Brownian motor in a granular medium

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In this work we experimentally study the behavior of a freely rotating asymmetric probe immersed in a vibrated granular medium. For a wide variety of vibration conditions the probe exhibits a steady rotation whose direction is constant with respect to the asymmetry. By changing the vibration amplitude and by filtering the noise in different frequency bands we show that the velocity of rotation depends not only on the RMS acceleration Γ, but also on the amount of energy provided to two separate frequency bands, which are revealed to be important for the dynamics of the granular medium: The first band governs the transfer of energy from the grains to the probe, and the second affects the dynamics by altering the viscosity of the vibro-fluidized material.

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I. INTRODUCTION

Exploiting random thermal motion for producing energy has been a longstanding dream, the impossibility of which in equilibrium systems was definitively established with the advent of thermodynamics showing that *perpetuum mobile* of the second kind is contrary to the second principle for any closed system. At the microscopic level this conclusion relies on molecular chaos as discussed by Smoluchowski and Feynman (see e.g. Refs. [1,2]), though the laws of thermodynamics do not prevent the possibility of extracting work from systems that are isothermic and in a stationary state *but not at equilibrium*. Such a state would require some sort of spontaneous symmetry breaking and constitutes the subject of Brownian motors [1].

Experimental realizations, generally named thermal ratchets, have been recently studied in different situations, ranging from the nano- to the microscale [3–5]. The interesting case of a thermal ratchet in which an out-of-equilibrium glass former acts as two thermal reservoirs at different temperatures has been numerically simulated recently [6]. Macroscopic realizations have been developed in the field of granular mechanics [7,8], a field of great relevance both industrially [due to the ubiquity of granular materials (GMs)] and theoretically (for the challenging properties and behavior generally exhibited by GMs [9]). Strictly speaking granular systems are athermal since the motion of the elementary constituents of the medium, the grains, is not affected by the ambient temperature. It may therefore seem contradictory to speak of thermal ratchets; however, by externally perturbing a sample, one can impart kinetic energy to the grains and so define a temperature [10], even though inelastic collisions and dissipation impede the emergence of a state analogous to thermal equilibrium.

Therefore, GMs can be used to implement macroscopic realizations of thermal ratchets, both experimentally [7,8,11] and theoretically [12–15]. Interesting models have been derived in which the nonzero mean angular velocity of the ratchet emerges naturally from the interaction with the granular material [16], though the model requires a very dilute granular gas. Another exemplary instance of a Brownian motor in a granular system may be seen in Ref. [17], where a ratchet with vanes of differing coefficient of normal restitution on either side is enclosed within a vibrated granular gas. The ratchet is seen to undergo spontaneous rotation as a consequence of unequal momentum transfer on the vanes' opposing sides.

With this last exception, where motion originates from mechanical asymmetry, the experimental realizations cited above were principally targeted toward the observation of spontaneous collective oriented motion of the grains, whereas in this work we wish to focus on the spontaneous motion of an external intruder with geometrical asymmetries *in a dense granular material in the absence of any collective granular motion*. Numerical simulations [13] and theoretical analyses [15,16] have demonstrated that such a phenomenon is possible, at least in granular gas.

Here we present novel results showing that the chaotic granular motion can indeed propel the asymmetric probe in a persistent direction. More specifically, the immersed asymmetric probe, under suitable conditions of fluidization and viscosity, exhibits consistent and significant motion in a constant direction with respect to its asymmetry. Under different conditions the system can exhibit collective motion of the grains (e.g., convection); however, in these cases the symmetry breaking is performed by the GM, and so the direction of asymmetry of the probe is irrelevant.

II. EXPERIMENTAL SETUP

The main components of the experimental apparatus are the granular medium, the shaker, and the probe, as shown in Fig. 1. The signal to the shaker is supplied by a function generator and is amplified and filtered with an attenuation slope of 24 dB/oct on both sides. The vibration was applied in the range from 10 to 1000 Hz, and the system acceleration ($0 < \Gamma < 4$, normalized to gravity g) is measured by an accelerometer fixed to the vibrating stage of the shaker. The probes, each 3 cm tall, are immersed to a depth H in the GM, measured from the granular surface to the bottom of the probe. The GM is composed of glass beads of diameter 2 mm $\pm$ 10% which approximately half fill a beaker, 90 mm diameter and 120 mm tall.
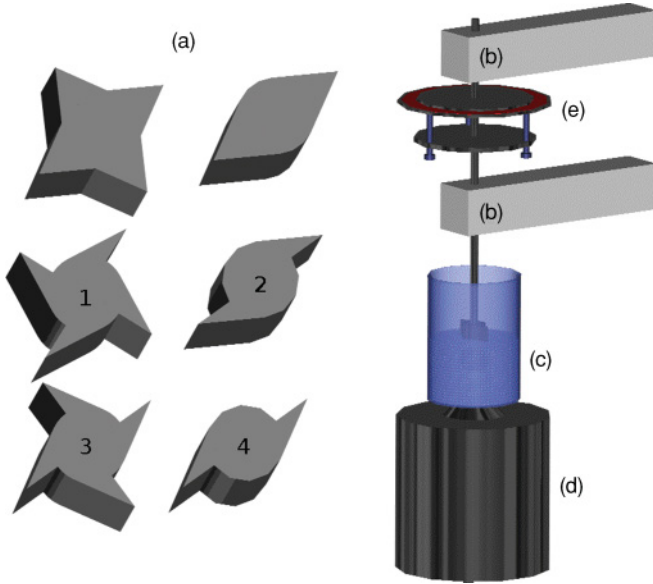
FIG. 1. (Color online) Example of the symmetric and asymmetric probes used (a). The probe is supported vertically by stops that bear on fixed teflon sleeves (b) allowing free rotation and is inserted into a beaker (c) containing the beads. The beaker is vibrated by a shaker (d) (Bruel and Kjaer type 4809), and the instantaneous angular position of the probe is registered by a rotary encoder (e) with resolution $1/500°$.

III. GENERAL RESULTS

It is well known that when a GM is shaken with a periodic signal, a wide range of coherent motions can arise [18]. Generally speaking, such motions are due to a dynamic symmetry breaking in the system [8] and vary from the presence of isolated and discontinuous collective movements of a minority of beads to major effects such as inclination of the surface or formation of a single vortex in which the entire GM rotates. In our system these coherent motions typically arise when the shaker is excited by a pure square or sine wave in a range of frequencies up to about 150–200 Hz. For example, well-defined internal convective motion was observed that caused the probe to rotate until it stops at a specific angle, irrespective of the starting point. Alternatively, in the presence of vortices the probe was seen to rotate indefinitely in a constant direction at roughly a constant velocity, with no dependence on either the presence or the direction of the probe asymmetry—the probe clearly follows the macroscopic collective motion of the granular material. However, above 200 Hz or with white-noise excitation, only chaotic motion of beads was observed with no collective effect, while over 1000 Hz the granular material seems no longer to respond to the agitation even for $\Gamma \gg 1$. Hence white noise was applied to the shaker in order to eliminate any convective or collective motion of the GM, and this allowed us to apply energy also below 200 Hz without triggering collective motion. The use of nonperiodic signals requires a correct definition of Γ [10]:

$$\Gamma = \frac{\sqrt{2\langle \ddot{z}^2(t)\rangle}}{g} = \frac{\sqrt{4\pi^2 \sum_f f^2 \langle |\zeta(f)|^2\rangle}}{g}, \quad (1)$$

in which $\langle \ldots \rangle$ is a time average, $\ddot{z}(t)$ is the instantaneous acceleration of the granular container, and $\zeta(f)$ is the Fourier transform of $z(t)$. In the experiment, the acceleration is proportional to the current fed to the shaker, whose spectral density is constant in the case of white-noise excitation. Equation (1) reduces to the usual definition of Γ for sinusoidal excitations. Other definitions also exist that reduce to the usual Γ for periodic signals; however, the above choice is motivated by the fact that Eq. (1) is proportional to the overall energy provided to the granular medium, and furthermore that it yields the value $\Gamma = 1$ at an evident transition point in our experiments (this transition is also observed at the calculated value of $\Gamma = 1$ with pure sinusoidal excitation).

Using white noise in a large enough frequency range eliminates convective motions, and the granular exhibits a uniform and homogeneous behavior. Under these conditions it may be expected that no net rotation of the probe would be observed due to the random nature of the action of the granular material on the probe surface. Contrarily, however, there is an almost regular rotation of the probe when $\Gamma > 1$, the direction of which depends on the orientation of the probe asymmetry. No net rotation was observed using the symmetric probes. The response of the probe at various excitation intensities and frequency ranges has been studied with the result that the probe velocity is dependent on the upper and lower frequency limits of the shaking signal. The probe immersion depth H into the GM has also been varied.

A. Rotation velocity

Figure 2 presents a general result illustrating the asymmetry-dependent mean rotation velocity $\langle \omega \rangle$ of the probe as a function of the vibration intensity Γ. Below $\Gamma = 1$ the velocity $\langle \omega \rangle = 0$. There is a clear transition at $\Gamma = 1$ at which point ω initially increases rapidly before reaching an asymptotic value that is dependent on the probe used (e.g., probes with four teeth rotate faster than those with two) and the direction of its asymmetry—inverting the probe asymmetry results in a simple inversion of ω. Notably, symmetric probes show $\langle \omega \rangle = 0 \forall \Gamma$.
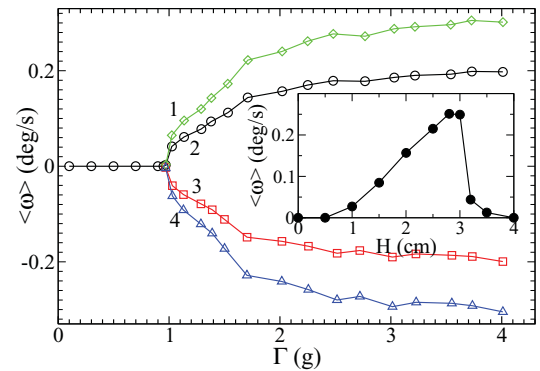

FIG. 2. (Color online) Probe average velocity as a function of the shaking acceleration for the two− and four-teeth asymmetric probes (numbered 1 to 4 as in Fig. 1). The vibration signal is white noise filtered in the range 15–300 Hz. Inset: Average rotation velocity ω as a function of the immersion depth H of the probe at $\Gamma = 2$. At $H = 3$ cm the probe is fully immersed in the medium.

In the inset to Fig. 2 the average velocity of the probe for various immersion depths H of the probe into the granular material is shown. For small $H < 1$ cm there is no rotation, and we assume that the action of the GM on the probe is insufficient to overcome friction in the support.

Increasing H causes the probe surface to engage more completely with the GM, and the velocity increases roughly linearly until $H \simeq 2.5$ cm. At this point the probe is almost fully immersed in the medium, and the velocity reaches a brief plateau. Further immersion $H > 3$ cm causes the rotation velocity to fall quickly to 0. This behavior suggests that the region of the GM that applies the spontaneous torque to the probe is close to the surface.

B. Filtering bands

The role of frequency in the activation of the spontaneous motion is elucidated by changing the intensity and frequency range of the applied white noise. It is seen in this way that the behavior of the system is principally determined by the energy provided in two major frequency bands. We consider a first series of experiments in which we apply a single-frequency band to the GM by applying a bandpass filter to white noise from f_1 to f_2 Hz: first, by altering $15 < f_1 < 60$ Hz with fixed $f_2 = 300$ Hz, and then by fixing $f_1 = 15$ Hz and altering $300 < f_2 < 900$ Hz. As the frequency band is widened, the overall energy provided to the system would increase; to investigate therefore only the change in frequency, the signal intensity is reduced in order to maintain a constant overall energy input, proportional to Γ (the probe behaves similarly to Fig. 2 as a function of Γ).

For different f_1 and fixed Γ the probe displays a different mean angular velocity $\langle \omega \rangle$. Values taken at $\Gamma = 4$ are shown in Fig. 3 as a function of f_1, the lower bandpass cutoff. It is clear that $\langle \omega \rangle$ decreases slowly from $f_1 = 15$ to 30 Hz. From 30 to 40 Hz there is a consistent jump, which corresponds to a previously observed absorption peak in the apparatus response function [19], due to the granular material efficiently absorbing energy from the vibration and so mobilizing the probe. For
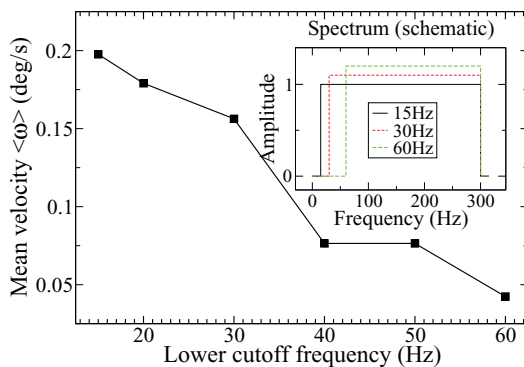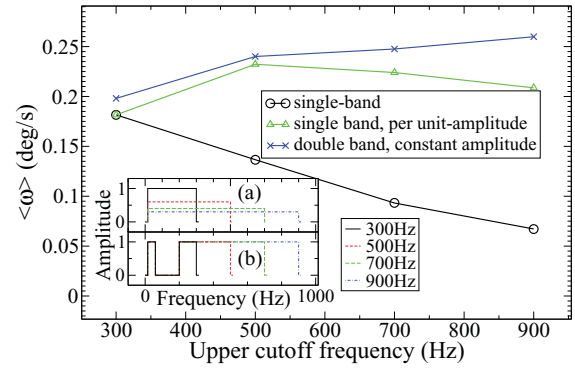
FIG. 4. (Color online) The value of the probe velocity (measured at $\Gamma = 4$) while varying the upper cutoff f_2 of the bandpass filter. This was performed both with a single band, maintaining the overall signal energy by decreasing the amplitude as appropriate, and with a double band at constant amplitude (see inset). In the first case, the velocity decreases (circles and black line) displaying a roughly linear behavior with respect to the amplitude (triangles and green line). In the second case, increasing sevenfold the energy in the upper band produces only a slight increase in the mean probe velocity (crosses and blue line); this is in contrast to the result in Fig. 3 where a reduction of approximately 15% resulted in a 75% reduction in the velocity.

$f_1 > 40$ Hz $\langle \omega \rangle$ continues to decrease slowly. Importantly, this jump between 30 and 40 Hz is independent of the value of the higher cutoff frequency f_2 (in this example fixed at 300 Hz, though fixed at up to 900 Hz in other experiments). Thus it can be concluded that the frequency band from roughly 30 to 40 Hz is crucial to generating motion of the probe.

We now fix $f_1 = 15$ Hz and increase the upper bandpass cutoff frequency $300 < f_2 < 900$ Hz, still maintaining a constant overall energy input by reducing the signal amplitude as necessary, as schematized in the inset to Fig. 4. The main panel of Fig. 4 demonstrates that the mean velocity actually *decreases* as the frequency band is widened (black circles). This may seem contrary to Fig. 2 but in fact is a direct consequence of Eq. (1) and the reduction in signal intensity, specifically in the 30–40 Hz band, which is required to maintain a constant Γ. In fact, if one divides the velocity obtained by the amplitude applied (relative to that with $f_2 = 300$ Hz), one obtains an almost constant velocity (green triangles), indicating linearity of ω with Γ.

To overcome this effect, we consider a final series of experiments in which the amplitude of the signal sent to the shaker is kept constant. In order to supply sufficient energy in the bands which interested us (15–60 and 200–900 Hz), it was necessary to apply a band reject from 60 to 200 Hz; the spectrum applied to the shaker is schematized in the inset to Fig. 4. In this manner a constant energy input in the lower band is maintained while the upper band can be independently widened.

In Fig. 4 we show the variation of the asymptotic probe velocity as the upper band is widened from 300 to 900 Hz (blue ✕). The velocity is seen to increase only slightly despite the large increase in the bandwidth.

When taken together with the observation that motion ceases when energy is removed from the lower-band *irrespective of the upper band*, then the logical conclusion is that the

FIG. 3. (Color online) The asymptotic value of the probe velocity (i.e., $\langle \omega \rangle$ at $\Gamma = 4$) while varying the lower cutoff f_1 of the bandpass filter from 15 to 60 Hz, necessitating an increase in the filter amplitude in order to maintain a constant Γ. The mean velocity decreases approximately proportional to f_1 with a step between 30 and 40 Hz, coincident with an absorption peak previously observed in the apparatus's response function [19].

upper frequency band is not responsible for the spontaneous torque causing the rotation. The slight increase in velocity, therefore, may be ascribed to a reduction in the viscosity of the GM; such a phenomenon has already been observed in Ref. [10]. Indeed a high-frequency vibration merely weakens the contacts between grains, which, however, remain caged in place by their neighbors, whereas a low-frequency, high-amplitude motion actually imparts macroscopic motion to the grains. The viscous torque M_g can be written as the product of the probe viscous coefficient in the GM ν_B and the probe velocity ω:

$$M_g = -\nu_B \omega, \tag{2}$$

where ν_B is explicitly dependent on the excitation band, and in fact is inversely proportional to Γ [10]:

$$\nu_B = \frac{a[\zeta(f)]}{\Gamma}, \tag{3}$$

where $a[\zeta(f)]$ is some function of the spectrum applied to the shaker and dictates the viscous response of the GM for a given value of Γ. Future work will be aimed at measuring the torque M_g to which the probe is subjected, without which it is not possible to quantify $a(\zeta)$. Thus we infer that, when the lower cutoff is increased above $f_1 = 40$ Hz, Γ is drastically reduced, which both increases the viscosity and reduces the spontaneous torque F generated by the GM. When the upper cutoff is increased, however, even if Γ remains almost constant, a decrease in viscosity is observed due to the presence of the term $a[\zeta(f)]$ in Eq. (3).

IV. DISCUSSION

In summary, the origin of the observed behavior can be traced back to both the mechanical absorption of energy by the GM at low frequency and its fluidization at high frequency. In the case of $\Gamma < 1$ rotation does not occur as the GM is insufficiently mobilized and maintains a high viscous coefficient ν_B. The reduction of ν_B with $\Gamma > 1$ confers sufficient mobility to the probe, and so rotation occurs. It is clear from Fig. 4 that a slight decrease of ν_B, due to the widening band, produces a slight increase in the probe velocity (blue curve).

During experimentation, the granular medium is clearly seen to gain considerable freedom near the top surface, and the beads closest to the faces of the probe tend to move away from the probe, reaching zones of the granular surface, which hinder the probe rotation less. In doing so the beads leave the probe radially and, because of the asymmetry, contribute some net momentum to the probe and trigger rotation. The gained momentum is smaller for the faces parallel to the radii of the probe, so the direction of rotation is that in which these faces move ahead. The hypothesis that surface grains, with their greater freedom, are responsible for the spontaneous rotation of the probe is consistent with the observation that a completely immersed probe, no longer in contact with the top surface, ceases to rotate Fig. 2 (inset). In this respect the results are different from those reported in Ref. [20] for the simulation and observation [21] of a Brownian motor driven by bacteria.

Given the results obtained above, the following equation of motion for the probe may be hypothesized:

$$I\dot{\omega} = F - R - \nu_B \omega + N, \tag{4}$$

where I is the moment of inertia of the rotating elements (the probe, its support, and any dragged GM [22]), F is the net torque exerted by the GM on the probe, R is the frictional torque of the support measured to be constant, $\nu_B \omega$ represents the viscous resistance torque due to the granular [10,23], and N is a noise term indicating fluctuations of F and is assumed to have zero time average, $\langle N(t) \rangle = 0$. Future investigations will be devoted to the measure of ν_b and $N(t)$.

V. CONCLUSIONS AND OUTLOOK

The main achievement of the experiment reported here has been the experimental observation of the Brownian motor effect at a macroscopic, centimeter, scale. The observations are in general agreement with other works related to "granodynamic" viscosity [10,23] and computer simulations [24]. The GM absorbs energy efficiently at 30–40 Hz, similar to that found by numerical experiments [24], and when excited in this band provokes spontaneous rotation of the immersed asymmetric probe. Excitation at higher frequency reduces the viscosity of the GM, but does not in itself trigger rotation.

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