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Statistical-mechanics approach to wide-band digital communication

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The emerging popular scheme of fourth generation wireless communication, orthogonal frequency-division multiplexing, is mapped onto a variant of a random field Ising Hamiltonian and results in an efficient physical intercarrier interference (ICI) cancellation decoding scheme. This scheme is based on Monte Carlo (MC) dynamics at zero temperature as well as at the Nishimori temperature and demonstrates improved bit error rate (BER) and robust convergence time compared to the state of the art ICI cancellation decoding scheme. An optimal BER performance is achieved with MC dynamics at the Nishimori temperature but with a substantial computational cost overhead. The suggested ICI cancellation scheme also supports the transmission of biased signals.

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Nowadays wireless communication systems are required to support a wide range of high data rate applications ranging from real-time video transfers to file downloads. These meticulous requirements demand constant improvement to the ever-increasing use of communication devices.

An interesting bridge between coding theory and statistical mechanics of disordered systems was set forth two decades ago in a seminal work by Sourlas [1]. He demonstrated that there was a mathematical equivalency between error-correcting codes (ECC) and finding the equilibrium properties of spin-glass models. Since then, physicists have shown how methods in statistical physics can complement and improve results obtained in the coding theory literature (e.g., Refs. [2–6]). A decade ago the link between statistical physics and information theory was extended to encompass wireless communications methods and especially the third generation mobile communication known as code-division multiple access (CDMA). The CDMA scheme was studied from the standpoint of statistical mechanics, while drawing on its relationship to a variant of the Hopfield model [7].

Orthogonal frequency-division multiplexing (OFDM) technology has attracted growing interest for the development of high data rate applications. It is used, for instance, in Wi-Fi devices and is the basis for 4G cellular phones. In OFDM systems, the entire available band-width is divided into N narrow bandwidths, and a block of N data symbols is modulated on N corresponding subcarriers which are orthogonal to each other. Since in the mobile radio environment the relative movement between transmitters and receivers causes Doppler frequency spreads, the orthogonality between the subcarriers is destroyed and intercarrier interference (ICI) occurs [9]. As a result, the bit error rate (BER) rapidly increases [10,11] and an efficient ICI cancellation scheme is mandatory to support high data rates.

Previous contributions utilized Hopfield network to mitigate ICI effects [12]. However, the relationship of the OFDM system to physics has yet to be made clear in the literature. In this paper we first introduce a link between the OFDM scheme and physics, by mapping it to a variant of the random field Ising Hamiltonian [13–16]. Although very simple to state, an analytical solution to the random field model can only be found for a few special cases. Nevertheless, it enables us to present the development of a new efficient ICI cancellation scheme based on Monte Carlo (MC) dynamics [17] at zero temperature as well as at the Nishimori temperature [18].

System model. (see Fig. 1) The OFDM system consists of N subcarriers, each of which has a symbol, $\mathbf{X}=[X_1 \ldots X_N]^T$, where $(\cdot)^T$ denotes the transpose of the random vector within the bracket. The symbols are converted into time-domain samples by using the N point inverse fast Fourier transform (IFFT) operation resulting in $\mathbf{x}=[x_1 \ldots x_N]^T=\mathbf{F}^{\dagger}\mathbf{X}$, where $\mathbf{F}$ is the N-point FFT matrix and $(\cdot)^{\dagger}$ denotes the complex-conjugate transpose of the argument within the bracket.

The received signals are $\mathbf{y}=[y_1 \ldots y_N]^T=\mathbf{H}\mathbf{x}+\eta$, where $\mathbf{H}$ is a known $N \times N$ time-domain channel matrix and η is a complex additive-white Gaussian noise (AWGN) vector taken from the complex normal distribution $\mathcal{CN}(0,\sigma^2)$.

Applying the N-point fast Fourier transform (FFT) on both sides of the received signals, the frequency-domain representation of the received signals is $\tilde{\mathbf{Y}}=\mathbf{F}\mathbf{y}=\mathbf{S}\mathbf{X}+\tilde{\mathbf{n}}$, where $\mathbf{S}=\mathbf{F}\mathbf{H}\mathbf{F}^{\dagger}$ and $\tilde{\mathbf{n}}=\mathbf{F}\eta$ are the frequency-domain channel matrix and noise vector, respectively. For a time-invariant or

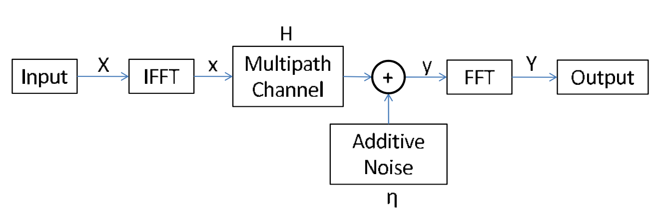

FIG. 1. (Color online) Encoding/decoding schemes of a prototypical OFDM communication system.

quasistatic channel, where the channel impulse response is assumed to be fixed over one OFDM symbol, the time-domain channel matrix, $\mathbf{H}$, is circulant, where its circulant property is achieved via a cyclic prefix [23]. As a result, the frequency-domain channel matrix, $\mathbf{S}$, is diagonal. However, for the time-varying case, $\mathbf{H}$ is not circulant, hence $\mathbf{S}$ is no longer diagonal and the signal energy is dispersed to the off-diagonal elements of the channel matrix, $\mathbf{S}$. The input-output relation for the time-varying case in the frequency-domain is

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{n}, \tag{1}$$

where $W_{ij}=\frac{S_{ij}}{S_{ii}}$ and $n_i=\frac{\tilde{n}_i}{Sii}$ are the normalized frequency-domain channel matrix and noise vector respectively $(i,j=1,2,\ldots N)$.

The correct source bias $\mathbf{m}=[m_1\ldots m_N]^T$ is assumed to be known by the receiver. As a result, each element of the input is taken from a Bernoulli process with a nonzero bias of mean $\mathbf{m}\in[-1,1]$. We represent the transmitted bits and the noise as vectors with $2N$ elements, $\mathbf{X}=[\mathbf{X}^R\ \mathbf{X}^I]$ and $\mathbf{n}=[\mathbf{n}^R\ \mathbf{n}^I]$, respectively, where $\mathbf{X}^R/\mathbf{X}^I$ and $\mathbf{n}^R/\mathbf{n}^I$ stand for the real/imaginary parts of the input and noise, respectively. Similarly, the normalized frequency-domain channel matrix is represented as a $2N\times 2N$ matrix, $\mathbf{W}=\begin{pmatrix}\mathbf{W}^R & -\mathbf{W}^I\\ \mathbf{W}^I & \mathbf{W}^R\end{pmatrix}$, where $\mathbf{W}^R/\mathbf{W}^I$ are the real/imaginary parts of the matrix, respectively. The above method makes it possible to map any complex matrix-vector representation to one over the real.

Physical equivalent of OFDM. Generally in communication channels the goal is to estimate the transmitted symbols $\mathbf{X}$ from the received symbols $\mathbf{Y}$. Hence the main quantity of interest is $P(\mathbf{X}|\mathbf{Y})$, where the microstates of the thermal system are equivalent to the hidden values of the channel's input $\mathbf{X}$. A comparison of the channel's *a posteriori* probability distribution with the Boltzmann distribution law results in

$$\frac{e^{-\beta\mathcal{E}}}{\mathcal{Z}} = P(\mathbf{X}|\mathbf{Y}) = \frac{P(\mathbf{X})P(\mathbf{Y}|\mathbf{X})}{\sum_{\mathbf{X}} P(\mathbf{X})P(\mathbf{Y}|\mathbf{X})}, \tag{2}$$

where $\mathcal{Z}=\Sigma_{\mathbf{X}}P(\mathbf{X})P(\mathbf{Y}|\mathbf{X})$ is the partition (normalization) function, $\beta=(k_BT)^{-1}$ is the inverse temperature, k_B is the Boltzmann constant which is arbitrarily set at $k_B=1$, and the *a posteriori* probability of Eq. (1) is given by [8]

$$P(\mathbf{Y}|\mathbf{X}) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2}\exp\left[-\frac{(\mathbf{W}\mathbf{X}-\mathbf{Y})^2}{2\sigma^2}\right]. \tag{3}$$

Substituting Eq. (3) in Eq. (2), one finds that $-\beta\mathcal{E}=\ln[P(\mathbf{X})]-\frac{\ln[2\pi\sigma^2]}{2}-\frac{(\mathbf{W}\mathbf{X}-\mathbf{Y})^2}{2\sigma^2}$ and the Hamiltonian can be written as a function of $\mathbf{X}$ only (see S1 in [24])

$$\beta\mathcal{E} \triangleq \frac{1}{\sigma^2}\left(\frac{1}{2}\mathbf{X}^T\mathbf{J}\mathbf{X}-\mathbf{h}^T\mathbf{X}\right), \tag{4}$$

where $\mathbf{h}=\mathbf{Y}^T\mathbf{W}+\sigma^2\tanh^{-1}(\mathbf{m})$ and $\mathbf{J}=\mathbf{W}^T\mathbf{W}$.

The OFDM Hamiltonian (4) is a variant of the random field Ising model where the interactions between the transmitted bits and X_j, are determined by the channel's properties. Furthermore, in contrast to the typical random field Ising model, the random field in Eq. (4) is a function of the received bits (spins), correlated with the transmitted bits X_i and the channel's properties. Although the sign of the field varies among bits, its correlations with the transmitted information enable the construction of an efficient decoder.

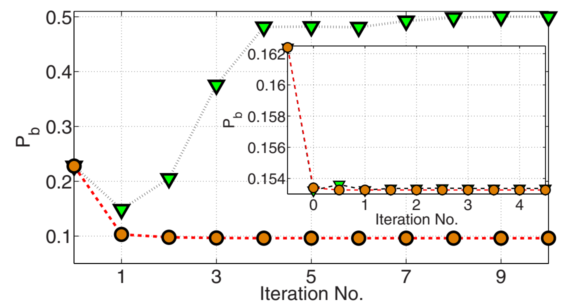

FIG. 2. (Color online) BER performance, P_b, for the proposed MC ICI cancellation scheme (○) and for the existing ICI cancellation (▽) as a function of the number of iterations. Simulations were run for $N=1024$, $\sigma=1$, and $\epsilon=0.4$. The inset depicts results for $\epsilon=0.1$. Simulation results are averaged over a sufficiently large ensemble of 5000 OFDM systems. Error-bars are smaller than the symbol size. The line is simply a visual guide.

The proposed decoder is simply a MC dynamics of Eq. (4), based on a sequential updating scheme. The simplest physical ICI decoder is achieved by zero-temperature MC dynamics which is based on the observation that minimizing $\beta\mathcal{E}$ [Eq. (4)] corresponds to the maximization of the *a posteriori* probability (MAP) which is known to be the optimal scheme for minimizing the block error probability.

Results. The efficiency of the proposed decoding scheme is demonstrated for a banded and circular frequency-domain channel matrix used in the literature to identify a prototypical OFDM channel and defined by a vector $\mathbf{S}$ with the following elements [25,26]

$$S_j = \sum_{p=1}^{M}\frac{h_p\sin[\pi(j-1)+\epsilon_p]}{N\sin\left\{\frac{\pi}{N}[(j-1)+\epsilon_p]\right\}} \times \exp\left\{i\pi\left(1-\frac{1}{N}\right)[(j-1)+\epsilon_p]\right\}, \tag{5}$$

where $j=1,2\ldots N$, M is the total number of propagation paths and $i\triangleq\sqrt{-1}$. h_p and $\epsilon_p\in[0,0.5]$ are the Rayleigh distributed typical amplitude and the normalized frequency offset (Doppler frequency shifts) of the pth path, respectively.

For simplicity of the discussion below we assume that each subcarrier transmits a binary symbol, $X_j^R\in\pm 1$ whereas $\mathbf{X}^I=0$.

Bit error rate (BER) performance as a function of the number of iterations, for the zero-temperature MC of Eq. (4) with $N=1024$, $\sigma=1$, and $\epsilon=0.1,0.4$ is depicted in Fig. 2 and is compared to the BER of the existing ICI cancellation scheme [27]. The BER performance in both ICI cancellation schemes is similar for low Doppler frequency shifts, $\epsilon=0.1$ (inset of Fig. 2). However, for higher Doppler frequency shifts, $\epsilon=0.4$ (where the ICI is more dominant), the proposed physical scheme clearly demonstrates robust BER perfor-

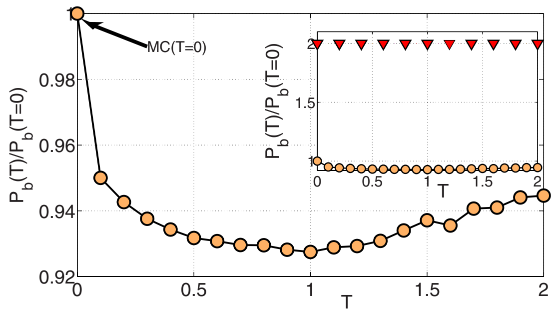

FIG. 3. (Color online) Normalized BER, $P_b(T)/P_b(T=0)$ as a function of the temperature T, for $N=10$, $\epsilon=0.4$, and $\sigma=1.5$. Simulation results (○) are averaged over 10 000 OFDM systems. Results clearly indicate an improved BER performance at the Nishimori temperature ($T=1$) compared to the zero-temperature MC ($T=0$) and the existing ICI cancellation scheme (▽-marks in the inset). Error bars are smaller than the symbol size.

mance, while the existing ICI cancellation scheme fails, BER ∼0.5. Note the rapid convergence rate of the proposed scheme, independent of ϵ.

It is widely known that the bit error probability is minimized by maximizing the marginal of the correct posterior distribution, which generally corresponds to decoding at the Nishimori temperature [18]. This argument was exemplified for ECC [19,20], proven in [21] and demonstrated from the Bayesian statistics point of view in [22].

We applied the Nishimori temperature MC dynamics for the proposed ICI cancellation scheme, which leads to accurate evaluation of the maximizer of the posterior marginal (MPM) estimator [22], generally known as the optimal estimator for minimizing the bit-wise error probability, and demonstrates the BER performance improvement over the zero-temperature MC ICI cancellation decoding scheme. By contrast, in terms of computation, the Nishimori temperature MC scheme costs much more than the zero-temperature MC. Therefore, in practice, the choice of the two strategies depends on how much computational time is allowed.

Considering Eq. (4), as we normalize $\mathbf{J}$ and $\mathbf{h}$ by σ^2, we define the Nishimori inverse temperature to be $\beta_N=1$. Figure 3 presents the normalized BER performance, $P_b(T)/P_b(T=0)$ as a function of the temperature T, for $N=10$, $\epsilon=0.4$ and $\sigma=1.5$. The minimal BER is obtained at the Nishimori temperature, $T=1$.

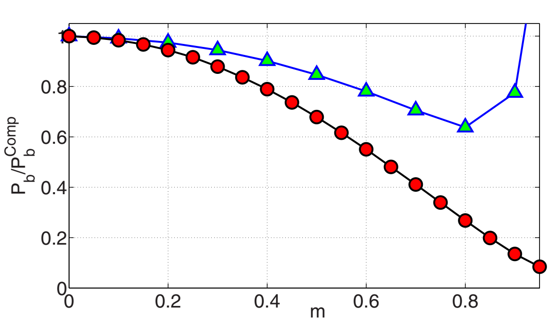

FIG. 4. (Color online) BER performance ratio of the proposed scheme to the alternative coding scheme, P_b/P_b^{Comp}, for a given bandwidth (○-marks) and for a given σ (△-marks). The ratio is plotted as a function of the bias m for $N=1024$, $\epsilon=0.1$, and $\sigma=1.5$. Simulation results are averaged over 5000 OFDM systems. Error bars are smaller than the symbol size.

Another major advantage of the proposed scheme is its ability to detect biased sources directly, with no need for source compression [28], since the random field of Eq. (4) comprises the biased sources $\mathbf{m}$. We compared the bit error probability, $P_b(m,\sigma)$, of the proposed scheme to the bit error probability, $P_b^{comp}(m=0,\sigma^{comp})$, achieved by ordinary detection of the transmission of (optimally) unbiased compressed source bits. Because after optimal compression no bias-based improvement is possible (i.e., $m=0$), a fair comparison between these two schemes is given by the following ratio:

$$\frac{P_b}{P_b^{comp}} = \frac{P_b(m,\sigma)}{P_b^{comp}[m=0,\sigma^{comp}=\sigma\sqrt{H_b(m)}]} = \frac{H_b(m)P_b(m,\sigma)}{P_b[m=0,\sigma^{comp}=\sigma\sqrt{H_b(m)}]}, \tag{6}$$

where $H_b(m)=-\frac{1+m}{2}\log_2(\frac{1+m}{2})-\frac{1-m}{2}\log_2(\frac{1-m}{2})$ denotes the binary source's entropy. The superiority of the proposed scheme is denoted by a ratio of less than 1.

The comparison of the compressed/uncompressed bits stream transmission scenarios was made under the following assumptions: (a) an optimal source compression and (b) a given bandwidth for the transmission of the bits stream. The first assumption implies that a single error in detecting a compressed bit leads on average to $1/H_b(m)$ errors in the uncompressed information. The second assumption reflects the fact that effectively the noise level of the compressed data is diminished by a factor $\sqrt{H_b(m)}$.

Figure 4 (○-marks) presents simulation results for the BER ratio [Eq. (6)] as a function of the bias m for $N=1024$, $\epsilon=0.1$, and $\sigma=1.5$. Interestingly, applying the proposed joint source-channel scheme was superior to the optimal separation (compression) scheme almost throughout the whole range of bias values.

Another possible way to compare the proposed joint source-channel scheme and the separation (compression) scheme is to keep the parameter σ constant, while assuming an optimal source compression. The BER of proposed scheme remains the same, while the BER performance of the source coding is calculated as follows. The compressed message in encoded back into the original size using a rate Rate$=H_b(m)$. The BER at the receiver, P_b^{MC}, serves as input to an optimal decoder of a binary symmetric channel (BSC) with a flip rate $f=P_b^{MC}=P_b(m=0,\sigma)$. The residue error of the separation scheme, P_b^{Comp}, is given by the equation for the channel capacity of the BSC

$$H_b(m) = \frac{1-H_2(f)}{1-H_2(P_b^{\mathrm{Comp}})}. \tag{7}$$

Figure 4 (△-marks) depicts typical simulation results of the BER ratio [Eq. (7)] as a function of the bias m. The superiority of the proposed scheme relative to the optimal compression alternative still remains valid. In this context, it should be kept in mind that while standard compression exhibits a substantial *delay* and demands an excessive computational cost from both the transmitter and receiver, the proposed joint source-channel scheme does not affect the transmitter.

An important factor in the implementation of the proposed scheme is its computational cost. A comparison indicates that both the complexities of the proposed scheme (see S2 in [24]) and the existing ICI cancellation schemes [27] are on the order $\mathcal{O}(N)$ per bit per iteration.

Finally it should be noted that the MC algorithm for the proposed scheme is not suitable for implementation in electrical circuits, which may reduce its practical utility. An interesting extension of the proposed scheme would be to derive a practical belief-propagation based algorithm, which can be implemented in parallel in electrical circuits, thus providing a solution for the TAP mean field approach, as proposed in [29]. However, the difference in the statistical properties of W from that of [29] demands further research for the implementation.

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