other side of Eq. (29), this equation becomes the nonlinear Schrödinger equation with time-dependent sign-changing nonlinearity.