# Intermittent pathways towards a dynamical target

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In this paper, we investigate the quest for a single target, which remains fixed in a lattice, by a set of independent walkers. The target exhibits fluctuating behavior between a trap and an ordinary site of the lattice, whereas the walkers perform an intermittent kind of search strategy. Our searchers carry out their movements in one of two states, between which they switch randomly. One of these states (the exploratory phase) is a symmetric nearest-neighbor random walk and the other state (the relocating phase) is a symmetric next-nearest-neighbor random walk. By using the multistate continuous-time random-walk approach we are able to show that for dynamical targets, the intermittent strategy (despite the simplicity of the kinetics chosen for searching) improves detection, in comparison to displacements in a single state. We have obtained analytic results, which can be numerically evaluated, for the survival probability and for the lifetime of the target. Thus, we have studied the dependence of these quantities both in terms of the transition probability that describes the dynamics of the target and in terms of the parameter that characterizes the walkers' intermittency. In addition to our analytical approach, we have implemented Monte Carlo simulations, finding excellent agreement between the theoretical-numerical results and simulations.

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## I. INTRODUCTION

Search problems have recently experienced a rapid growth and motivated a great deal of work in many varied situations (see Ref. [1] and references therein): fishermen and shoals, prey and predators, two molecules in the course of a reaction, a protein that looks for an specific site on DNA strand, and medical drugs and illnesses. Thus, search problems span a wide range of domains and fields. When the target is fixed at a given location in space, the search problem is equivalent to the trapping problem, that is, the situation where a set of walkers independently diffuse in space until one of them is caught by the trap.

Among different forms of search strategies [1–3], the so-called intermittent strategies—which combine a phase of relocation (where the searcher may or may not be capable of capturing the target) with a phase of search (where target capture is always allowed)—have been proven relevant and able to be optimized at various scales. Intermittent motion occurs in a wide array of living organisms from protozoans to mammals. It has been observed that numerous animal species switch between two distinct types of behavior while foraging or searching for shelter or a mate [4–6]. At a microscopic scale, we find intermittent motion, for example, in the binding of a protein to specific sites on DNA for regulating transcription, as is the case when a protein has the ability to diffuse in one dimension by sliding along the length of the DNA, in addition to its diffusion in bulk solution [7–10].

In Refs. [11–13] a theoretical model for the search kinetics of a hidden target was presented, assuming that each searcher could be in either of two states of diffusion. It was shown that intermittent strategies always improve target detection in comparison with single-state displacement. An important aspect in the searching dynamics that has recently been studied [12,14] is to consider not only the different states in the motion of walkers, but also what happens with the trapping process when additional dynamical effects are taken into account. For instance, in Ref. [12,14] the sighting range and the smell capacity of predators was considered as a sort of additional search ability.

The aim of this study is to complete and extend previous results [11–13] and to present another relevant case for modeling real search problems: the inclusion of a fluctuating behavior in the target. These fluctuations can modify and prevent a successful encounter between a searcher and the target. This behavior may be due to the internal evolution of the trap or due to its interaction with a changing environment. For instance, in a chemical context, the activation or deactivation of a reagent can be caused by external factors (photons, solvent molecules, etc) [15]. In biological contexts, the dynamical behavior of the target is also a determinant; for example, reactions occurring within biomembranes require some geometrical configurations in the biomolecule structure to be completed. The absence of these configurations inhibits the reaction, whereas stochastic changes in the molecule geometry can let it take place. Even the delivery of drug in medical treatments can involve blocking chemical reactions in order to boost the delivered medicine's effectiveness [16].

Reference [17] introduced a generalization of the trapping model that allows encounters between particles of two kinds, $A$ and $B$, with or without annihilation, depending on the internal state of particle $A$. Particle $A$, which is identified as the trap in this work, has two states: an active one in which the annihilation takes place, and an inactive one in which it behaves as a regular site. The $B$ particles are our walkers. In this work, we take a step forward in modeling search problems by the formulation of a unified framework that comprises the dynamical behavior of the trap and the intermittent search strategy performed by the walkers. We exploit the theory of multistate random walk (RW) [18], we use the concepts of survival probability (SP) and mean target lifetime (MTL), and we establish the connection to the first passage time (FPT) corresponding to the problem of one walker.

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The outline of this paper is as follows. The next section presents our model and gives the basic definitions and concepts. Also, this section describes the analytical approach to the trapping process. Section III presents the main results for the SP and the MTL of the target through a comparison between the numerical evaluation of our analytical framework and Monte Carlo simulations. In Sec. IV we discuss our conclusions. Finally, in Appendix A we develop the analytical calculations of Sec. III, corresponding to infinite chains and rings, whereas in Appendix B we consider the high transition regime for the trap.

## II. ANALYTICAL APPROACH

### A. The model

We restrict our work to chains (finite and infinite) and assume that the dynamical trap is held fixed at the origin of the lattice. A set of walkers, uniformly distributed along the chain, starts the "search" at $t = 0$. At the trap site, the following situations may occur:

(i) The trap is in an active status, that is, it works (the first walker reaching the trap is caught with probability one, that is, perfect trapping).

(ii) The trap is in a passive status and stays that way until the searcher leaves it, that is, the trap behaves like any other chain site and capture cannot be carried out.

(iii) The trap is in a passive status but changes its condition before the searcher leaves, that is, the capture is also performed.

We denote the internal states of the trap by $i = 1$ (active status) and $i = 2$ (passive status). On the other hand, we assume that each predator can make two types of motion on the lattice:

(i) *Exploration.* RW with symmetric jumps to first-nearest neighbors, with transition probability per unit time $\lambda$.

(ii) *Relocalization.* RW with symmetric jumps to second-nearest neighbors, also with transition probability per unit time $\lambda$.

We also assume that the walkers' dynamics and the dynamical behavior of the trap are independent.

The proposed composite process can be described by the coupled master equations,

$$\frac{\partial P_{1,i_0}(\vec{s},t|\vec{s}_0,0)}{\partial t} = \mathbb{A} P_{1,i_0}(\vec{s},t|\vec{s}_0,0) + \gamma_2\, P_{2,i_0}(\vec{s},t|\vec{s}_0,0) - \gamma_1\, P_{1,i_0}(\vec{s},t|\vec{s}_0,0), \tag{1}$$

$$\frac{\partial P_{2,i_0}(\vec{s},t|\vec{s}_0,0)}{\partial t} = \mathbb{A} P_{2,i_0}(\vec{s},t|\vec{s}_0,0) + \gamma_1\, P_{1,i_0}(\vec{s},t|\vec{s}_0,0) - \gamma_2\, P_{2,i_0}(\vec{s},t|\vec{s}_0,0), \tag{2}$$

where $P_{i,i_0}(\vec{s},t|\vec{s}_0,t=0)$ is the conditional probability of the walker being at site $\vec{s}$ with the trap in state $i$ at time $t$, given that it was at site $\vec{s}_0$ with the trap in state $i_0$ at $t = 0$. For simplicity, we have restricted the activation-deactivation process of the trap to time-exponential density functions with parameters $\gamma_i$, that is, $\gamma_i$ is the probability transition rate of the trap to make a transition from its state $i$ to the other state. The dynamical evolution of the walkers, taking into account its intermittency, is described by the operator $\mathbb{A}$. Particularly, for a chain, we get

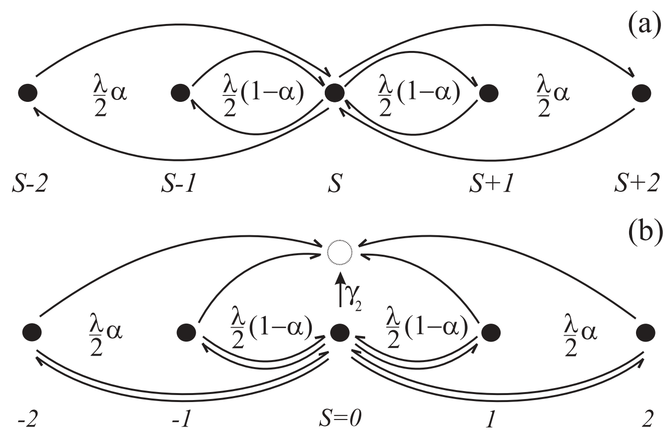


FIG. 1. (a) Schematic transitions of the walker to and from site $s$ (away from the trap: $s \neq -1,0,1$) and (b) Walker transitions to and from $s = 0$ (trap site). A walker dwelling at site 0 could be trapped with rate $\gamma_2$ (the probability transition rate for activation of the trap). The dynamics of the trap is independent of the dynamics of walkers.

$$[\mathbb{A}]_{s,s'} = \frac{\lambda}{2}[(1-\alpha)(\delta_{s,s'-1} + \delta_{s,s'+1}) + \alpha(\delta_{s,s'-2} + \delta_{s,s'+2}) - 2\,\delta_{s,s'}], \tag{3}$$

where $\alpha$ is the parameter that regulates the walker's frequency intermittency and $\lambda$ its diffusion constant (see Fig. 1).

### B. The trapping process

We will focus on the SP of the dynamical target, that is, the probability that the target remains undetected up to a time $t$, and its closely related quantity, the MTL [19], which compute the time in which the first walker reaches the target under the appropriate circumstances of capture. We define $F_{1,i_0}(\vec{0},t|\vec{s}_0,0)$ as the first passage time density through the site $\vec{0}$ at time $t$, when capture is possible, given that the searcher was at $\vec{s}_0$ with the target in state $i_0$ at time $t = 0$. In the way of Ref. [16], we introduce the notion of generalized state, which takes into account the position of the walker and the state of the target, $(\vec{s},i)$. The connection between FPT density at $(\vec{0},1)$ at time $t$ from $(\vec{s}_0,i_0)$, $F_{1,i_0}(\vec{0},t|\vec{s}_0,0)$, and the conditional probability $P_{i,i_0}(\vec{s},t|\vec{s}_0,t=0)$ is established by

$$\hat{F}_{1,i_0}(\vec{0},u|\vec{s}_0,0) = \frac{\hat{P}_{1,i_0}(\vec{0},u|\vec{s}_0,0)}{\hat{P}_{1,1}(\vec{0},u|\vec{0},0)}, \tag{4}$$

which is the known Siegert formula [20], generalized to internal states [21]. We are denoting the Laplace transform of a function of $t$ by a caret over the corresponding function. Thus, for example,

$$\hat{P}_{i,i_0}(\vec{s},u|\vec{s}_0,0) = \mathcal{L}\{P_{i,i_0}(\vec{s},t|\vec{s}_0,0)\} = \int_0^\infty e^{-ut}\, P_{i,i_0}(\vec{s},t|\vec{s}_0,0)\, dt\,.$$

When trapping occurs independently of the initial state of the target, the SP in the presence of only one walker may be written (if $\vec{s}_0 \neq \vec{0}$) as

$$\Phi_1(\vec{0},t|\vec{s}_0,0) = 1 - \sum_{i_0=1}^{2} \theta_{i_0} \int_0^t F_{1,i_0}(\vec{0},\tau|\vec{s}_0,0)\, d\tau, \quad (5)$$

where $\theta_{i_0}$ is the probability of the initial state of the target. Thus, the target is initially active with probability $\theta_1$ or inactive with probability $\theta_2$. The SP at time t, $\Phi_N(t)$, of the dynamical target at the origin in the presence of $N$ independent walkers that diffuse on an $M$-sites lattice can be written in terms of the SP in the presence of only one walker, $\Phi_1(\vec{0},t|\vec{s}_0,0)$, as [19]

$$\Phi_N(t) = \left(1 - \frac{1}{M-1} \sum_{\vec{s}_0 \neq \vec{0}} [1 - \Phi_1(\vec{0},t|\vec{s}_0,0)]\right)^N, \quad (6)$$

where we have assumed a uniform probability distribution for the initial position of the walkers, i.e, the probability that a given walker is initially at a particular site $\vec{s}_0 (\neq \vec{0})$ is $(M-1)^{-1}$. Notice that we explicitly exclude the possibility of having a walker at the position of the target at $t = 0$. In the bulk limit, $N \to \infty$, $M \to \infty$, with $N/M \to \rho$, where the constant $\rho$ is the concentration of walkers, we get

$$\Phi_\rho(t) = \exp\left(-\rho \sum_{\vec{s}_0 \neq \vec{0}} [1 - \Phi_1(\vec{0},t|\vec{s}_0,0)]\right). \quad (7)$$

The MTL is defined in finite domains as [19]

$$T_N = \int_0^\infty \Phi_N(t)\, dt, \quad (8)$$

and in the bulk limit as

$$T_\rho = \int_0^\infty \Phi_\rho(t)\, dt. \quad (9)$$

We left for the appendixes the detailed calculations of the magnitudes presented in this section for the cases of infinite chains and rings of $M$ sites.

### III. RESULTS

In this section, we illustrate the general framework presented in the preceding section. We consider one-dimensional systems and give some general ideas to interpret the obtained results. The inverse Laplace transform involved in the analytical expressions, given in the appendixes, is evaluated numerically [22] for obtaining concrete results and then we establish a comparison with independent Monte Carlo simulations.

A brief review of our simulation methodology is appropriate at this point. We uniformly distribute the searchers (with probability $\rho$ per site) in a one-dimensional lattice with periodic boundary conditions. The target is placed at the origin of the lattice. The propagation of the searchers in the presence of a dynamical target is implemented as follows: Each searcher is assigned an internal clock (all start synchronized at time $t = 0$) that is updated according to its waiting time probability distribution. For the activation-deactivation process of the target a similar procedure to the searchers is used: the target is assigned its own internal clock that is updated with time-exponential density functions with parameters $\gamma_1$ and $\gamma_2$. We define an indicator function that records the needed information: if the target was captured up to a certain time (for the SP) and whether the target was captured and the time in which this happened (for the mean target lifetime). A randomly chosen walker takes a step to its nearest neighbors with probability $(1-\alpha)$ or its next-nearest neighbors with probability $\alpha$ and left or right with equal probability (1/2). We check if the trapping conditions are fulfilled, and if they are, we stop the dynamics, update our indicator function, and generate a new ensemble of walkers. If the trapping conditions are not fulfilled, we continue the dynamics by taking another randomly chosen walker. Again, if trapping occurs, the indicator function is updated and the dynamics stopped; if not, the walk continues. The output of interest of each realization is, for the SP, whether it was captured up to a certain (predefined) time, and the time of capture for the mean target lifetime.

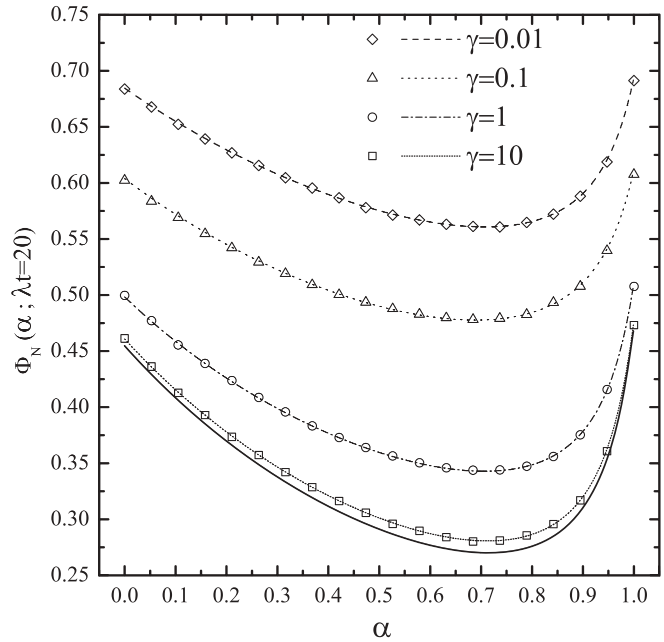


FIG. 2. Analytical-numerical calculations (lines) and Monte Carlo simulations (symbols) for the SP, $\Phi_N(\alpha;t)$, up to time $t = 20$ for different target transition rates ($\gamma$). (diamonds) $\gamma = 0.01$, (triangles) $\gamma = 0.1$, (circles) $\gamma = 1$, and (squares) $\gamma = 10$. We have also included for comparison the static trap case (thick solid line).

In this section, we show the numerical results obtained from the analytical expressions for the infinite chain (see Appendix A1) and the finite ring (see Appendix A2). For both the infinite and finite cases a searchers' concentration $\rho = 0.1$ was used (for the finite case the concentration is defined by $\rho = N/M$). All the finite chains considered in the figures correspond to a ring of $M = 20$ sites, with the exception of Fig. 4, where different sizes ($M$) of the ring are explicitly stated. All times are given in units of the inverse of the diffusion constant ($1/\lambda$). It is worth commenting that when we talk about target transition rates $\gamma_i$, these are low (high) relative to the diffusion constant $\lambda$. An equivalent interpretation can be made if we

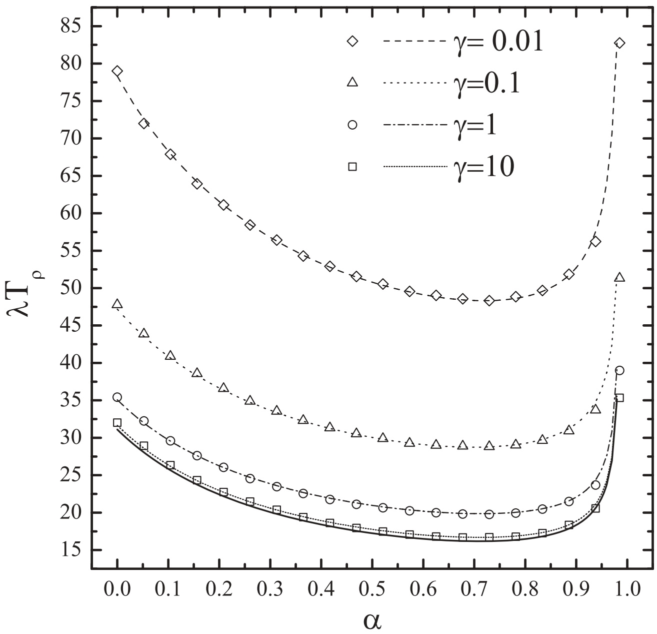


FIG. 3. Analytical-numerical calculations (lines) and Monte Carlo simulations (symbols) for the mean target lifetime (MTL) $T_\rho$, for different target transition rates ($\gamma$). (diamonds) $\gamma = 0.01$, (triangles) $\gamma = 0.1$, (circles) $\gamma = 1$, and (squares) $\gamma = 10$. We have also included for comparison the static trap case (thick solid line).

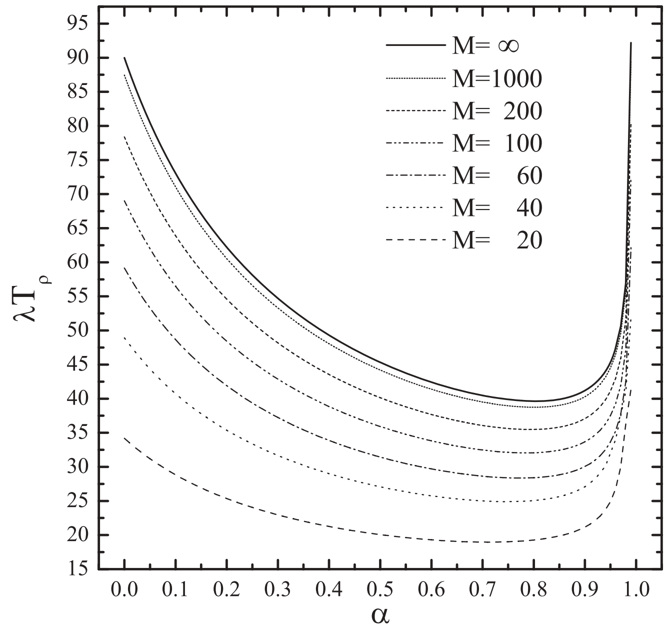


FIG. 4. Analytical-numerical calculations for the mean target lifetime (MTL) for a fixed target transition rate ($\gamma = 1$) as a function of the walker intermittency parameter $\alpha$ and for different sizes $M$ of the chain. From bottom to top, $M = 20,40,60,100,200,1000$, and $M = \infty$ (thick solid line).

consider the target mean sojourn time in state $i$ as $\gamma_i^{-1}$; this will be long (short) on the time scale determined by the propagator $\mathbb{A}\,(\lambda^{-1})$. In the following we will consider symmetric transition rates for the activation-deactivation process of the target, $\gamma_1 = \gamma_2 = \gamma$.

In Fig. 2 we present curves (for the finite case) corresponding to $\Phi_N(\alpha,t)$ for a fixed evolution time $t = 20$. Notice how the intermittent search can improve the detection probability, that is, minimize the SP of the target, compared with the single-state search ($\alpha \sim 0$, $\alpha \sim 1$). As a comparison we also include the "static trap" case, that is, the target is always active. As can be seen from the figure, an optimal value for $\alpha$ can be found for each target transition rate $\gamma$ chosen. Even though all curves present a similar behavior, it is apparent that the transition rate $\gamma$ plays an important role. The ratio between the maximum value of the SP (at $\alpha = 1$) and its minimum is almost of 80% (120%) for $\gamma = 0.1$ ($\gamma = 0.01$). At high values of $\gamma$, the "static trap" case is approached.

The curves shown in Fig. 3 correspond to the MTL, in the finite case, as a function of the walker intermittency parameter $\alpha$, for different target transition rates $\gamma$. As is clear from the figure, the results show the same trend as the SP (Fig. 2), also revealing a remarkable rise in MTL for low values of the target transition rates $\gamma$. It could be inferred from the curves that with a modest transition rate value ($\gamma = 0.1$) the target could almost double its lifetime expectancy while a high "activity" of the target ($\gamma > 1$) leads it to the static case. Note that although MTL has less information than the SP, it is a simple and efficient tool for characterizing the proposed search scheme.

Figure 4 depicts the behavior of the MTL for a fixed target transition rate ($\gamma = 1$) as a function of the walker intermittency parameter $\alpha$ and for different sizes ($M = 20,40,60,100,200,1000,\infty$) of the chain. In all cases a concentration of searchers $\rho = 0.1$ was used. Notice how the finite chain (ring) approaches the infinite chain even for not too large values of $M$ . As can be seen from the figure, the minimum in MTL is maintained for all system sizes, which constitutes a robust property of the intermittent search approach.

In Fig. 5 we consider the behavior of the SP, $\Phi_N(\alpha;t)$, in the high transition regime of the dynamic target for a fixed evolution time $t = 20$. In this limit, the behavior of the SP approaches an imperfect trap (see Appendix B), with $\nu = (\gamma_1 + \gamma_2)\gamma_2/\gamma_1$ a "measure of the imperfection" of the trapping process. When $\nu \to 0$ there is no trapping and if $\nu \to \infty$ perfect trapping is achieved. Notice how the dynamical trapping resembles the imperfect case even for not too large values of $\gamma_i$. It is worth mentioning the excellent agreement between the analytical-numerical results and the Monte Carlo simulations.

## IV. CONCLUSIONS

We have presented a simple model for the search kinetics of a set of walkers performing intermittent motion in a quest for a dynamical target. The model is based only on a RW, and our results complement and extend previous related results given in Refs. [11–13]. However, this model differs from those mentioned. In our previous work, the first searcher that finds the target captures it with probability one (we denominate that situation the "static trap" in this work). In the present work,

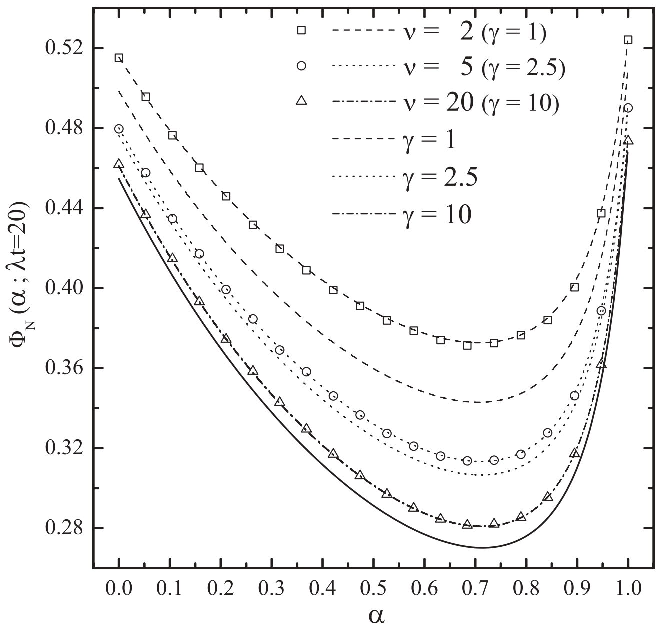


FIG. 5. Analytical-numerical calculations (lines) and Monte Carlo simulations (symbols) for the SP, $\Phi(\alpha;t)$, up to time $t = 20$, in the high transition regime of the target. Lines with symbols corresponds to the "imperfect" (high transition) case and the same type of line (without symbols) for the dynamical case. For instance (squares) $\nu = 2$, $\gamma = 1$; (circles) $\nu = 5$, $\gamma = 2.5$, and (squares) $\nu = 20$, $\gamma = 10$. We have also included for comparison the static case (thick solid line).

a walker-target encounter does not necessarily end in capture, but depends rather on the state of the dynamic target.

We have considered the target's survival probability (at a fixed time) and the target's lifetime, and also studied the dependence of these quantities on both the transition probability ($\gamma_i$) between the states of the target and the parameter that characterizes the walker's intermittency ($\alpha$). Thus, we have established that the SP is a nonmonotonic function of $\alpha$ for a wide range of the transitions probabilities $\gamma_1$ and $\gamma_2$, showing that intermittent strategies still improve target detection when compared with single-state displacement. This confirms the utility of the intermittent search approach [23], even in the case of a dynamical target.

We introduced the MTL and its connection with the SP was established. As was the case for SP, MTL was also a nonmonotonic function of $\alpha$ for several values of the transition probabilities $\gamma_i$, adequately depicting the improvement provided by the intermittent search strategy. Although MTL carries less information than the SP, it has shown to be an efficient global optimizer for search strategies using intermittent motion. In all cases the agreement between analytical-numerical results and Monte Carlo simulations was quite good.

Thus, we have fulfilled our goal of presenting a simple model, based only in diffusion, that captures in an unified framework the dynamical behavior of the target and the intermittent search strategy performed by the walkers. The present scheme is both simple enough to be studied analytically and rich enough to be able to mimic the influence of the target's dynamics in the capture process, and it shows that intermittency is always favorable for optimizing the search.

The present model of intermittent search can be generalized in several directions: higher dimensions, continuous systems, non-Markovian target dynamics, etc. All of these aspects will be the subject of future work.


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### APPENDIX A

Here we describe some essential details of the calculations in Sec. II B. We focus on the $P_{i,i_0}(s,t|s_0,t=0)$, which are the building blocks for the SP and MTL. Given that we now apply our model only to chains, we drop the vector notation.

#### 1. Infinite chain

The solution of Eq. (1), with the particularization of Eq. (3), for an infinite (homogeneous) chain, can be given following the guidelines of Refs. [18,24,25]. The exploitation of the indicated formalism leads us to analytic closed expressions for $P_{i,i_0}(s,t|s_0,t=0)$ in the Fourier-Laplace space. However, as we are interested in $P_{1,i_0}(s,t|s_0,t=0)$ (we take $i = 1$ as the active status of the target) we only show the solution for this case. The relevant transformed results read

$$\hat{P}_{1,1}(k,u) = \frac{1}{\Gamma}[\gamma_2 \hat{P}^0(k,u) + \gamma_1 \hat{P}^0(k,u+\Gamma)],$$
$$\hat{P}_{1,2}(k,u) = \frac{1}{\Gamma}[\gamma_2 \hat{P}^0(k,u) - \gamma_2 \hat{P}^0(k,u+\Gamma)], \quad \text{(A1)}$$

where $\Gamma = \gamma_1 + \gamma_2$, $P^0(k,u) = 1/[u - \mathbb{A}(k)]$ is the Fourier-Laplace transform of the conditional probability $P^0(s,t|s_0, t=0)$ corresponding to the intermittent walker being at site $s$ at time $t$, given that it was at site $s_0$ at $t = 0$ (without trap), and $\mathbb{A}(k) = \lambda[(1-\alpha)\cos k + \alpha \cos 2k - 1]$ is the Fourier transform of the evolution operator $\mathbb{A}$ given by Eq. (3).

We are interested in obtaining results for any initial state of the target or trap. Therefore is useful to evaluate the average,

$$\sum_{i_0} \hat{P}_{1,i_0}(k,u)\,\theta_{i_0} = \hat{P}_{1,1}(k,u)\,\theta_1 + \hat{P}_{1,2}(k,u)\,\theta_2$$
$$= \frac{1}{\Gamma}(\gamma_2 \hat{P}^0(k,u) + (\gamma_1\theta_1 - \gamma_2\theta_2)\hat{P}^0(k,u+\Gamma)), \quad \text{(A2)}$$

where $\theta_1$ is the target probability of being initially active, and $\theta_2$ that of being inactive and satisfying $\theta_1 + \theta_2 = 1$. As usual, we choose for $\theta_i$ the equilibrium probabilities [16] $\theta_1 = \gamma_2(\gamma_1+\gamma_2)^{-1}$, $\theta_2 = \gamma_1(\gamma_1+\gamma_2)^{-1}$. The Fourier inversion of Eq. (A2) could be calculated in an exact way resulting in

$$\sum_{i_0} \hat{P}_{1,i_0}(s,u|s_0,0)\,\theta_{i_0} = G\left(\frac{\eta_1^{|s-s_0|}}{\sqrt{x_1^2-1}} + \frac{\eta_2^{|s-s_0|}}{\sqrt{x_2^2-1}}\right), \quad \text{(A3)}$$

where $\eta_1 = x_1 - \sqrt{x_1^2 - 1}$, $\eta_2 = x_2 + \sqrt{x_2^2 - 1}$, $G = \gamma_2/2\lambda\Gamma\alpha(x_1 - x_2)$, and

$$x_{1,2} = -\frac{1-\alpha}{4\alpha} \pm \frac{1}{2}\sqrt{\left(\frac{1-\alpha}{2\alpha}\right)^2 + 2\,\frac{u+\lambda(1+\alpha)}{\lambda\alpha}}.$$

Averaging Eq. (A3) over the starting positions (uniformly distributed) of the walker we arrive at

$$\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{P}_{1,i_0}(s,u|s_0,0)\,\theta_{i_0} = \frac{G}{\sqrt{x_1^2-1}}\frac{2\eta_1}{1-\eta_1} + \frac{G}{\sqrt{x_2^2-1}}\frac{2\eta_2}{1-\eta_2}. \tag{A4}$$

Taking $s=0$, $s_0=0$, and considering Eqs. (A3) and (A4), we can write

$$\begin{aligned}\sum_{s_0\neq 0} &\mathcal{L}\{[1-\Phi_1(0,t|s_0,0)]\}\\ &= \frac{1}{u}\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{F}_{1,i_0}(0,u|s_0,0)\,\theta_{i_0}\\ &= \frac{1}{u}\frac{1}{\hat{P}_{1,1}(0,u|0,0)}\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{P}_{1,i_0}(0,u|s_0,t=0)\,\theta_{i_0}. \end{aligned}\tag{A5}$$

Equation (A5) constitutes one of our main results and allows us derive SP from Eq. (7) and MTL from Eq. (9), after taking the inverse Laplace transform. However, despite being able to obtain analytical results in Laplace space for Eq. (A5), its length and complexity made the analysis a difficult task. The analytical inversion of the Laplace transform of the results seems to be beyond our possibilities, so we have used a numerical procedure [22] for its calculation in Sec. III.

### 2. Ring of $M$ sites

For the finite case, we take the results from the preceding section, and obtain for the ring a solution in the form [26]

$$\hat{P}^M_{i,i_0}(s,u|s_0,t=0) = \sum_{l=-\infty}^{\infty} \hat{P}_{i,i_0}(s+lM,u|s_0,t=0)\,. \tag{A6}$$

In order to evaluate the sum proposed in Eq. (A6), we focus our attention on one term of Eq. (A3). Given that for all $l \neq 0$, $|l|\,M > (s-s_0)$ and if $l<0$ $|(s-s_0)+l\,M| = -(s-s_0) - l\,M$, we get

$$\begin{aligned}\sum_{l=-\infty}^{\infty} \eta_1^{|s-s_0+lM|} &= \eta_1^{|s-s_0|} + \left(\eta_1{}^{|s-s_0|} + \eta_1^{-|s-s_0|}\right)\sum_{l=1}^{\infty}\eta_1^{lM}\\ &= \frac{1}{1-\eta_1^M}\left(\eta_1^{|s-s_0|} + \eta_1^{M-|s-s_0|}\right).\end{aligned}\tag{A7}$$

Working in a similar way with the other terms, we obtain the complete solution of Eq. (A6), for the state of capture ($i=1$) as

$$\sum_{i_0} \hat{P}^M_{1,i_0}(s,u|s_0,0)\,\theta_{i_0} = \sum_{l=-\infty}^{\infty}\sum_{i_0}\hat{P}_{1,i_0}(s+lM,u|s_0,0)\,\theta_{i_0} = G\left(\frac{\eta_1^{|s-s_0|}+\eta_1^{M-|s-s_0|}}{\sqrt{x_1^2-1}\left(1-\eta_1^M\right)} + \frac{\eta_2^{|s-s_0|}+\eta_2^{M-|s-s_0|}}{\sqrt{x_2^2-1}\left(1-\eta_2^M\right)}\right). \tag{A8}$$

For a uniform distribution of walkers along the ring the following expression is useful:

$$\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{P}^M_{1,i_0}(s,u|s_0,0)\theta_{i_0} = G\left(\frac{2}{\sqrt{x_1^2-1}}\frac{\eta_1-\eta_1^M}{(1-\eta_1^M)(1-\eta_1)} + \frac{2}{\sqrt{x_2^2-1}}\frac{\eta_2-\eta_2^M}{(1-\eta_2^M)(1-\eta_2)}\right). \tag{A9}$$

With Eq. (A8) evaluated in $s=0$, $s_0=0$, and taking into account (A9), we can write

$$\begin{aligned}\sum_{s_0\neq 0} &\mathcal{L}\{[1-\Phi_1(0,t|s_0,0)]\}\\ &= \frac{1}{u}\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{F}^M_{1,i_0}(0,u|s_0,0)\,\theta_{i_0}\\ &= \frac{1}{u}\frac{1}{\hat{P}^M_{1,1}(0,u|0,0)}\sum_{\substack{i_0=1,2\\ s_0\neq 0}} \hat{P}^M_{1,i_0}(0,u|s_0,t=0)\,\theta_{i_0}. \end{aligned}\tag{A10}$$

From Eq. (A10), the SP [Eq. (6)], $\Phi_N(t)$, and the MTL [Eq. (8)], $T_N$, are obtained. However, as in the preceding section, the size and complexity of Eq. (A10) makes the inversion of the Laplace transform beyond our possibilities, so we need to use a numerical procedure [22] for obtaining the concrete results presented in Sec. III.

## APPENDIX B

In this appendix we consider the high transition regime in dynamical trapping, that is, the behavior of the SP in the limit $\Gamma \gg \lambda$. The calculation may be carried out starting with the Laplace transform of Eq. (5):

$$\hat{\Phi}_1(0,u|s_0,0) = \frac{1}{u}\left(1 - \sum_{i_0} F_{1,i_0}(0,u|s_0,0)\,\theta_{i_0}\right). \tag{B1}$$

Let us consider the second term on the right-hand side of Eq. (B1)

$$\begin{aligned}\sum_{i_0} \hat{F}_{1,i_0}(0,u|s_0,0)\theta_{i_0} = \gamma_2\,\hat{P}^0(0,u|s_0,0)/\\ (\gamma_2\hat{P}^0(0,u|0,0) + \gamma_1\hat{P}^0(0,u+\Gamma|0,0)).\end{aligned}\tag{B2}$$

In the considered limit ($\Gamma \gg \lambda$), $\hat{P}^0(0,u+\Gamma|0,0) \sim 1/\Gamma$ [27], then

$$\sum_{i_0} \hat{F}_{1,i_0}(0,u|s_0,0)\,\theta_{i_0} \simeq \frac{\gamma_2 \hat{P}^0(0,u|s_0,0)}{\gamma_2 \hat{P}^0(0,u|0,0) + \gamma_1/\Gamma} \simeq \frac{\nu \hat{P}^0(0,u|s_0,0)}{1+\nu \hat{P}^0(0,u|0,0)}, \tag{B3}$$

where $\nu = \Gamma\gamma_2/\gamma_1$. Notice that Eq. (B3) adequately provides the limits of perfect trapping ($\gamma_2/\gamma_1 \gg 1$, i.e., $\nu \to \infty$) $\sum_{i_0} \hat{F}_{1,i_0}(0,u|s_0,0)\,\theta_{i_0} = \hat{P}^0(0,u|s_0,0)/\hat{P}^0(0,u|0,0)$ and no target or trap present ($\gamma_2/\gamma_1 \ll 1$, i.e., $\nu \to 0$) $\sum_{i_0} \hat{F}_{1,i_0}(0,u|s_0,0)\,\theta_{i_0} = 0$. Using Eqs. (B3) and (B1) we finally obtain

$$\hat{\Phi}_1(0,u|s_0,0) \simeq \frac{1}{u}\left(1 - \frac{\nu \hat{P}^0(0,u|s_0,0)}{1+\nu \hat{P}^0(0,u|0,0)}\right). \tag{B4}$$

This result resembles the general case when detection of the target upon encounter is less than certain, that is, imperfect trapping [3]. From Eq. (B4) and using the same procedure from Appendix A 2, the SP [Eq. (6)], $\Phi_N(t)$, and the MTL [Eq. (8)], $T_N$, could be evaluated for the present regime.


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