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# Experimental measurement of efficiency and transport coherence of a cold-atom Brownian motor in optical lattices

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The rectification of noise into directed movement or useful energy is utilized by many different systems. The peculiar nature of the energy source and conceptual differences between such Brownian motor systems makes a characterization of the performance far from straightforward. In this work, where the Brownian motor consists of atoms interacting with dissipative optical lattices, we adopt existing theory and present experimental measurements for both the efficiency and the transport coherence. We achieve up to 0.3% for the efficiency and 0.01 for the Péclet number.

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Brownian motors (BM's) are devices that can rectify noise into work or directed motion in the absence of external forces. They are of interest for the understanding of fundamental principles in statistical physics and thermodynamics, and several studies have shown that they play a crucial part in transport phenomena in nature; see, for example, [1–3]. Since BM's utilize noise, they can work in regions where the inherent noise is large compared to other interactions. Applications of BM's, therefore, reach into the nanoscales, where they make ideal tools for powering up nanomachines [4–6]. Recent reviews of the subject can be found in [7–10].

Of particular interest for any motor is the quantification of its efficiency, usually defined as the ratio of produced work to input energy. Due to the peculiar nature of the energy source of BM's, determination of efficiency is not straightforward. There have been several theoretical discussions on the efficiency of BM's [11–15], and different performance characteristics have been discussed in [16]. We present here experimental measurements of two performance characteristics of a BM realized with ultracold atoms in double optical lattices [17]: the efficiency, that is, the fraction of input power driving the directed motion, and the transport coherence, or the Péclet number, that is, the comparison between the drift and the diffusion. Usually, the efficiency is defined in terms of the amount of work obtained from the motor against a load. As no load is present in our case, we instead follow the convention [11,12] of defining "useful energy" as the energy needed to drive the directed motion of the atoms against friction. It has also been argued that including the dissipation due to friction against the directed motion provides a better definition of efficiency even when a load is present [11].

For a BM to be able to function, it has to (i) present an asymmetry [18] and (ii) be out of thermal equilibrium [19]. In most cases, the symmetry breaking arises either from a time-asymmetric periodic driving force with zero average (rocked ratchet), or by flashing an asymmetric potential (flashed ratchet). However, as shown in our system [17], rectification can be achieved by switching between two symmetric potentials.

The model for the BM used in our experiment was introduced in [20]. Briefly, particles with mass $m$ move in two symmetric potentials, $U_1 = A_1 \cos(kx)$ and $U_2 = A_2 \cos(kx + \phi)$, phase shifted by $\phi$, and are randomly transferred between the two with unequal transfer rates $\Gamma_{1\to2} \neq \Gamma_{2\to1}$. In addition, the particles experience a friction force $-\alpha_i \dot{x}$ and a diffusive force $\xi_i(t)$ in either lattice $i = 1,2$. This gives the equations of motion

$$m\ddot{x} = -\nabla_x U_i(x) - \alpha_i \dot{x} + \xi_i(t). \tag{1}$$

Here, $\xi_i(t)$ satisfies the relations $\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_i(t') \rangle = 2D_i\delta(t - t')$. Thus the atom is both subject to work from the potential $U_i$ and to fluctuations and dissipation given by the diffusion coefficient $D_i$ and the friction coefficient $\alpha_i$.

For an atom moving in a single periodic potential, the long-time average of the work goes to zero, and the atoms reach a steady state with kinetic temperature $D_i/\alpha_i$. This changes when it is transferred between the potentials, changing instantaneously its potential energy. The total work on an atom is therefore equal to the changes in potential energy summed over all jumps between the potentials. For identical potentials ($A_1 = A_2, \phi = 0$), no energy is gained by an atom transferred between the potentials; see Fig. 1(a). In this situation, both potentials satisfy the symmetry condition $U_i(-x) = U_i(x)$, which entails that $\langle \dot{x} \rangle = -\langle \dot{x} \rangle$, and hence no BM effect is possible [21]. Introducing a nonzero $\phi$ between the lattices, the system still possesses glide reflection [22], but because of the unequal transfer rates between the potentials, there is no symmetry condition requiring $\langle \dot{x} \rangle = 0$, and therefore there will in general be a rectification [18]. An exception is the point $\phi = \pi$, where again $U_i(-x) = U_i(x)$, leading to zero current, with the input energy gained from the transfer between potentials only appearing as a heating of the atoms.

Following the discussion in [12], we can derive a detailed balance relation for the input power $P_{\text{in}}$ acting on a particle governed by the equation of motion (1),

$$P_{\text{in}} = \frac{\overline{\alpha}\,\overline{p}^2}{m^2} + \frac{\overline{\alpha\delta p^2}}{m^2} - \frac{N\overline{D}}{m}. \tag{2}$$

Here $\overline{\alpha}$, etc., signifies the time average (over the two lattice states), and $\delta p(t)$ is the variation of the momentum $p(t)$ around its long-time average $\overline{p}$. The first and second terms

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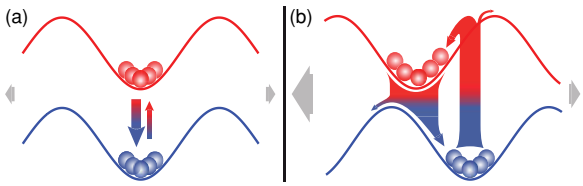


FIG. 1. (Color online) Schematic drawing of the process driving our BM. In each potential, an inherent friction and diffusion is present. Vertical arrows indicate the transfer between the potentials, horizontal ones the total diffusion. (a) $\phi = 0$: atoms are transferred between the lattices without any gain in potential energy. No symmetry is broken and no drift is induced. (b) $\phi = 3/2\pi$: the symmetry is broken and the transfer adds energy to the system, as a leftward drift.

on the right-hand side of Eq. (2) are the energy loss due to dissipation associated with the directed and the random motion, respectively. The third term represents the energy gained through diffusive processes internal to either of the two lattices, as represented by the constants of momentum diffusion $D_i$. The factor $N$ is the dimensionality of the system, in our case $N = 3$, and $\delta p^2 = \delta p_x^2 + \delta p_y^2 + \delta p_z^2 \simeq 3\delta p_z^2$. In our system, the temperature might be slightly different in the different directions [23]. However, this approximation will only have a minor effect on the results.

In [11,12], the efficiency of a BM without a load is defined as the energy dissipated by friction acting against the directed motion over the total energy input, that is,

$$\eta = \frac{\overline{\alpha}\,\overline{p}^2/m}{P_{\text{in}}} = \frac{\overline{\alpha}\,\overline{p}^2/m}{\overline{\alpha}\,\overline{p}^2/m + \overline{\alpha\delta p^2}/m - N\overline{D}}. \tag{3}$$

Assuming that $\overline{\alpha}$ is uncorrelated to the lattice state, so that $\overline{\alpha\delta p^2} = \overline{\alpha}\overline{\delta p^2}$, this expression is simplified to

$$\eta = \frac{\overline{p}^2/m}{\overline{p}^2/m + \overline{\delta p^2}/m - N\overline{D}/\overline{\alpha}}. \tag{4}$$

Simulations indicate that, even for a relatively large difference in the friction coefficients between the two lattices ($\alpha_2 = \alpha_1/2$), this assumption introduces an error in $P_{\text{in}}$ of only about 2%.

The experiment has been described in detail in [17,24,25]. In short, we use laser cooling to trap and cool cesium atoms and transfer them into a double optical lattice [24,25]. These are potentials realized from the interference pattern of laser beams due to a second-order interaction between the induced atomic dipole moment and the periodic light fields [26]. The two potentials correspond to two different hyperfine levels, $F = 3$ and 4, of the electronic ground state of cesium. Each atom will be transferred between the two potentials at random times through optical pumping, with rates for transfer, scattering, and cooling set by the parameters of the laser fields (intensity and detuning). To collect data, we use absorption imaging to measure the mean momentum $\overline{p}$ as well as the size of the atomic cloud. The imaging is done in the horizontal plane to avoid any effects of gravity [27]. To access the mean momentum spread (the kinetic temperature) $\overline{\delta p^2}$ in Eq. (4), we use a time-of-flight technique that enables fast and accurate measurements of the distribution of the momentum, $\delta p_z^2$ [28].

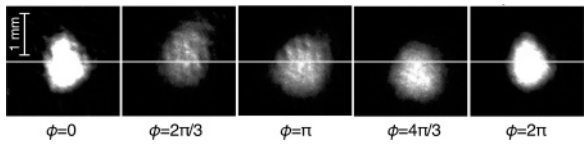


FIG. 2. Absorption images of atoms in the double optical lattice for five different relative spatial phases where the potential depth is 200 $\mu$K. The atoms are kept in the lattices for 150 ms. For $\phi = 0$ and $2\pi$, the drift is zero and the diffusion is small. For $\phi = \pi$, the drift is also zero but the diffusion is much larger. Images 2 and 4 show maximum drifts in the two different directions.

In the experiment we, adjust the potential depths by controlling the intensities in the lattice beams such that $A_1 = A_2$. To assess the quantity $\overline{D}/\overline{\alpha}$, we study the system at $\phi = 0$, where the potentials are identical and there is no BM effect since the transfer between the potentials does not change the energy of the system ($P_{\text{in}} = 0$). From the energy balance (2), we then obtain, in agreement with the equipartition theorem,

$$\overline{E_{\text{kin}}}|_{\phi=0} = \left.\frac{\overline{\delta p^2}}{2m}\right|_{\phi=0} = \frac{N}{2}\frac{\overline{D}}{\overline{\alpha}}. \tag{5}$$

The association of $\overline{D}/\overline{\alpha}$ with the kinetic temperature at $\phi = 0$ assumes that the diffusion and friction constants are independent of the relative phase of the lattices. Following the standard model of Sisyphus cooling [29], our model (1) assumes that diffusion and friction are spatially homogeneous. It should be noted, though, that in a more accurate model these coefficients are dependent on the position $x$ of the atom in the lattice. The spatial distribution of atoms in either lattice will have some dependence on $\phi$, which will translate into a dependence on the spatially averaged friction and diffusion coefficients. This is ignored in our model, introducing a degree of approximation in Eq. (4) for efficiency.

Absorption images were taken for five different potential depths. In Fig. 2, we show typical raw data for atoms kept 150 ms in the lattices. A clear drift is seen in images 2 and 4, while images 1, 3, and 5 show no drift, as expected. Images such as Fig. 2 have been taken for $0 \leqslant \phi \leqslant 2\pi$ for potential depths between 40 and 200 $\mu$K, and the analyzed data can be

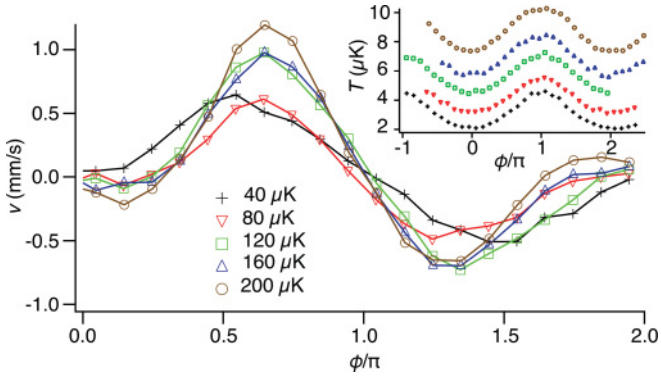


FIG. 3. (Color online) The drift velocity vs the phase shift for a lattice holding time of 150 ms. Inset: the kinetic temperature for the same data.

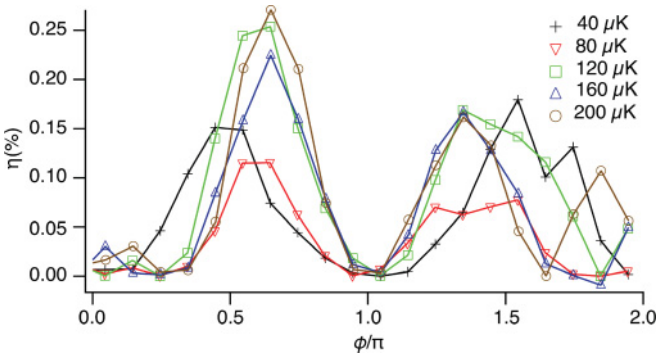


FIG. 4. (Color online) The efficiency, according to Eq. (4), of the Brownian motor as a function of the relative spatial phase for five different potential depths. The greatest efficiency is achieved for $\phi$ equal to $2\pi/3$ and $4\pi/3$ and drops to zero for $\phi = \pi$.

seen in Fig. 3 in terms of the drift velocity. The induced drifts are expected to be symmetric around $\phi = \pi$. However, slightly larger drifts are observed for $\phi = 2\pi/3$ than for $\phi = 4\pi/3$, most likely due to experimental limitations in the alignment and the intensity balance of the lattice beams. Also shown in Fig. 3 is the kinetic temperature for the same parameters. We find that the baseline of the kinetic temperature increases with the potential depth, while the amplitude of its variation with $\phi$ is roughly unchanged. As discussed earlier, the $\phi$-dependent kinetic temperature is represented by the second term on the right-hand side of Eq. (2), while the baseline (or kinetic temperature at $\phi = 0$) is represented by the third term. Hence their difference is the variation of the kinetic temperature with $\phi$. Our data show that this variation is approximately the same for different potential depths. Hence, the greater efficiency for larger potential depths is mainly due to the increase in the drift momentum. Using the data from Fig. 3 in Eq. (4), we obtain the efficiency as a function of $\phi$, where the maximum efficiency is close to 0.3%; see Fig. 4.

An alternative way to characterize the rectified motion is by the coherence of the transport, where the linear transport is compared to the diffusion. This can be quantified using the Péclet number [16],

$$\mathrm{Pe} \equiv \frac{|\langle v \rangle| l}{\tilde{D}_{\mathrm{eff}}}, \tag{6}$$

where $l$ is a characteristic length of the system, in our case the lattice constant, and $\tilde{D}_{\mathrm{eff}}$ is the effective spatial diffusion given by

$$\tilde{D}_{\mathrm{eff}} \equiv \lim_{t \to +\infty} \frac{\langle x^2(t) \rangle - \langle x(t) \rangle^2}{2t}. \tag{7}$$

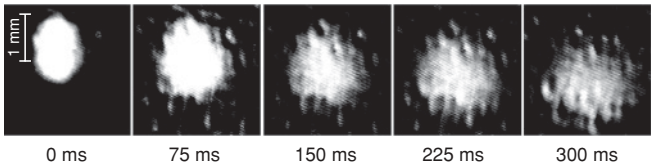


FIG. 5. Time evolution of the atomic sample for a potential depth of 200 $\mu$K. Both the drift and the diffusion are seen.

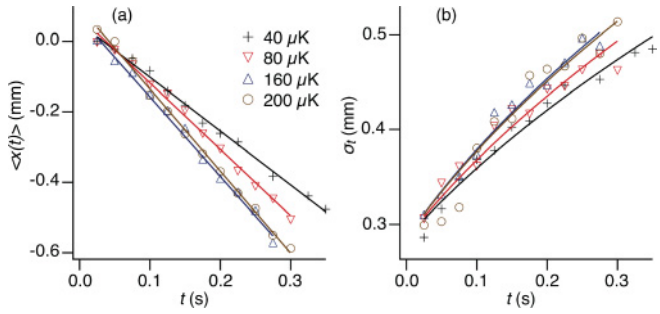


FIG. 6. (Color online) (a) Position of the center of mass of the atomic sample. (b) Root-mean-square radius of the atomic sample as a function of holding time for four different potential depths. The center of mass moves linearly as expected [17], and indicates faster drifts for higher potential depths. The size of the cloud grows with time due to diffusion according to Eq. (8).

For atoms in dissipative optical lattices, where thermal fluctuations play an important role, $\tilde{D}_{\mathrm{eff}}$ becomes the spatial diffusion constant $\tilde{D} = \langle [\delta x(t) - \delta x(0)]^2 \rangle / (2t)$, where $\delta x(t) = x(t) - \langle x(t) \rangle$ [30]. This quantity can be calculated from the expansion of the atomic cloud in the optical lattices, where the size of the cloud is given by

$$\sigma_t = \sqrt{\sigma_0^2 + 2\tilde{D}t}, \tag{8}$$

with $\sigma_t$ the root-mean-square radius at time $t$.

In order to quantify the performance in terms of the Péclet number, series of absorption images of the time evolution of the atomic cloud, such as shown in Fig. 5, have been taken. The phase is set to achieve maximum drift ($\phi = 2\pi/3$).

In Fig. 6(a), a series of such images have been analyzed and the drift is plotted against the holding time in the lattice. In Fig. 6(b), the width of the sample is shown against the holding time, from which the diffusion constant $\tilde{D}_{\mathrm{eff}}$ can be extracted by fitting to Eq. (8). Combining the result with the measured average velocity of the sample, according to Eq. (6), gives the Péclet number for different potential depths; see Fig. 7.

In conclusion, we have adopted existing theory and presented experimental results for two measures of the performance of a Brownian motor, namely the efficiency and the

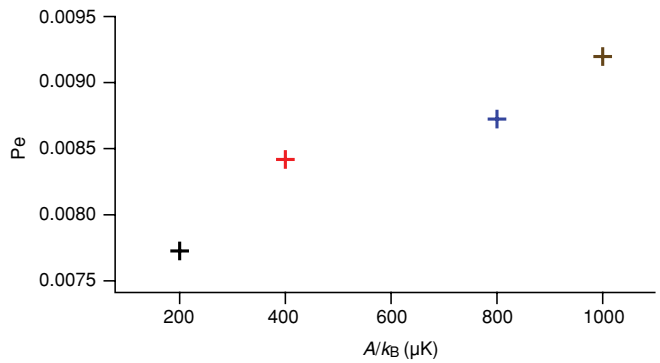


FIG. 7. (Color online) The measured Péclet number, Eq. (6), as a function of potential depth. The data indicate a greater coherence in the transport for higher potential depths.

Péclet number, in a system of ultracold atoms in a double optical lattice. The results indicate trends that give higher efficiency and transport coherence for deeper potentials, and are in agreement with the values of the Péclet number that were predicted for similar systems [31]. Although our BM prototype differs from other BM's, the fundamental principles are the same, and hence these kinds of characteristic measurements allow for interesting comparisons between BM systems from different fields.

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[1] R. D. Vale and R. A. Milligan, Science **288**, 88 (2000).
[2] M. Schliwa and G. Woehlke, Nature (London) **422**, 759 (2003).
[3] R. D. Astumian, Science **276**, 917 (1997).
[4] M. G. L. van den Heuvel and C. Dekker, Science **317**, 333 (2007).
[5] T. E. Mallouk and A. Sen, Sci. Am. **300**, 72 (2009).
[6] J. Wang, ACS Nano **3**, 4 (2009).
[7] P. Hänggi and F. Marchesoni, Rev. Mod. Phys. **81**, 387 (2009).
[8] P. Hänggi, F. Marchesoni, and F. Nori, Ann. Phys. (Leipzig) **14**, 51 (2005).
[9] P. Reimann, Phys. Rep. **361**, 57 (2002).
[10] R. D. Astumian and P. Hänggi, Phys. Today **55**(11), 33 (2002).
[11] I. Derényi, M. Bier, and R. D. Astumian, Phys. Rev. Lett. **83**, 903 (1999).
[12] D. Suzuki and T. Munakata, Phys. Rev. E **68**, 021906 (2003).
[13] L. Machura, M. Kostur, P. Talkner, J. Łuczka, F. Marchesoni, and P. Hänggi, Phys. Rev. E **70**, 061105 (2004).
[14] P. Reimann and P. Hänggi, Appl. Phys. A **75**, 169 (2002).
[15] F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).
[16] H. Linke, M. T. Downton, and M. J. Zuckermann, Chaos **15**, 026111 (2005).
[17] P. Sjölund, S. J. H. Petra, C. M. Dion, S. Jonsell, M. Nylén, L. Sanchez-Palencia, and A. Kastberg, Phys. Rev. Lett. **96**, 190602 (2006).
[18] P. Curie, J. Phys. III (Paris) **3**, 393 (1894).
[19] R. P. Feynman, R. B. Lieghton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963), Vol. 1.
[20] L. Sanchez-Palencia, Phys. Rev. E **70**, 011102 (2004).
[21] P. Reimann, Phys. Rev. Lett. **86**, 4992 (2001).
[22] R. Kanada and K. Sasaki, J. Phys. Soc. Jpn. **68**, 3759 (1999).
[23] J. Jersblad, H. Ellmann, and A. Kastberg, Eur. Phys. J. D **22**, 333 (2003).
[24] H. Ellmann, J. Jersblad, and A. Kastberg, Phys. Rev. Lett. **90**, 053001 (2003).
[25] H. Ellmann, J. Jersblad, and A. Kastberg, Eur. Phys. J. D **22**, 355 (2003).
[26] G. Grynberg and C. Mennerat-Robillard, Phys. Rep. **355**, 35 (2001).
[27] M. Zelan, H. Hagman, K. Karlsson, C. M. Dion, and A. Kastberg, Phys. Rev. E **82**, 031136 (2010).
[28] H. Hagman, P. Sjölund, S. J. H. Petra, M. Nylén, A. Kastberg, H. Ellmann, and J. Jersblad, J. Appl. Phys. **105**, 083109 (2009).
[29] S. Chu, Rev. Mod. Phys. **70**, 685 (1998); C. Cohen-Tannoudji, *ibid.* **70**, 707 (1998); W. D. Phillips, *ibid.* **70**, 721 (1998).
[30] L. Machura, M. Kostur, F. Marchesoni, P. Talkner, P. Hänggi, and J. Łuczka, J. Phys. Condens. Matter **17**, S3741 (2005).
[31] A. B. Kolton and F. Renzoni, Phys. Rev. A **81**, 013416 (2010).