Amplitude-frequency fluctuations of the seasonal cycle, temperature anomalies, and long-range persistence of climate records

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The presence of long-term persistence of climate records on scales from 2 to 15 yr has been reported in the literature, even if the universality of this result is controversial. In the present paper results from monthly temperature records measured for about 250 yr in Prague and Milan are reported. Because of the nonlinear and nonstationary character of temperature time series the seasonal contribution has been identified through the empirical mode decomposition. We find that the seasonal component of the climate records is characterized by some time scales showing both amplitude and phase fluctuations. By using a more suitable definition of temperature anomalies, and thus excluding persistence effects due to seasonal oscillations and trends, the occurrence of long-term persistence has been investigated through the detrended fluctuation analysis. Our results indicate persistence on scales from 3 to 10 yr with similar values for the detrended fluctuation analysis indices.

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I. INTRODUCTION

A complete characterization of atmospheric variability in the time domain is important for the climate forecasting. It is useful to quantify, within reasonably narrow limits, the potential extent of global warming, to downscale the global effect, in order to understand the local implications of global change, and to provide empirical constrains to the global models, for which the reproductive power is a fundamental request for interpreting climate change predictions. In particular, investigations on temporal correlations can help us to characterize the persistence of weather and to relate it to the different climate regimes. Climate fluctuations have been detected, on all time scales, from annual to Milankovitch periods. The presence of two distinct power laws in the spectrum of surface temperature series, breaking at about 100 yr, indicates interdependence among the various scales $[1,2]$ $[1,2]$ $[1,2]$ $[1,2]$ and the presence of some persistence or memory in the process that generates fluctuations. It is well established that the weather is persistent on short times, and this kind of short-range persistence typically breaks on 1 week, a time period that corresponds to the average duration of general weather regime [[3](#page-6-2)]. On larger scales the occurrence of persistence can be related to physical mechanisms operating at such characteristic times. In particular the weather is persistent during the period in which a very stable high-pressure system is established over a particular region remaining in place for several weeks, the so called "blocking" $[4]$ $[4]$ $[4]$. On a monthly scale, persistence could be linked to solar flare intermittency $[5]$ $[5]$ $[5]$ or, at even longer terms, a source for weather persistence might be a slowly varying external forcing such as the sea surface temperatures $\lceil 3 \rceil$ $\lceil 3 \rceil$ $\lceil 3 \rceil$. On the scale of seasons, one of the most pronounced phenomena is the El Niño southern oscillation occurrence, every \sim 3-5 yr, which strongly affects the weather over the tropical Pacific as well as over North America $[6]$ $[6]$ $[6]$.

The presence of long-term persistence in climate system is already debated $[7]$ $[7]$ $[7]$. It is commonly investigated through the estimate of the detrended fluctuation analysis (DFA) scaling exponent δ . This parameter represents a statistical index related to the dynamics of fluctuations of a stochastic process. In particular DFA applied to some temperature data sets showed that a scaling law with a roughly universal exponent δ \approx 0.65 exists in the range of time scales from 2 to 15 yr [[8](#page-6-7)]. The coupling of atmospheric and oceanic processes could be involved in setting of long-range persistence with the same exponent for the weather stations in different climatic zones and time regimes $\lceil 8 \rceil$ $\lceil 8 \rceil$ $\lceil 8 \rceil$. The effects of this coupling, underlined in the context of interdecadal and century-scale climate oscillations $[9]$ $[9]$ $[9]$, is one of the core matter in climatology $[10]$ $[10]$ $[10]$. More recently the claimed universality has been questioned [[11](#page-6-10)] since a value $\delta \approx 0.5$ seems to be present over continental lands while higher values $\delta \approx 0.65$ have been found over the coastline. This has been attributed to slowly varying external forcing such as the presence of ocean, where $\delta \approx 1$, or even big reservoirs of water [[11](#page-6-10)]. Even if the value of δ has been used as a statistical tool to distinguish among different climatic regions $[12]$ $[12]$ $[12]$, the search for a systematic behavior of scaling exponents with distance from the sea and/or latitude fails $\left[3,13,14\right]$ $\left[3,13,14\right]$ $\left[3,13,14\right]$ $\left[3,13,14\right]$ $\left[3,13,14\right]$, and the claimed universality of persistence is a matter of scientific debate $[15,16]$ $[15,16]$ $[15,16]$ $[15,16]$. Furthermore, comparisons with global climate models lead to contrasting results. A test performed on seven state-of-the-art global models failed to reproduce the scaling behavior of six long temperature records by underestimating the long-range persistence of the atmosphere $[17,18]$ $[17,18]$ $[17,18]$ $[17,18]$. On the contrary it has been demonstrated that long-range correlations can be reproduced by numerical models when the coupling with ocean $\lceil 11 \rceil$ $\lceil 11 \rceil$ $\lceil 11 \rceil$ or volcanic forcing $[19]$ $[19]$ $[19]$ is properly taken into account. We have to remark that persistence analysis can be relevant to the current debate over the global warming to distinguish the "anthropogenic signal" from the fluctuations due to the natural variability of the geophysical system $[20]$ $[20]$ $[20]$. In fact, to overcome this problem, empirical data are complemented with simulated ones obtained from global circulation models $\lceil 21 \rceil$ $\lceil 21 \rceil$ $\lceil 21 \rceil$. Hence, the presence or absence of persistence represents a very useful test for the competing climate models and to verify the basic assumptions underlying them $[8]$ $[8]$ $[8]$. On the basis of some detailed data analysis, it has been emphasized [[22](#page-6-21)] that scale-free statistics can be hardly supported by climatic fluctuations, that is, the value of persistence exponent δ obtained from usual data sets is not constant but instead depends on the scale. Lanfredi *et al.* [[22](#page-6-21)] described possible dynamical scenarios, associated to the fractal behavior of δ , through a simple bivariate Markovian model able to account for the apparent scale invariance.

It is quite clear that, since the analysis of long-term persistence on temperature records has led to contrasting results, more precise analyses of temperature time series are required before suggesting dynamical paradigms useful for climate modeling and for the assessment of climate change. The question is how the persistence that might be generated by very different mechanisms on different time scales can be quantified. The answer to this question is not easy. In fact, correlations, and in particular long-term correlations, can be masked by trends that can be generated by anthropic processes, e.g., by the well-known urban warming. Even uncorrelated data in the presence of long-term trends may look like correlated ones; and, on the other hand, long-term correlated data may look like uncorrelated data influenced by a trend. Hence, in order to calculate the degree of persistence, we need to distinguish between trends and correlations. Usually, to eliminate trends in the temperature data sets, the temperature anomalies are calculated. In climate studies, the temperature anomalies are commonly defined with respect to the seasonally varying mean value. Namely, given a sequence of daily temperatures T_i , the anomalies ΔT_i are defined as the differences

$$
\Delta T_i = T_i - \langle T_i \rangle,\tag{1}
$$

where $\langle T_i \rangle$ is the temperature mean value for the *i*th calendar day, averaged over a significantly large sample of data. In such a definition of anomaly, there is the implicit assumption that the seasonal annual cycle is constant, and it is generated by a set of stationary processes. Clearly, since the response of the climate system to external forcing is nonlinear, the validity of the previous assumption is often questionable. Hence, the classical definition of anomaly could not be adequate to the complex physics of the system; and, consequently, the persistence estimation, along with the proposed physical mechanisms introduced to explain the correlation, may be misleading or even erroneous. Moreover, definition ([1](#page-1-0)) cannot take into account for seasonal trends or irregularities observed as changes in both amplitude $\lceil 23 \rceil$ $\lceil 23 \rceil$ $\lceil 23 \rceil$ and phases $[24–26]$ $[24–26]$ $[24–26]$ $[24–26]$ in the annual cycle of surface temperature. These are related to the complex nonlinear response of the atmosphere, land, and oceans to the periodic forcing provided by the annual motion of the Earth around the Sun $\left[24,26\right]$ $\left[24,26\right]$ $\left[24,26\right]$ $\left[24,26\right]$ or changes in albedo, soil moisture, and short-wave forcing $[25]$ $[25]$ $[25]$. In addition the effect of long-term climate change, if any, should also be included in the definition of anomaly. As claimed by Thomson $[24]$ $[24]$ $[24]$ "Anomaly series used in climate research that have been deseasonalized by subtracting monthly averages need to be recomputed. The best method for doing this is not obvious."

In this paper, we will investigate two related topics. The first is to reexamine the concept of Earth's temperature anomalies, compatible with a seasonal cycle that can be modulated in both amplitude and frequency. This is done by using the empirical mode decomposition (EMD) $[27]$ $[27]$ $[27]$, a technique that decomposes a nonstationary time series through intrinsic mode functions (IMFs) and allows a deseasonalization by using a partial sums of EMD modes. In this way, by analyzing two historical temperature records, we defined a seasonal cycle which could vary both in amplitude and phase. As a second issue, we calculate the degree of persistence in temperature records by using a definition of anomaly obtained through the EMD. We will show that, in this way, a similar degree of persistence of the climate system can be detected, at least up to some few years, for both data sets.

II. DEFINITION OF SEASONAL BEHAVIOR AND TEMPERATURE ANOMALIES USING EMD

The periodicities involved in the data set, and their relative amplitudes, have been identified through the EMD, a technique developed to process nonstationary data $\lceil 27 \rceil$ $\lceil 27 \rceil$ $\lceil 27 \rceil$ and successfully applied in many different contexts $[28-31]$ $[28-31]$ $[28-31]$, including geophysical systems $[32-35]$ $[32-35]$ $[32-35]$. In the EMD framework, a time series $T(t)$ is decomposed into a finite number *n* of oscillating IMFs as

$$
T(t) = \sum_{j=0}^{n-1} \theta_j(t) + r_n(t).
$$
 (2)

The IMFs $\theta_j(t)$ are a set of basis functions obtained from the data set under analysis by following the "sifting" procedure described by Huang *et al.* [[27](#page-6-26)]. This process starts by identifying local minima and maxima of the raw signal $T(t)$. The envelopes of maxima and minima are obtained through cubic splines, and the mean between them, namely, $m_1(t)$, is then calculated. The difference between the raw time series and the mean series, $h_1(t) = T(t) - m_1(t)$, represents an IMF only if it satisfies two criteria: (i) the number of extrema and zero crossings does not differ by more than 1 and (ii) at any point the mean value of the envelope defined by the local maxima and the envelope defined by the local minima are zero. If $h_1(t)$ does not support the criteria, the previous steps are repeated by using $h_1(t)$ as raw series; and $h_{11}(t) = h_1(t)$ $-m_{11}(t)$, where $m_{11}(t)$ is the mean of the envelopes, is generated. This procedure is repeated *k* times until $h_{1k}(t)$ satisfies the IMF's properties. The sifting procedure should be applied with care since at the extreme limit the process could generate a pure frequency modulated IMF of constant amplitude. To guarantee that the IMF components contain enough physical sense with respect to both amplitude and frequency modulations, a criterion to stop the sifting process has been introduced $[27]$ $[27]$ $[27]$. A kind of standard deviations, calculated using two consecutive siftings, is defined as

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$$
\sigma = \sum_{t=0}^{N} \left[\frac{|h_{1(k-1)}(t)| - |h_{1k}(t)|}{h_{1(k-1)}^2(t)} \right],
$$
\n(3)

and the iterative process is stopped when σ is smaller than a threshold value σ_{thres} , which in our case is chosen as 0.3 [[27](#page-6-26)]. The first IMF, $\theta_1(t) = h_{1k}(t)$, contains the shortest time scale of the process and has a zero local mean. The function $r_1(t) = T(t) - \theta_1(t)$ is called the first residue, and it is analyzed in the same way as just described thus obtaining a new IMF $\theta_2(t)$ and a second residue $r_2(t)$. The process continues until θ_k or r_k is almost zero everywhere or when the residue $r_k(t)$ becomes a monotonic function from which no more IMF can be extracted. At the end of the procedure the original time series is decomposed into *n* empirical modes ordered with increasing characteristic time scale and a residue $r_n(t)$ which can be either the trend or a constant. Each $\theta_j(t)$ has its very own time scale and represents a zero mean oscillation experiencing amplitude and frequency modulations. That is, each IMF $\theta_j(t)$ is not restricted to a particular frequency, but it experiences both amplitude and frequency modulations, namely, it can be written as $\theta_j(t) = A_j(t) \cos \Phi_j(t)$, where $A_j(t)$ and $\Phi_j(t)$ represent, respectively, the amplitude and the phase of the *j*th mode. This kind of decomposition is local, com-plete, and in fact orthogonal [[27,](#page-6-26)[36](#page-6-31)]. The residue $r_n(t)$ in Eq. ([2](#page-1-1)) describes the mean trend, "if any." The statistical significance of information content for each IMF, with respect to a white noise, can be checked by applying the test by $\lceil 37 \rceil$ $\lceil 37 \rceil$ $\lceil 37 \rceil$. The EMD approach is surely more appropriate when dealing with non-stationary and even nonlinear data, like temperature records. In these cases averages of the data, critically depending on the chosen number of points, could cancel some of relevant features in the original signal and reduce temporal resolution.

The orthogonality property of EMD modes can be exploited to reconstruct the signal through partial sums in Eq. (2) (2) (2) $[27,35,38]$ $[27,35,38]$ $[27,35,38]$ $[27,35,38]$ $[27,35,38]$. In fact each IMF captures the empirical dynamical behavior of a single independent mode of the system, namely, each *j* mode captures a single aspect of the complex dynamics. This means that it makes sense to split the EMD modes of temperature signals into three parts, namely, a seasonal contribution $S(t)$, the temperature anomalies $\Delta T(t)$, and the residual $r_n(t)$:

$$
T(t) = S(t) + \Delta T(t) + r_n(t).
$$
 (4)

By looking at the set of IMFs, described by the index *j* =0,1,...,*n*− 1, we can define two mutually orthogonal sets of indices *s* and *r*, such that each $j \in s \oplus r$. Then, by partial sums, we can reconstruct the seasonal contribution by using only the subset *s*, that is,

$$
S(t) = \sum_{j \in s} \theta_j(t),\tag{5}
$$

while the remaining IMFs are used to define temperature anomalies

$$
\Delta T(t) = \sum_{j \in r} \theta_j(t). \tag{6}
$$

Of course, due to the complexity of the system, the sets *r* and

FIG. 1. Time history of temperatures for the Milan data set (upper panel) and the Prague data set (lower panel). Temperatures are given in °C.

s cannot be defined *a priori*; rather they will be suitably chosen by looking at the time behavior of the various IMFs.

To check this approach, two long temperature records of Prague and Milan from the European Climate Assessment & Data set, available online at $\lceil 39 \rceil$ $\lceil 39 \rceil$ $\lceil 39 \rceil$, have been analyzed in this paper. The data sets report the daily maximum temperatures for 230 yr (from 1775 up to 2005) and 240 yr (from 1763 up to 2003), respectively. Since in this paper we are interested in long-term correlations we consider the monthly averaged samples. The time history of both temperature records is reported in Fig. [1.](#page-2-0) The EMD applied to both data sets results in a sequence of 11 IMFs and a residue, for each record, shown in Fig. [2.](#page-3-0) As it can be seen, high values of *j* correspond to larger temporal scales. The residue r_{11} corresponds to a monotonic increase in temperature in the last 2 centuries of about 1.5 °C for Prague and 5 °C for Milan probably due to the phenomenon of urbanization, namely, a local temperature increase due to the growth of urban areas. This contribution is well isolated by the EMD from the other oscillating modes related to long-term correlations. Figures $3(a)$ $3(a)$ and $3(b)$ show the Fourier power spectra of the raw data. A dominant 1-yr peak, associated to the seasonal cycle, is present in both records along with high-frequency harmonics. It must be remarked that the peak from Fig. $3(b)$ $3(b)$ is broader than the corresponding peak in Fig. $3(a)$ $3(a)$, probably indicating that the seasonal cycle of the Prague data set should involve some time scales.

To obtain the temperature anomalies, the set r in Eq. (6) (6) (6) must be accurately defined. To this purpose let us discuss the behavior of the IMFs. A careful look at the time evolution of IMFs from the two data sets indicates substantial differences and reveals a characteristic dynamical behavior for the Prague sample. For both data sets the first IMF θ_0 , corresponding to the lower time scale, describes monthly fluctuations. This mode is not associated to subharmonic of the seasonal cycle; rather it could describe the weather vagaries on monthly scales. The highest amplitude modes θ_1 , character-

FIG. 2. Time evolution of the IMFs θ_i for both data sets. Left column refers to the Milan data set, and right column refers to the Prague data set.

ized by a mean period of about 1 yr, can be associated to the yearly seasonal temperature fluctuations. Let us focus on EMD modes $j=1$ and $j=2$ from Prague shown in Fig. [4](#page-4-0) over a restricted time interval of about 25 yr [Figs. $4(a)$ $4(a)$ and $4(b)$, respectively]. The IMF $\theta_1(t)$ does not show a regular annual oscillation revealing a sudden reduction in amplitude around 1940. On the other hand, $\theta_2(t)$ is characterized by a mean period of about 1.4 yr, and its amplitude is amplified by about 10 around 1940. Irregular intervals within $\theta_1(t)$ indicate that the whole seasonal contribution is not captured by this mode. We conjecture that a regular seasonal cycle could be obtained when both $\theta_1(t)$ and $\theta_2(t)$ are summed up, as shown in Fig. $4(c)$ $4(c)$.

The observed behavior indicates that the seasonal fluctuations are far from being stationary but are associated to some time scales because of the presence of both amplitude and phase modulations. In this case, the seasonal contribution is not isolated in a single EMD mode, but it needs the two IMFs to be completely recovered. The Milan record does not show the above effect, and the seasonal contribution is contained only in the mode $j=1$. It must be remarked that the time behavior of Prague $\theta_1(t)$ should belong to what is know in the literature as "mode mixing" effect, namely, when a

FIG. 3. Upper panels report the Fourier power spectra of (a) Milan and (b) Prague temperature records. Middle panels report the Fourier spectra of seasonal oscillation given by (c) the EMD mode $j = 1$ for Milan and (d) the sum of modes $j = 1$ and $j = 2$ for Prague. Lower panels represent the Fourier spectra of the temperature anomalies for both (e) Milan and (f) Prague. The numbers over the peaks indicate the corresponding periodicity in months.

single IMF consists of signals of widely separated scales [[27](#page-6-26)]. This feature of EMD, associated to the occurrence of intermittency, is troublesome in signal processing where the main purpose is the signal cleanliness. In this framework, to effectively separate IMFs without mixed scales, the noiseassisted method named ensemble EMD (EEMD) has been developed $\lceil 30 \rceil$ $\lceil 30 \rceil$ $\lceil 30 \rceil$. This approach consists of sifting an ensemble of white noise-added signals and treats the mean as the final true result. White noise series should cancel out in the averaging process, when a sufficient number of different realizations are used, thus reducing the chance of mode mixing. The spread of the seasonal cycle over two modes is detected also through EEMD procedure as shown in Ref. [[35](#page-6-30)]. This indicates that the need of two modes to describe the seasonal cycle does not depend on the used technique, but could be related to nonstationarity and intermittency of the temperature records probably attributable to physical reasons. To this purpose, a discussion about the physical origin of the irregular intervals observed in the seasonal IMFs can be found in Ref. $[26]$ $[26]$ $[26]$. According to this data analysis, the irregular seasonal phase could be induced by the local changing of balance between direct insolation and the net energy received by the Earth, with the intermittency occurrence being modulated by the Earth's nutation. Since the temperature

FIG. 4. Time history of the IMF (a) $j=1$ and (b) $j=2$, for a time period of about 25 yr around an irregular seasonal period for the Prague data set. In the lower panel we report the sum of both modes $[(a)$ and (b) , respectively].

records are very sensitive to the local conditions, the modulation due to the Earth's nutation is detected only when a statistical analysis, over a significant number of stations, is performed $[26]$ $[26]$ $[26]$. EMD and EEMD provide an equivalent result for the purpose of anomaly calculation since we have to cut off the modes $j=1$ and $j=2$ from the partial sums. In this paper we chose to discuss the EMD results since the EEMD, by introducing the white noise-added signals, could cancel the information about the intermittency, thus cutting off some physical information. Moreover, IMFs, because of their orthogonality, have a more direct physical interpretation with respect to EEMD modes.

It is important to remark that, as our results indicate, some care must be taken in defining anomalies since nonstationary periods could be present in the temperature records and could also affect the regular seasonal oscillation. In these cases the classical definition of temperature anomalies, as that given in Eq. (1) (1) (1) , is not more suitable. Once the set *r* has been defined, our definition of temperature anomalies ([6](#page-2-1)) should be more suitable to accurately take into account these nonstationary periods.

Taking care of the above considerations, the most natural way to define temperature anomalies is to consider the contribution of all EMD modes, but the properly defined seasonal oscillation. This contribution, as we said before, is different for the two analyzed data sets. This means that the set r in Eq. ([6](#page-2-1)) represents the collection of EMD modes such that $r = \{j | 0 \le j \le 11\} \ominus \{j = 1\}$ for the Milan data set, while *r* $=\{j \mid 0 \leq j \leq 11\} \ominus \{j=1,2\}$ for the Prague data set. With this choice the seasonal cycle, that could present amplitude and phase variations, is excluded by the definition of temperature anomalies.

Fourier power spectra of $S(t)$ from Milan [Fig. $3(c)$ $3(c)$] and Prague $\left[\text{Fig. 3(d)} \right]$ $\left[\text{Fig. 3(d)} \right]$ $\left[\text{Fig. 3(d)} \right]$ records indicate how the EMD isolate the seasonal contribution. The Fourier spectra of $\Delta T(t)$ for both Milan and Prague records are reported in Figs. $3(e)$ $3(e)$ and $3(f)$.

FIG. 5. Autocorrelation functions of temperature anomalies for Milan (white symbols) and Prague (black symbols) as a function of the lag time.

It can be noted that the plot of Fig. $3(e)$ $3(e)$, referring to Milan, shows a peak at about 9 months indicating interannual periodicity. The peak disappears when $\theta_2(t)$ is included into *s*, thus indicating that, also for the Milan record, the seasonal fluctuations are not properly stationary. By observing that (i) the 9-month peak amplitude in the Fourier space is about two orders of magnitude lower than the maximum power and (ii) the mean amplitude of the mode $j=2$ for Milan is of the same order of magnitude of $\theta_3(t)$ from Prague data set, we can say that the Milan temperature anomalies defined above can be considered as deseasonalized at very good approximation.

The time decay of the autocorrelation functions of temperature anomalies $\Delta T(t)$, for both data sets, is rather slow (see Fig. [5](#page-4-1)). In particular, there exists an evident 3-month correlation due to the mode $j=0$ which, as we said before, describes interannual periodicities. In fact, by excluding the mode $j=0$ from the set r , the 3-month autocorrelation disap-pears (Fig. [6](#page-4-2)). In both cases, the temperature anomalies for the Milan data set show a slower decay. In fact the first autocorrelation coefficient which falls within the 95% confidence interval for zero correlation (~ 0.04) is estimated for lag times of \sim 10 months for Prague and \sim 112 months for Milan.

FIG. 6. Autocorrelation functions of temperature anomalies for Milan (white symbols) and Prague (black symbols) as a function of the lag time. Temperature anomalies are defined by excluding subannual fluctuations.

FIG. 7. The scaling function $F_2(n)$ [(a) and (b)] and logarithmic derivative dy [(c) and (d)] as a function of the temporal scale *n* for both the Milan and Prague data set. Straight lines correspond to linear fits (see the text for details).

III. ANALYSIS OF PERSISTENCE OF TEMPERATURE ANOMALIES

To investigate the persistence in the analyzed records, we implemented the DFA, a commonly used tool for minimizing externally induced nonstationary effects describable in the form of low-order polynomials. The DFA algorithm, applied to the temperature anomalies ΔT_i , consists of some standard steps. First of all from a sequence of anomalies of length *N* we extract the so-called profile

$$
y_k = \sum_{i=1}^k \Delta T_i.
$$
 (7)

The profile y_k is divided into boxes of equal length *n*, and in each box a polynomial curve of order *p* has been fitted thus obtaining the local trend $y_k^p(n)$. The variance of the detrended signal $\sigma_k^{(p)}(n) = y_k - y_k^p(n)$ is calculated for each box, and the average of the variances over all boxes is defined as

$$
F_p(n) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [\sigma_k^{(p)}(n)]^2}.
$$
 (8)

A power-law relationship $F_p(n) \sim n^{\delta_p}$, in a certain region of scales *n*, indicates a degree of persistence with exponent δ_p . The scaling exponent $\delta_p = 1/2$, obtained for uncorrelated Brownian-like stochastic processes, separates a persistent process with $\delta_p > 1/2$ from an antipersistent process where δ_p < 1/2. Both the scaling range and the scaling exponent in general depends on *p*. According to previous analysis [[15](#page-6-14)[,16](#page-6-15)[,22](#page-6-21)] we use the DFA2, namely, $p=2$ in Eq. ([8](#page-5-0)). To reduce the noise level the standard "sliding window" technique, where local trend removal and variance computation were performed by choosing each possible starting value for a given box *n*, has been applied. DFA2 curves for both records, after removing the seasonal cycle, are shown in Figs. $7(a)$ $7(a)$ and $7(b)$ for Milan and Prague, respectively. Scaling can be established for both records on time scales between 3 and 10 yr, namely, within the persistence range indicated by [[16](#page-6-15)]. The scaling exponent δ , calculated by the means of linear fit, assumes the values of $\delta = 0.72 \pm 0.01$ for Milan and $\delta = 0.78 \pm 0.01$ for Prague. This result implies long-term persistence, in temperature records, for time scales between 3 and 10 yr. In order to check the strength of this result it is interesting to focus on local details which could be lost in the log-log $F_2(n)$ plot. Where the power-law fit n^{δ} is right, the local logarithmic derivative *dy* $=$ *d* log₁₀ *F*₂ $(n)/d$ log₁₀ *n* is constant, and its value corresponds to δ . Figures [7](#page-5-1)(c) and 7(d) show the behavior of *dy*, calculated by using a third-order derivative scheme, as a function of $log_{10} n$. The local derivatives are highly variable mainly at large time scales, where nonlinearity is particularly impressive. However, for both data sets, in the same range of scales, namely, 3–10 yr, defined above, *dy* is almost constant. In particular, we found $\delta \approx 0.74 \pm 0.22$ for Milan and δ $\approx 0.78 \pm 0.38$ for Prague. Note that the larger values of uncertainty are due to the greater accuracy of the local derivative in δ calculation. We have to remark that, by using the classical definition (1) (1) (1) of temperature anomalies for the same records, a linear range for the logarithmic derivative is hardly detectable $\lceil 22 \rceil$ $\lceil 22 \rceil$ $\lceil 22 \rceil$.

IV. CONCLUSIONS

In this paper the scaling evolution of temperature anomalies has been investigated by comparing two historical temperature records lasting about 250 yr measured at Milan and Prague stations. The EMD analysis has been used to give a more suitable definition of temperature anomalies by recovering a set of orthogonal modes called IMFs. The anomalies have been thus obtained through partial reconstructions by excluding IMFs associated to obvious persistence effects such as the urban warming and the seasonal cycle. EMD allows us to show that the seasonal contribution for the Prague data set is spread over two different modes, thus indicating that the fluctuations on yearly scales are far from being stationary and are characterized by the superposition of various time scales. On the other hand, the Milan seasonal fluctuations behave in a more regular way and are isolated in a single EMD mode. By defining the seasonal contribution in the right way and deseasonalizing and detrending the temperature time series, the resulting temperature anomalies show the same degree of persistence over a reduced range of scales, at variance to what has been reported in the literature [[15,](#page-6-14)[16](#page-6-15)], where different values of persistence have been found from some temperature records. Our results are not compatible also with the recent results of Lanfredi *et al.* [[22](#page-6-21)] about the absence of persistence over a reasonable range of scales. In our case the presence of the well-defined scaling behavior is due to the right computation of temperature anomalies which correctly remove the amplitude-phase modulated seasonal component and the local trend. This gives rise to the existence of long-term persistence from about 3 to 10 yr with a scaling exponent similar for both data sets. The coupling of atmospheric and oceanic processes $[8]$ $[8]$ $[8]$ could represent one of the possible causes of the found longrange persistence for both stations. Other phenomena—like,

e.g., El Niño $[6]$ $[6]$ $[6]$ —are unlikely because of the geographical position of the analyzed stations. It must be remarked that by using only $\theta_1(t)$ to define the seasonal cycle in the Prague data set, namely, if we choose $r = \{j | 0 \le j \le 11\} \ominus \{j = 1\}$, a lower value $\delta \approx 0.67 \pm 0.01$ for the scaling exponent is obtained. This indicates that a suitable definition of anomalies is fundamental when persistence effect in climate is investi-

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gated. As a conclusion, temperature anomalies defined through EMD seem to be more suitable, with respect to the classical approaches, at least to estimate the complex seasonal cycle or trend within temperature records. Of course discussions about the universality of persistence exponents needs a statistical analysis involving a large number of stations which will be the subject of an extended future work.

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