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Stochastic theory of the Stokes parameters in randomly twisted fiber

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We present the stochastic approach of the polarization state of an electromagnetic wave traveling through randomly twisted optical fiber. We treat the case of the weak randomness. When the geometric torsion of the fiber is distributed as a Gaussian law, we can write explicitly the Fokker-Planck equation for the Stokes parameters of the wave, and find the complete solution of the polarization-state distribution.

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I. INTRODUCTION

Understanding and controlling wave propagation in disordered media has been one of the major issues in condensed matter physics for the last decade [1–3]. The problem has been investigated for all kind of waves (scalar sound waves, vectorial electromagnetic waves [4], spinorial quantum waves such as electrons [5], etc). It is now established that the classical Boltzmann-like theory is not enough to describe the behavior of weakly disordered media, particularly because of localization phenomenon [6]. In this spirit, propagation of vector polarized waves in weakly disordered media has been actively studied when randomness is caused by impurities [7–9]. In this case, the electric permittivity tensor can be written phenomenologically as a random matrix, and the problem is treated either through numerical simulations [10], Brownian theory on the Poincaré sphere [11], or in the mathematical framework of the random matrices [12].

In a recent paper [13], we proposed an approach of the stochastic theory of polarized electromagnetic wave in random birefringent media, using the functional integral technique. This way appeared to be powerful enough to obtain the exact Fokker-Planck equation for the Stokes parameters in a straightforward way. The purpose of the present paper is to apply this general formalism to a specific optical medium of wide technological interest: the randomly twisted optical fiber without losses. Up to now, regularly twisted or coiled optical fibers have been investigated in various aspects [14–18], but the particular role of the randomness remains theoretically challenging. A series of works by Malykin *et al.* [19] deals with optical fibers as sequences of homogeneous segments with Poisson-distributed lengths. It corresponds to the case of low density of inhomogeneities (here "inhomogeneity" corresponds either to a chemical impurity, to a structural defect, or to a localized mechanical stress), and the elastic modulus of the fiber is so large that the segment in between two consecutive impurities is of constant torsion. On the other hand, we study here the case where the density of impurities is so large that the twist is fluctuating on a microscopic scale. Then, we consider the fiber torsion as a continuous random variable, and we use the functional integral technique to obtain analytical results about the stochastic behavior of the polarization state.

II. GENERAL FORMULATION

A. The twisted waveguide

We consider a wave propagating along the smooth curve $(\mathcal{C})$. The lengthwise distance along the curve $(\mathcal{C})$ is defined as the value of the curvilinear coordinate s. We define the vector field $\mathbf{t}$ tangent to $(\mathcal{C})$ in every point.

The natural frame along $(\mathcal{C})$ is the orthonormal triad of tangent, normal, and binormal unit vectors $(\mathbf{t},\mathbf{n},\mathbf{b})$, also known as the Frenet-Serret frame. The relations

$$\frac{d\mathbf{t}}{ds} = \kappa\,\mathbf{n}; \quad \frac{d\mathbf{n}}{ds} = -\kappa\,\mathbf{t} + \tau\,\mathbf{b}; \quad \frac{d\mathbf{b}}{ds} = -\tau\,\mathbf{n} \tag{1}$$

hold, with κ the curvature and τ the torsion, which both may depend on the coordinate s.

Generally, $\mathbf{t}$ is an anisotropy axis of the material, such that the electric permittivity takes the form $\hat{\epsilon} = \epsilon_{tt}\mathbf{t}\otimes\mathbf{t} + \hat{\epsilon}_{\perp}$, where ϵ_{tt} has a real value, and the 2×2 tensor $\hat{\epsilon}_{\perp}$ applies in the plane locally orthogonal to the tangent vector $\mathbf{t}$:

$$\hat{\epsilon}_{\perp} = \begin{pmatrix} \epsilon_{nn} & \epsilon_{nb} \\ \epsilon_{bn} & \epsilon_{bb} \end{pmatrix}.$$

$\hat{\epsilon}_{\perp}$ is a Hermitian tensor for the medium to be transparent.

B. The slowly varying envelope approximation

The Maxwell equations for the electric component $\mathbf{E}$ of the electromagnetic field of frequency ω is

$$\nabla\times(\nabla\times\mathbf{E}) = \omega^2\,\hat{\epsilon}\,\mathbf{E} \tag{2}$$

in a medium of electric permittivity $\hat{\epsilon}$. The magnetic permeability is set to unity. We assume the slowly varying envelope approximation (SVEA) to be valid [20], that is

$$\mathbf{E} = e^{i\phi}\mathbf{F}, \tag{3}$$

where the vector $\mathbf{F}$ experiences slow spatial variations (on typical lengths l), while ϕ is a real phase varying fast in the space (on the wavelength $\lambda \ll l$). The local wave vector is defined as $\mathbf{k} = \nabla\phi$. Using the quasi-isotropic approximation [21], which is applicable in the case of small anisotropy, $\phi = k_0\int n\,ds$, with n the local refractive index of the fiber. Within this framework, the wave vector is in the direction of the tangent vector $\mathbf{t}$, that is, $\mathbf{k} = k\mathbf{t}$.

Moreover, $\mathbf{F}^\star \cdot \mathbf{F} = $ constant, since the medium is supposed to be without losses. Then, considering the transverse vector $\mathbf{F}$ initially perpendicular to $\mathbf{t}$ for $s = 0$, this vector remains in the plane $(\mathbf{n}, \mathbf{b})$: $\mathbf{F} = f_n \mathbf{n} + f_b \mathbf{b}$ for all $s > 0$, and the Maxwell equation (2) allows us to write down a couple of equations for the components (f_n, f_b):

$$\frac{2i}{k}\frac{d}{ds}\begin{pmatrix} f_n \\ f_b \end{pmatrix} = \begin{pmatrix} -a & -b + ic + 2i\tau/k \\ -b - ic - 2i\tau/k & a \end{pmatrix}\begin{pmatrix} f_n \\ f_b \end{pmatrix},$$

with the real anisotropy coefficients

$$a = \frac{\epsilon_{nn} - \epsilon_{bb}}{2\epsilon_0'}, \quad b = \frac{\epsilon_{nb} + \epsilon_{bn}}{2\epsilon_0'}, \quad c = i\frac{\epsilon_{nb} - \epsilon_{bn}}{2\epsilon_0'}. \tag{4}$$

In these equations, the average electric permittivity in the plane $(\mathbf{n},\mathbf{b})$ is $\epsilon_0' = (\epsilon_{nn} + \epsilon_{bb})/2$. The coefficients a,b,c are considered here as constant functions of s, though a more general case can be treated as well.

The torsion τ appears naturally as a coupling between the two polarizations.

In the case where the twist is due to mechanical stress, the coefficient c should depend linearly (for the small deformations) on the actual value of τ [22]. It would result in the same previous formulas, with the coefficient 2 in front of $i\tau/k$ changed in a material-dependent elasto-optic constant.

III. THE MAXWELL EQUATION AS A TWO-STATES SCHRÖDINGER EQUATION

We introduce the alternative basis $(\mathbf{e}_+,\mathbf{e}_-)$, which is formally similar to the circular-polarization basis in the Euclidean frame:

$$\mathbf{e}_\pm = \frac{1}{\sqrt{2}}(\mathbf{n} \pm i\mathbf{b}) = A^t \begin{pmatrix} \mathbf{n} \\ \mathbf{b} \end{pmatrix},$$

where the unitary matrix A is

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}.$$

Thus, considering the two-components vector $\mathbf{\Psi} = A^\dagger \mathbf{F}$, we get

$$\frac{2i}{k}\frac{d\mathbf{\Psi}}{ds} = \left[-a\hat{\sigma}_1 - b\hat{\sigma}_2 - \left(c + \frac{2\tau}{k}\right)\hat{\sigma}_3\right]\mathbf{\Psi}, \tag{5}$$

where we used the traceless Pauli matrices

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and their transforms $A^\dagger \hat{\sigma}_1 A = \hat{\sigma}_2$, $A^\dagger \hat{\sigma}_2 A = \hat{\sigma}_3$, $A^\dagger \hat{\sigma}_3 A = \hat{\sigma}_1$.

In (5), the right-hand term can be conveniently written as $\hat{H}\mathbf{\Psi}$ with $\hat{H} = -a\hat{\sigma}_1 - b\hat{\sigma}_2 - (c + 2\tau/k)\hat{\sigma}_3$ the Hamiltonian operator.

We introduce the Stokes parameters $\{S_j\}_{j=1,\ldots,3}$, as the average values of the Pauli matrices, namely [23,24]

$$S_j = \mathbf{\Psi}^\dagger \hat{\sigma}_j \mathbf{\Psi}.$$

Since $\mathbf{\Psi}^\dagger \cdot \mathbf{\Psi} = \mathbf{F}^\star \cdot \mathbf{F} = $ constant, the three-dimensional Stokes vector $\mathbf{S} = (S_1,S_2,S_3)$ has a constant norm. It is normalized as $\mathbf{S}^2 = 1$. Using the Ehrenfest theorem in the Schrödinger-type equation (5), and the commutation relation $[\hat{\sigma}_1,\hat{\sigma}_2] = 2i\hat{\sigma}_3$ (and cyclic permutations), one finds the evolution equation for the Stokes vector $\mathbf{S}$ under the form [14,25]

$$\frac{d\mathbf{S}}{ds} = k\,\mathbf{S} \times \mathbf{G}, \tag{6}$$

with $\mathbf{G}$ the pseudomagnetic field written as

$$\mathbf{G} = (a,b,c + 2\tau/k) = -\nabla_{\mathbf{s}} H,$$

with $\nabla_{\mathbf{s}} H$ the gradient of the classical Hamiltonian $H = -aS_1 - bS_2 - (c + 2\tau/k)\,S_3$ in the $\mathbf{S}$ space. For completion, we fix the initial condition to $\mathbf{S}_0 = (1,0,0)$ to make the later expressions simple.

Qualitatively, we can guess from Eq. (6) polarization rotation along the twisted waveguide curve. It is in agreement with the classical result of rotation of the polarization in a chiral birefringent material [26].

IV. THE TWISTED FIBER

As a first special case, we assume that the fiber has no intrinsic birefringence [i.e., $(a,b,c) = (0,0,0)$]. The pseudomagnetic field $\mathbf{G} = [0,0,2\tau(s)/k]$ is the consequence of the only geometric torsion, and the equations of motion for the Stokes parameters are

$$\frac{dS_1}{ds} = 2\tau(s)S_2; \quad \frac{dS_2}{ds} = -2\tau(s)S_1; \quad \frac{dS_3}{ds} = 0. \tag{7}$$

These equations can also be derived using the Rytov equations for the components f_n, f_b [27]. It results in the rotation of the Stokes vector on the Poincaré sphere around the third axis:

$$\mathbf{S} = [\cos 2\psi(s),\ \sin 2\psi(s),0], \tag{8}$$

with the orientation angle

$$\psi(s) = -\int_0^s \tau(s')ds'. \tag{9}$$

The result $S_2/S_1 = \tan 2\psi(s)$ is in agreement with the parallel transport law for the wave electric vector, as derived under the form $f_b/f_n = \tan\psi(s)$ by Berry [18].

These equations of motion do not depend on the actual value of k, thus the change in the polarization state does not depend on the wavelength.

V. THE RANDOMLY TWISTED FIBER

With the result (8) at hand, we address the question of the random torsion on the evolution of the Stokes vector. Let us suppose that the torsion $\tau(s)$ is fluctuating around an average function $\bar{\tau}(s)$ according to the Gaussian distribution with the width μ [28]:

$$P[\tau(s)] = A\exp\left\{-\int_0^s \frac{1}{2\mu}[\tau(s') - \bar{\tau}(s')]^2 ds'\right\}, \tag{10}$$

with A the normalization constant.

The average value of $\exp[i\psi(s)]$, with ψ the function defined in (9), is calculated as the functional integral

$$\langle e^{2i\psi(s)}\rangle = A\int \exp\left[-2i\int_0^s \tau(s')ds'\right] \times \exp\left\{-\int_0^s \frac{1}{2\mu}[\tau(s')-\bar{\tau}(s')]^2 ds'\right\}\mathcal{D}[\tau(s)] = e^{2i\langle\psi(s)\rangle}\exp(-2\mu s). \quad (11)$$

It results in the exponentially decreasing averaged Stokes vector

$$\langle \mathbf{S}\rangle = e^{-2\mu s}[\cos 2\langle\psi(s)\rangle, \sin 2\langle\psi(s)\rangle, 0],$$

where the average orientation angle is

$$\langle\psi(s)\rangle = -\int_0^s \bar{\tau}(s')ds'.$$

Similarly, the autocorrelation of the actual Stokes vector behaves like

$$\langle \mathbf{S}(s)\mathbf{S}(0)\rangle = e^{-2\mu s}\cos[2\langle\psi(s)\rangle], \quad (12)$$

and the associated power spectral density $\Phi(f) = \int_{-\infty}^{\infty}\langle \mathbf{S}(s)\mathbf{S}(0)\rangle \exp(-2i\pi f s)ds$ is generally that of a Brownian noise, $\Phi(f) \sim 1/f^2$.

VI. STOCHASTIC BEHAVIOR OF THE POLARIZATION STATE

We consider now the case where the *average* torsion $\bar{\tau}$ is independent of s. This assumption may be connected with the statistical homogeneity of the medium, and most of the applications deal with such a general assumption.

Many results can be obtained exactly in this case. For example, the power spectral density associated with the autocorrelation of the Stokes parameters (12) is

$$\Phi(f) = \frac{\mu(\mu^2+\bar{\tau}^2+\pi^2 f^2)}{[\mu^2+(\bar{\tau}-\pi f)^2][\mu^2+(\bar{\tau}+\pi f)^2]},$$

which well behaves as $\sim 1/f^2$ for the large values of f. More importantly, we will see that the Fokker-Planck equation can be written explicitly and solved analytically. It gives a rare example where the stochastic behavior is fully available.

A. Direct calculation of the Stokes-vector probability distribution

The value of the S_3 component of the Stokes vector is a constant, independent of the torsion [see Eq. (7)]. Here $S_3 = 0$, since we assume the initial state $\mathbf{S}_0 = (1,0,0)$.

The propagator K linking the Stokes-vector state at the coordinate s to its value at the origin is generally [28]

$$K[\mathbf{S}(s)|\mathbf{S}_0] = \int \prod_s \delta\left(\frac{d\mathbf{S}}{ds} - k\mathbf{S}\times\mathbf{G}\right) \times \exp\left\{-\int_0^s \frac{1}{2\mu}[\tau(s')-\bar{\tau}(s')]^2 ds'\right\} \times \mathcal{D}[\tau(s)]\mathcal{D}(\mathbf{S}). \quad (13)$$

Writing $\mathbf{S}(s) = (\cos 2\psi, \sin 2\psi, 0)$, with the unknown function ψ of the arc-length s, one can perform the integration in (13) over all the functions τ, following the same formalism and derivation as detailed in Ref. [13]. To explain briefly the technique we use Eq. (7) to rewrite the integrand $\delta(d\mathbf{S}/ds - k\mathbf{S}\times\mathbf{G})\mathcal{D}(\mathbf{S})$ appearing in (13), in terms of the orientation angle ψ, namely $\delta(d\psi/ds+\tau)\mathcal{D}[\psi(s)]$. Then, integration over the ensemble of all the functions $\tau(s)$ leads to

$$K[\mathbf{S}(s)|\mathbf{S}_0] = \int \exp(-\mathcal{S})\mathcal{D}[\psi(s)],$$

with the action $\mathcal{S}$:

$$\mathcal{S} = \frac{1}{2\mu}\int_0^s \left(\frac{d\psi}{ds}+\bar{\tau}\right)^2 ds'. \quad (14)$$

The most probable path is given by the condition $\delta\mathcal{S} = 0$, that is $d^2\psi/ds^2 = 0$. Then, the first derivative of ψ is a constant, and can be written as $d\psi/ds = (\psi + n\pi)/s$, with ψ the value at the coordinate s. The winding number n is the number of times the path goes past the value ψ over the range $[0,s]$. This leads to the formula

$$K[\mathbf{S}(s)|\mathbf{S}_0] = \frac{1}{\sqrt{2\pi\mu s}}\sum_{n=-\infty}^{\infty}\exp\left[-\frac{s}{2\mu}\left(\frac{\psi+n\pi}{s}+\bar{\tau}\right)^2\right].$$

Because only the angle ψ is variable in this geometry, the propagator $K[\mathbf{S}(s)|\mathbf{S}_0]$ coincides with the probability, say $P[\psi;s]$, to get the value ψ, modulo π, at the location s. Then, using the reciprocal Jacobi θ-function identity

$$\sqrt{\alpha}\sum_{n=-\infty}^{\infty} e^{-\pi\alpha(n+z)^2} = \sum_{n=-\infty}^{\infty} e^{-\pi n^2/\alpha + 2i\pi n z},$$

valid for any $\alpha > 0$ and complex z, we can write the propagator under the form

$$P[\psi;s] = \frac{1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} e^{-2\mu s n^2}\cos[2n(\psi+\bar{\tau}s)], \quad (15)$$

which is a Jacobi-θ function [29] $\{$namely $\pi P[\psi;s] = \vartheta_3[\psi+\bar{\tau}s, \exp(-2\mu s)]\}$. Also, the distribution could conveniently be rewritten in terms of the reduced adimensional variable μs and parameter $\bar{\tau}/\mu$. The function (15) is plotted in Fig. 1 for the case $\bar{\tau}/\mu = 0$.

One can note that, calculating the average value $\langle e^{2i\psi}\rangle = \int_{-\pi/2}^{\pi/2} e^{2i\psi}P[\psi;s]\,d\psi$, one recovers the result

$$\langle e^{2i\psi}\rangle = e^{-2i\bar{\tau}s}\exp(-2\mu s),$$

which is a particular case of (11). More generally, all the moments of $\mathbf{S}$ can readily be calculated analytically. One finds for example for the variances $\langle S_1^2\rangle - \langle S_1\rangle^2 = \frac{1}{2}[1-\exp(-4\mu s)][1-\exp(-4\mu s)\cos(4\bar{\tau}s)]$, $\langle S_2^2\rangle - \langle S_2\rangle^2 = \frac{1}{2}[1-\exp(-4\mu s)][1+\exp(-4\mu s)\cos(4\bar{\tau}s)]$. When $\mu s \ll 1$, which should be the usual case, the variances behave as $\sim 8\mu^2 s^2$ and $\sim 4\mu s$, respectively. In this case, the variance of S_2 is much larger than the variance of S_1, and its measurement could give access to the value of μ which governs the torsion fluctuations.

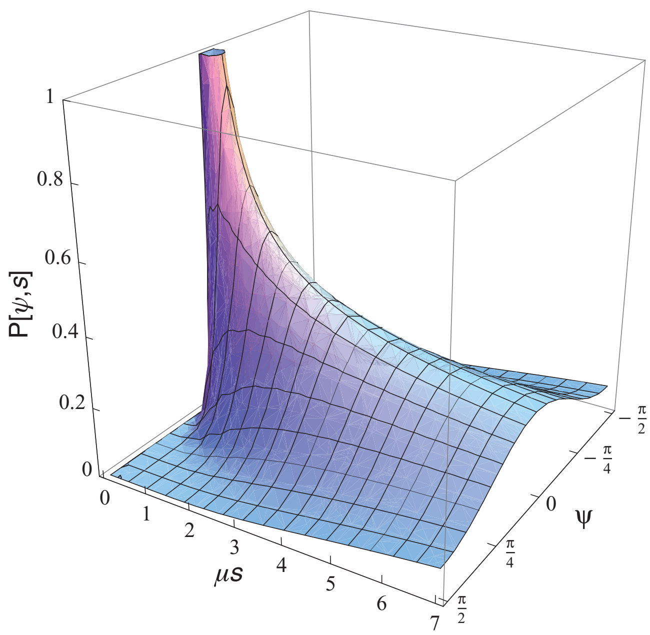

FIG. 1. (Color online) Probability distribution function of the orientation angle ψ, for increasing values of the reduced curvilinear coordinate μs. This example is for a Gaussian distribution of the torsion, with $\bar{\tau} = 0$ and positive standard deviation μ. For $\mu s = 0$, the $P[\psi; s]$ distribution is a δ-distribution centered in $\psi = 0$. When μs increases, the distribution $P[\psi; s]$ is given in (15) and resembles a Gaussian distribution with increasing value of the variance. The exact expression for the variance is given in (16).

Correlation functions or average values can be calculated from (15). For example, the quantity $\langle \psi^2 \rangle$ is known to be meaningful in the case of real fibers [30]. Here, it is as the function of s:

$$\langle \psi^2 \rangle = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \left[1 - e^{-2\mu s n^2} \cos(2ns\bar{\tau})\right], \qquad (16)$$

and it increases monotonically from 0 to $\pi^2/12$.

B. The general Fokker-Planck equation

To obtain the general Fokker-Planck equation for this problem, one can either consider the motion of a magnetic moment $\mathbf{S}$ in a magnetic field which is the sum of a constant field $\mathbf{G}_0 = (a,b,c + 2\bar{\tau}/k)$ and a random field $\mathbf{g} = [0,0,2(\tau - \bar{\tau})/k]$ of $\mathbf{0}$ mean

$$\frac{d\mathbf{S}}{ds} = k\,\mathbf{S} \times (\mathbf{G}_0 + \mathbf{g}), \qquad (17)$$

or as a magnetic moment in the constant magnetic field and submitted to the random torque $\mathbf{R}$:

$$\frac{d\mathbf{S}}{ds} = k\,\mathbf{S} \times \mathbf{G}_0 + \mathbf{R}, \qquad (18)$$

with the constraint $\mathbf{S}^2 = 1$.

Both approaches are known to be equivalent [31], leading to the same Fokker-Planck equation. Indeed, $\mathbf{R}$ is a vector field of $\mathbf{0}$ mean and variance $4\mu^2$, independent of the values of $\mathbf{S}$, and it can be treated as uncorrelated Gaussian vectorial random field.

Using the Brownian theory applied to Eq. (17) [32] or the functional integral technique applied to Eq. (18) [13,31], one can derive the exact *general* (i.e., for any anisotropy coefficients a,b,c, complying with the weak anisotropy assumption, and any initial condition $\mathbf{S}_0$) Fokker-Planck equation for the probability distribution $P[\mathbf{S}; s]$ of the Stokes parameters $\mathbf{S} = (S_1,S_2,S_3) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, of an electromagnetic wave propagating in a transparent fiber of random Gaussian torsion. It is

$$\frac{\partial P}{\partial s} = -\frac{1}{\sin\theta}\left(\frac{\partial H}{\partial \phi}\frac{\partial P}{\partial \theta} - \frac{\partial H}{\partial \theta}\frac{\partial P}{\partial \phi}\right) + \frac{\mu}{2}\nabla^2 P, \qquad (19)$$

with the classical Hamiltonian $H = -aS_1 - bS_2 - (c + 2\bar{\tau}/k)\,S_3$, and a,b,c given as in (4). The solution of the Fokker-Planck equation (19) for the case $(a,b,c) = (0,0,0)$ and the initial condition $\mathbf{S}(0) = (1,0,0)$ is (15).

VII. SUMMARY AND POSSIBLE DOMAINS OF APPLICATION

The behavior of the polarized electromagnetic wave in a medium with random birefringence has been formulated within the framework of the Fokker-Planck theory in the case of the randomly twisted optical fiber. Figure 2 shows an example of such a fiber. The vectors show the directions of the normal vector $\mathbf{n}$, and they visualize the random twist of the fiber.

The propagation medium is then essentially characterized by the torsion that is a geometric feature of the curve in space. In the present paper, we have shown how to calculate the probability distribution of the Stokes parameters on the Poincaré sphere in such a case, using a general technique that we developed in a previous paper [13]. In the case of fibers without any intrinsic birefringence, exact results on the statistics of the polarization state were obtained by examining the angular distribution of the Faraday rotation.

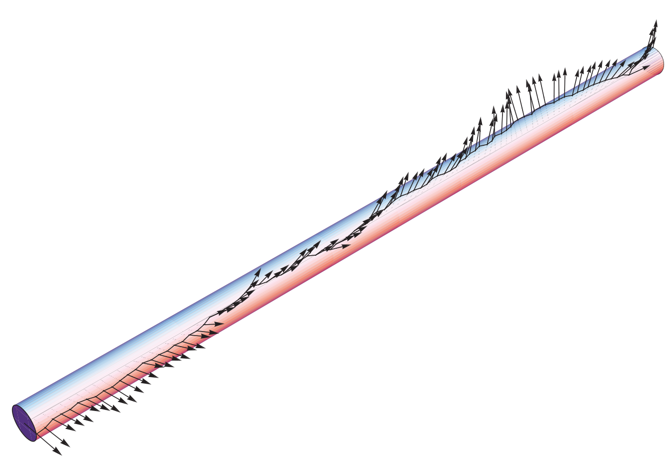

FIG. 2. (Color online) Sketch of a fiber with curvature $\kappa = 0$, null average torsion $\bar{\tau} = 0$, and random torsion τ distributed as a normal law with positive standard deviation μ. The cylinder represents the fiber, and the sequence of arrows shows 100 successive directions of the normal vector $\mathbf{n}$ along the fiber. The range $0 \leqslant \mu s \leqslant 10$ is shown.

This approach is relevant to the analysis of data on optical fibers with weakly randomized twisting. It might be the case for loose optical fibers in a fluid at rest (air or water, for example), the small random motion of the fluid resulting in random torsion of the fiber, through the fluid viscosity.

It could also be the case for the photonic crystal fibers, made of array of identical fibers [33]. The small deviations to the perfect fiber being analyzed through the statistics of the polarization states at a given fiber length.

Another case worthy to be investigated that way, is the propagation of the light through a weakly disordered transparent material. The sequence of scattering events leads to a number of various "optical paths" as through infinity of dissipative waveguides with random torsion. If the disorder is weak, only the paths close to the null average torsion (the straight line) are relevant, then the distribution of the polarization fluctuations should give insight of the statistics of the inhomogeneities.

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