Localized whistlers in magnetized spin quantum plasmas

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The nonlinear propagation of electromagnetic (EM) electron-cyclotron waves (whistlers) along an external magnetic field, and their modulation by electrostatic small but finite amplitude ion-acoustic density perturbations are investigated in a uniform quantum plasma with intrinsic spin of electrons. The effects of the quantum force associated with the Bohm potential and the combined effects of the classical as well as the spin-induced ponderomotive forces (CPF and SPF, respectively) are taken into consideration. The latter modify the local plasma density in a self-consistent manner. The coupled modes of wave propagation is shown to be governed by a modified set of nonlinear Schrödinger-Boussinesq-like equations which admit exact solutions in form of stationary localized envelopes. Numerical simulation reveals the existence of large-scale density fluctuations that are self-consistently created by the localized whistlers in a strongly magnetized high density plasma. The conditions for the modulational instability (MI) and the value of its growth rate are obtained. Possible applications of our results, e.g., in strongly magnetized dense plasmas and in the next generation laser-solid density plasma interaction experiments are discussed.

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I. INTRODUCTION

Having been discovered more than a century ago [1], whistler waves become one of the most important waves in plasmas. Such waves (also known as helicons in solid state plasmas) are low-frequency (lf) (in comparison with the electron-cyclotron frequency, ω_c) right-hand circularly polarized (RCP) electromagnetic (EM) waves guided almost along the external magnetic field in dense plasmas. Because of the increase of their group velocity with the frequency, $\omega < \omega_c/2$ (see, e.g., Ref. [2]), the lf waves arrive later giving rise a whistling down-effect observed at ground level. Stenzel in his classic paper [3] demonstrated experimentally the creation of magnetic field-aligned density perturbations excited by the ponderomotive force exerted by the EM whistlers.

Whistler waves are important not only in space plasmas due to wave-particle interactions, but also in laboratory plasmas as helicons for efficient plasma production as well as in dense astrophysical environments [4–10]. On the other hand, large amplitude whistlers propagating in a magnetized plasma can initiate a great variety of nonlinear effects, e.g., three-wave interactions, parametric instabilities [10], modulational instability and the subsequent soliton formation [4–6]. The latter which, in turn, causes local electron density enhancement or depletion in plasmas, are considered as a basis for understanding laser energy deposition in pellets [11], pulsar radiation interaction with the ambient magnetosphere [12], whistler wave propagation in solar winds [13] etc. Recent laboratory experiment [14] and observations from the Freja satellite [15] show the clear evidence for the formation of whistler envelope solitons accompanied by plasma density cavities. Moreover, electrons in Van Allen radiation belts can be accelerated to MeV energies within a short period by large amplitude whistlers [16]. The latter have recently been observed by the Cluster spacecraft [17], the STEREOS [16] and the THEMIS [18]. Furthermore, laboratory experiments [19] and theoretical confirmation [20] have demonstrated the existence of propagating whistler spheromaks with fields exceeding the ambient magnetic field. Whistlers also contribute to fast magnetic reconnection and plasma dynamics in two-beam laser-solid density plasma interaction experiments [21].

Recently, there has been a notably growing interest in investigating various quantum plasma effects in view of some experimental progresses in nanoscale plasmas [22,23], ultracold plasmas [24], spintronics [25], and plasmonics [26]. On the other hand, superdense quantum plasmas are omnipresent in compact astrophysical objects, e.g., the interior of massive white dwarfs, interior of Jupitors, magnetars etc. [27–29], as well as in the next generation intense laser-solid density plasma interaction experiments [30–32]. In dense plasmas, degenerate electrons follow Fermi-Dirac pressure law, and there are typically quantum force associated with the Bohm de Broglie potential, which produce wave dispersion at nanoscales [33–35]. Furthermore, the effects of the electron spin manifests itself in terms of a magnetic dipole force, as well spin precession, which can be exploited by transforming the Pauli equation to fluidlike variables [36,37]. More elaborate kinetic models has also been developed [38,39]. Hence the dynamics of electrons in Fermi degenerate plasmas will be affected not only by the Lorentz force, but also by the effects of quantum statistical pressure, the Bohm force as well as the effects due to intrinsic spin of electrons. We ought to mention that in a dense

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magnetized plasma there also exist spin waves, which can be excited by intense neutrino fluxes. Thus, nonlinear theories of EM waves, in particular whistlers in magnetized dense plasmas need to be developed in its own right accounting for all these quantum effects. Recently, the theory of the ponderomotive force in plasmas has been extended to account for the contribution from the intrinsic spin of electrons [40]. It has been demonstrated that an EM pulse can induce a spin-polarized plasma by this spin-ponderomotive force (SPF). Such force could also play an important role in the propagation of lf EM waves, e.g., whistlers, Alfvén waves.

Our objective here is to present a theoretical study of modulated whistler wave packets interacting nonlinearly with background lf density perturbations that are reinforced by the classical ponderomotive force (CPF) [5] as well as the SPF [40]. The role of the ion motion as well as the dispersive effects due to charge separation and the electron tunneling are also taken into account. We will include the field-aligned velocity perturbation (free electron streaming) associated with the lf motion, and in addition, generalize the related classical results that exist in the literature (see, e.g., Refs. [4,5]). The obtained results could be useful for understanding the propagation of localized EM whistlers which may emanate in the interior of magnetized white dwarfs, magnetars as well as in the next generation intense laser-solid density plasma experiments.

II. NONLINEAR EVOLUTION EQUATIONS

Let us consider the propagation of nonlinearly coupled EM whistlers and ion-acoustic (IA) density perturbations along a constant magnetic field $\mathbf{B}=B_0\hat{z}$ in a quantum electron-ion plasma where any equilibrium drift velocity is zero. In the modulational representation, the high-frequency (hf) EM wave field for the RCP whistlers is given by $\mathbf{E}=(\hat{x}-i\hat{y})E(z,t)\exp(ikz-i\omega t)+\text{c.c.}$, where $E(z,t)$ is the slowly varying (both in space and time) envelope of the whistler wave electric field and c.c. stands for the complex conjugate. Also, $\omega(k)$ represents the whistler wave frequency (number). The basic equations for the evolution of nonlinear whistlers then read [36,40,41].

$$\frac{\partial n_e}{\partial t}+\nabla\cdot(n_e\mathbf{v}_e)=0, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_e\,.\,\nabla\right)\mathbf{v}_e=-\frac{e}{m_e}(\mathbf{E}+\mathbf{v}_e\times\mathbf{B})-\frac{\nabla P_e}{m_e n_e}+\frac{\hbar^2}{2m_e^2}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)+\frac{2\mu}{m_e\hbar}\mathbf{S}\cdot\nabla B, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_e\cdot\nabla\right)\mathbf{S}=-\frac{2\mu}{\hbar}(\mathbf{B}\times\mathbf{S}), \tag{3}$$

where n_e, m_e, $\mathbf{v}_e$ denote the number density, mass and velocity of electrons, respectively, $\mathbf{B}$ is the magnetic field and P_e is the electron thermal pressure. Also, $\mathbf{S}$ is the spin angular momentum with its absolute value $|\mathbf{S}|=|S_0|\equiv\hbar/2$; $\mu=-(g/2)\,\mu_B$, where $g\approx 2.002\,319\,3$ is the electron g-factor and $\mu_B\equiv e\hbar/2m_e$ is the Bohr magneton. Equations (1)–(3) are then closed by the following Maxwell equations with $\nabla\cdot\mathbf{B}=0$.

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{4}$$

$$\nabla\times\mathbf{B}=\mu_0\left(\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}-en_e\mathbf{v}_e+\frac{2\mu}{\hbar}\nabla\times n_e\mathbf{S}\right). \tag{5}$$

Equations (1)–(3) represent the nonrelativistic evolution of spin–1/2 electrons, and are applicable even when different states with spin-up and spin-down (relative to the magnetic field) can be well represented by a macroscopic average. This may, however, occur in the regimes of very strong magnetic fields (or a very low temperature regimes), where generally the electrons occupy the lowest energy spin states. On the other hand, for a time-scale longer than the spin-flip frequency, the macroscopic spin state is well described by the thermodynamic equilibrium spin configuration, and in this case the above model can still be applied. However, such case in which the macroscopic spin state will be attenuated by a factor decreasing the effective value of $|\mathbf{S}|$ below $\hbar/2$, will not be considered further in the present work. As a consequence, our studies will be focused on the regime of strong magnetic fields and high density plasmas.

Taking the curl of Eq. (2) and using Eqs. (3)–(5) we readily obtain the following evolution equation for whistlers:

$$\begin{aligned}0=&\frac{e}{m_e}\frac{\partial\mathbf{B}}{\partial t}+\frac{\varepsilon_0}{en_e}\frac{\partial}{\partial t}\left(\frac{\partial^2\mathbf{B}}{\partial t^2}+\frac{1}{n_e}\nabla n_e\times\frac{\partial\mathbf{E}}{\partial t}\right)-v_{ez}\nabla\times\frac{\partial\mathbf{v}_e}{\partial z}\\&+\frac{1}{e\mu_0}\frac{\partial}{\partial t}\left[\frac{1}{n_e}\nabla\times(\nabla\times\mathbf{B})\right]-\frac{2\mu}{e\hbar}\frac{\partial}{\partial t}\left[\frac{1}{n_e}\nabla\times(\nabla\times n_e\mathbf{S})\right]+\frac{1}{m_e\mu_0 n_e}\nabla\times[(\nabla\times\mathbf{B})\times\mathbf{B}]+\frac{2\mu}{m_e\hbar}\nabla\\&\times(S^a\nabla B_a)-\frac{\varepsilon_0}{m_e n_e}\nabla\times\left(\frac{\partial\mathbf{E}}{\partial t}\times\mathbf{B}\right)-\frac{2\mu}{m_e\hbar n_e}\nabla\times[(\nabla\times n_e\mathbf{S})\times\mathbf{B}].\end{aligned} \tag{6}$$

In the linear theory, the whistler frequency ω and the wave number k are related by the following linear dispersion relation in the nonrelativistic limit (see for details, Ref. [2]):

$$n_R^2\left(1+\frac{\omega_\mu}{\omega-\omega_g}\right)=1-\frac{\omega_{pe}^2}{\omega(\omega-\omega_c)}, \tag{7}$$

where $n_R\equiv ck/\omega$ is the refractive index, $\omega_\mu=g^2|S_0|/4m_e\lambda_e^2$ is the frequency due to the plasma magnetization current and $\lambda_e\equiv c/\omega_{pe}$ is the electron skin depth with $\omega_{pe(i)}\equiv\sqrt{n_0e^2/\varepsilon_0 m_{e(i)}}$ denoting the electron (ion) plasma frequency. Also, $\omega_c=eB_0/m_e$ is the electron-cyclotron frequency and $\omega_g=(g/2)\omega_c$ is the electron spin-precession frequency.

The nonlinear dynamics of whistler wave envelopes under the modulation of electron density perturbations associated with the lf IA fluctuations and of the nonlinear frequency shift caused by the magnetic field-aligned free streaming of electrons with flow speed v_{ez}, can be described by the following nonlinear Schrödinger (NLS)-like equation which is obtained from the EM wave Eq. (6) as

$$i\left(\frac{\partial E}{\partial t}+v_g\frac{\partial E}{\partial z}\right)+\frac{v_g'}{2}\frac{\partial^2 E}{\partial z^2}-\Delta E=0, \tag{8}$$

where $E\equiv E_x-iE_y$, and the group speed, $v_g\equiv d\omega/dk$ (see Eq. (11) in Ref. [2]) and the group dispersion, $v_g'\equiv d^2\omega/dk^2$ of whistlers are given by

$$v_g=\left(\frac{2c^2k}{\omega_{pe}^2}+\frac{g^2\hbar k}{4m_e(\omega-\omega_g)}\right)\bigg/\left(\frac{2\omega}{\omega_{pe}^2}+\frac{\omega_c}{(\omega-\omega_c)^2}+\frac{g^2\hbar k^2}{8m_e(\omega-\omega_g)^2}\right), \tag{9}$$

$$v_g'=\frac{v_g}{k}\left[1-\frac{2kv_g^2}{\Lambda\omega_{pe}^2}\left(1-\frac{\omega_c\omega_{pe}^2}{(\omega-\omega_c)^3}\right)-\frac{g^2\hbar k^2 v_g}{4m_e\Lambda(\omega-\omega_g)^2}\left(2-\frac{v_g k}{\omega-\omega_g}\right)\right]. \tag{10}$$

The nonlinear frequency shift Δ is given by

$$\Delta=\frac{v_g}{\Lambda}\left[\frac{k\omega v_{ez}}{(\omega-\omega_c)^2}+\left(\frac{\omega}{\omega-\omega_c}+\frac{g^2\hbar k^2}{4m_e(\omega-\omega_g)}\right)N\right], \tag{11}$$

where $\Lambda=2c^2k/\omega_{pe}^2+g^2\hbar k/4m_e(\omega-\omega_g)$ and $N\equiv n_e/n_0$ is the relative perturbed density. By disregarding the spin contribution one can recover the previous results [4,5]. Note that the term $\propto v_{ez}$, representing the Doppler shift due to the plasma streaming along the external magnetic field, is no longer negligible, but may be comparable to the other nonlinear terms, and can thus change the sign of the nonlinearity as well. More precisely, both v_g' and Δ will change their sign depending on the frequency range to be considered as well as the contribution from the spin correction terms. Later, we will see that the change of sign is important for the formation of localized wave packets at different whistler frequencies. The quantities N and v_{ez} are related to each other by the electron continuity equation.

$$\frac{\partial N}{\partial t}+\frac{\partial v_{ez}}{\partial z}=0. \tag{12}$$

Note that the ponderomotive force due to the EM whistlers usually drives the lf (compared to the whistler wave frequency ω) density perturbations which propagate along the field lines with low-phase speed (compared to the electron thermal speed). Thus, the lf electrostatic modulation also satisfies the electron momentum equation

$$\frac{\partial v_{ez}}{\partial t}+\frac{e}{m_e}E_l+V_F^2\frac{\partial N}{\partial z}-\frac{\hbar^2}{4m_e^2}\frac{\partial^3 N}{\partial z^3}=\frac{e^2}{2m_e^2\omega^2}\left(\Gamma_1\frac{\partial|E|^2}{\partial z}-k\Gamma_2\frac{\partial|E|^2}{\partial t}\right), \tag{13}$$

where E_l is the lf part of the wave electric field and $V_F=\sqrt{k_BT_F/m_e}$ is the Fermi speed relevant for a high density plasma [42]. Here $T_F\equiv\hbar^2(3\pi^2n_0)^{2/3}/2k_Bm_e$ and k_B is the Boltzmann constant. The term $\propto\hbar^2$ is the quantum correction associated with the Bohm de Broglie potential. The ponderomotive force contributions are proportional to the constants Γ_1 and Γ_2 where

$$\Gamma_1=\frac{\omega}{\omega-\omega_c}+\frac{g^2\hbar k^2}{4m_e(\omega-\omega_g)},$$

$$\Gamma_2=\frac{\omega_c}{(\omega-\omega_c)^2}+\frac{g^2\hbar k^2}{4m_e(\omega-\omega_g)^2}. \tag{14}$$

in which the first terms appear due to CPF [5] and the second ones ($\propto\hbar$) are due to the SPF [40]. The equations for the cold ion motion involved in the lf IA perturbations are

$$\frac{\partial n_i}{\partial t}+n_0\frac{\partial v_{iz}}{\partial z}=0, \tag{15}$$

$$\frac{\partial v_{iz}}{\partial t}=\frac{e}{m_i}E_l, \tag{16}$$

$$\frac{\partial E_l}{\partial z}=\frac{e}{\varepsilon_0}(n_i-n_e). \tag{17}$$

Eliminating n_i, E_l, v_{iz} and disregarding the term $\propto m_e/m_i$, we obtain from Eqs. (13) and (15)–(17) the driven wave equation for lf perturbations of the Boussinesq-type as

$$\frac{\partial^2}{\partial t^2}\left(1-\lambda_F^2\frac{\partial^2}{\partial z^2}\right)N-c_s^2\frac{\partial^2 N}{\partial z^2}+\frac{\hbar^2}{4m_em_i}\frac{\partial^4 N}{\partial z^4}=\mu_1\frac{\partial^2}{\partial z^2}\left(\omega_{pi}^2+\frac{\partial^2}{\partial t^2}\right)|E|^2-\mu_2\frac{\partial^2}{\partial z\,\partial t}\left(\omega_{pi}^2+\frac{\partial^2}{\partial t^2}\right)|E|^2, \tag{18}$$

where $\mu_1=\varepsilon_0\Gamma_1/2n_0m_e$ and $\mu_2=\varepsilon_0k\Gamma_2/2n_0m_e\omega^2$, $c_s=\sqrt{k_BT_F/m_i}$ is the ion-acoustic speed and $\lambda_F=c_s/\omega_{pi}$ is the Fermi screening length for electrostatic oscillations.

Thus, we have a set of three coupled equations, namely, Eqs. (8), (12), and (18), modified from previous results by the SPF and quantum tunneling, which describes the nonlinear coupling of electron whistler waves with the field-aligned electrostatic density fluctuations. These equations can be recast by normalizing the variables according to $z\to z/\lambda_F$, $t\to t\omega_{pi}$, $E\to E/E_0$, $v_{ez}\to v_{ez}/c_s$, in which case we obtain

$$i\left(\frac{\partial E}{\partial t}+V_g\frac{\partial E}{\partial z}\right)+\frac{V_g'}{2}\frac{\partial^2 E}{\partial z^2}-\Psi E=0, \tag{19}$$

$$\frac{\partial N}{\partial t}+\frac{\partial v_{ez}}{\partial z}=0, \tag{20}$$

and

$$\left[\frac{\partial^2}{\partial t^2}\left(1-\frac{\partial^2}{\partial z^2}\right)-\frac{\partial^2}{\partial z^2}+H^2\frac{\partial^4}{\partial z^4}\right]N$$
$$=\lambda_1\frac{\partial^2}{\partial z^2}\left(1+\frac{\partial^2}{\partial t^2}\right)|E|^2-\lambda_2\frac{\partial^2}{\partial z\,\partial t}\left(1+\frac{\partial^2}{\partial t^2}\right)|E|^2, \tag{21}$$

where $E_0=\sqrt{2k_BT_Fn_0/\varepsilon_0}$, $V_g=v_g/c_s$, $V_g'=v_g'\omega_{pi}/c_s^2$, $\Psi=\Delta/\omega_{pi}$, $H=\hbar\omega_{pe}/2k_BT_F$ is the quantum coupling parameter, $\lambda_1=\omega_{pe}^2\Gamma_1/\omega^2$ and $\lambda_2=\omega_{pe}^2kc_s\Gamma_2/\omega^2$. Equations (19)–(21) contain the main results of the present work. In particular, previous results [4,5] can be recovered by disregarding the spin contribution $\propto\hbar$ as well as the particle dispersion $\propto H$ and considering, e.g., the isothermal equation of state (relevant for low or moderate density plasmas).

III. STATIONARY LOCALIZED SOLUTIONS

In this section we will investigate the properties of nonlinear whistlers by solving numerically the Eqs. (19)–(21) in the stationary frame $\xi=z-Mt$ (where $M\equiv V/c_s$). We will consider the parameter regimes for the density and the magnetic field for which the nonrelativistic fluid model is valid and SPF is comparable to the CPF. We will also see that the case in which SPF dominates over the CPF may correspond to the strongly magnetized superdense plasmas where relativistic treatment may be necessary. However, before going further to such discussions let us first consider the particular case in which the dispersion due to charge separtion (quasineutrality) is negligibe. The latter can be justified even when the spin effects dominate, i.e., $\chi\equiv\hbar k^2/m_e\omega\gg 1$ [2]. From the scaling

$$k^2\lambda_F^2\sim\left(\frac{V_F^2}{c^2}\right)\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{\omega^2}{\omega_{pe}^2}\right). \tag{22}$$

we find that the quasineutrality limit $k^2\lambda_F^2\ll 1$ holds in nonrelativistic ($V_F^2\ll c^2$) plasmas as long as $n_R\equiv ck/\omega>1$ and $\omega_{pi}<\omega<\omega_{pe}$. However, we will see that in a specific parameter regime, such restrictions can be valid for very lf ($\omega\ll\omega_c$) whistler modes. In this case, $B_0<B_Q\equiv 4.4\times 10^9$ T and $n_0\gg 10^{32}$ m^{-3} [2] with $T_F\gtrsim T_B\simeq\hbar\omega_c/k_B$. Moreover, when $\chi\gg 1$, the contribution from the term $\propto\lambda_2$ can be smaller than that $\propto\lambda_1$, since $|\lambda_2/\lambda_1|\sim(kc_s/\omega)(m_e\omega/\hbar k^2)\ll 1$. Thus, in the quasineutral regime, we obtain from Eqs. (19)–(21) the following NLS equation.

$$\frac{V_g'}{2}\frac{d^2E}{d\xi^2}+i(V_g-M)\frac{dE}{d\xi}+\bar{\Delta}|E|^2E=0, \tag{23}$$

together with

$$N=\Lambda|E|^2,\quad v_{ez}=M\Lambda|E|^2. \tag{24}$$

Then we can write Eq. (11) as $\Delta=-\bar{\Delta}|E|^2$, where $\bar{\Delta}$ is defined as

$$\bar{\Delta}\approx-\frac{\Lambda\omega v_g}{\Lambda\omega_{pi}(\omega-\omega_c)}\left(1+\frac{kV}{\omega-\omega_c}+\frac{\hbar k^2}{m_e\omega}\right), \tag{25}$$

where $\Lambda=(\lambda_1+\lambda_2M)/(M^2-1)$. Physically, the electrons experience a longitudinal force exerted by the front of the whistler pulse, and thereby gain a net energy. The electrons gain energy during the rising front of the pulse, but then slow down by the backward ponderomotivelike force. Moreover, electrons can approach the group velocity of the whistler when it reaches the pulse peak at the center. From Eq. (24), we find that this can be possible for $M^2\ll 1$, which may happen for a whistler frequency satisfying $\omega_c/2<\omega<\omega_c$ and for high density ($\sim 10^{36}$ m^{-3}) and strongly magnetized ($B_0\sim 10^8$ T) plasmas. In this case, the Fermi speed may exceed the group speed ($\sim c$). On the other hand, corresponding to the parameters as in Fig. 1 below, $M\gg 1$ and $\lambda_1+\lambda_2M\sim 0.01$, so that $N\sim|E|^2\times 10^{-7}$, $v_{ez}\sim|E|^2\times 10^{-5}$, and $\bar{\Delta}\sim 10^{-7}$. Again, note that slow electrons can freely move along the direction of the external magnetic field. The finite velocity perturbations would then induce an additional density change in order to maintain the conservation of particles (equation of continuity) under localized disturbances. Consequently, the total density variation in the frequency-shift becomes δN, where

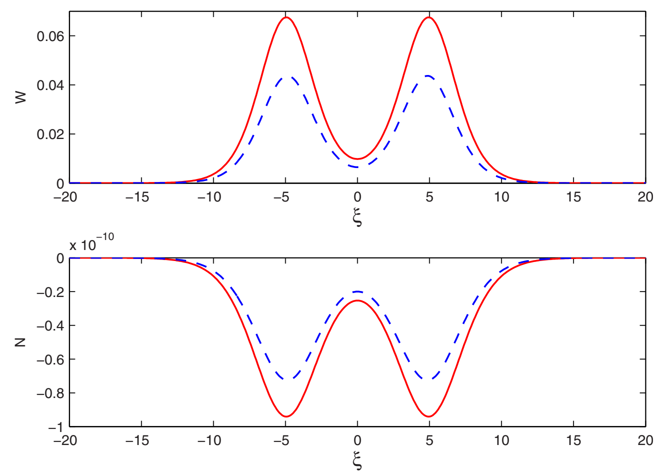

FIG. 1. (Color online) Whistler solitary solution of Eqs. (28) and (29) with associated electric field W (upper panel) and density perturbation N (lower panel) for $\Theta=0.8$ (solid line) and 0.82 (dashed line). The other parameter values are $n_0=10^{34}$ m^{-3}, $B_0=5\times 10^8$ T, $\omega_c/\omega_{pe}=15.6$, $\omega=0.4$, $M(\equiv v_g/c_s)=234.54$, $V_g'=-1.81$, $v_g=2.98\times 10^8$ m/s.

$$\delta\approx 1+\frac{kV}{\omega-\omega_c}+\frac{\hbar k^2}{m_e\omega}. \tag{26}$$

Clearly, δ changes sign whenever the third term $\propto\hbar$ in Eq. (26) dominates over the other two terms. Now, for lf propagation of whistlers, $\lambda_1<0$ and as in the previous section, $\lambda_2M(>0)$ is smaller compared to λ_1 when the spin contribution dominates. Thus, in the quasineutral lf regime, the density and velocity perturbations are positive and negative according as the whistler wave propagation is subsonic or supersonic [see Eq. (24)].

Furthermore, localized bright (dark) envelope solutions of Eq. (23) exist through the modulational instability (stability) when $\bar{\Delta}v_g'>0(<0)$. For lf waves ($\omega<\omega_c$), when $\hbar k^2/m_e\omega\gg 1$, $\lambda_1+\lambda_2M<0$, $v_g'<0$ and $\bar{\Delta}\gtrless 0$ according as $M\lessgtr 1$. Hence, a possible final state of the MI could be a supersonic (subsonic) bright (dark) solitonlike structure in a quasineutral spin quantum plasma. Equation (23) has an exact soliton solution (when $\bar{\Delta}$ and V_g' have the same sign) of the form

$$E(\xi) = E_m \operatorname{sech}[E_m \sqrt{\bar{\Delta}/V_g'}(\xi - \xi_0)], \tag{27}$$

where E_m, ξ_0 are constants. The other particular cases, namely the quasistationary lf density response (i.e., $\partial_t \to 0$) for which $\omega \ll \omega_{pi}$ [4] and the case of unidirectional propagation (near sonic envelope) in which the quasineutrality is not a valid assumption [7] will not be discussed here as those cases are not so relevant to the parameter regimes to be considered, instead we will focus on our main Eqs. (19)–(21).

Thus, we look for stationary solutions of Eqs. (19)–(21) in the stationary frame $\xi = z - Mt$. Here we assume E to be of the form $E = W(\xi)\exp(-i\Theta t)$, where W is a real function and Θ is a real constant. Then Eqs. (19)–(21) reduce to

$$\frac{V_g'}{2}\frac{d^2W}{d\xi^2} + W\Omega + \tilde{\Delta}NW = 0, \tag{28}$$

$$(-M^2 + H^2)\frac{d^2N}{d\xi^2} + (M^2 - 1)N = (\lambda_1 + \lambda_2 M)\left(M^2\frac{d^2W^2}{d\xi^2} + W^2\right), \tag{29}$$

where $\tilde{\Delta} = \bar{\Delta}/\Lambda$. We numerically solve the Eqs. (28) and (29) by Newton method with the boundary conditions N, W, $d_\xi^2 N$, $d_\xi^2 W \to 0$ as $|\xi| \to \infty$. We consider the density and magnetic field strength to vary as $n_0 \sim 10^{34} - 10^{36}\ \mathrm{m}^{-3}$ and $B_0 \sim 10^8$ T. Figure 1 illustrates the existence of double-hump localized whistler envelope accompanied with a density depletion for a set of parameters: $n_0 \sim 10^{34}\ \mathrm{m}^{-3}$, $B_0 \sim 5 \times 10^8$ T, $\omega = 0.4$, and $\Theta = 0.2$. The corresponding frequencies are $\omega_{pi} = 1.32 \times 10^{17}\ \mathrm{s}^{-1}$, $\omega_{pe} = 5.64 \times 10^{18}\ \mathrm{s}^{-1}$ and $\omega_c = 8.79 \times 10^{19}\ \mathrm{s}^{-1}$. Also, $M(\equiv v_g/c_s) = 234.54$, $\lambda_{De}(=\lambda_{Fe} \equiv V_{Fe}/\omega_{pe}) = 9.67 \times 10^{-12}$ m, and $V_{Fe} = 5.46 \times 10^7$ m/s. Thus, the whistlers have negative group dispersion with $V_g' = -1.81$. From the dispersion relation we obtain $k = 1.18 \times 10^{11}\ \mathrm{m}^{-1}$, which corresponds to whistlers with a wavelength of 5.3121×10^{-11} m, and the group speed is $v_g = 2.98 \times 10^8$ m/s. Furthermore, the nonlinear frequency shift is obtained as $\tilde{\Delta} = 0.85$. The density depletion is observed quite small due to large group velocity (compared to the sound speed) of the whistler waves.

In another illustration (Fig. 2) with a higher magnetic field, we observe a dark-soliton-like structure correlated with a density hump. The amplitude of the solitary pulse decreases as the magnetic field increases. In Fig. 3 we have presented the solitary structures when the density is very high ($n_0 \sim 10^{36}\ \mathrm{m}^{-3}$). This basically corresponds to the case when $\hbar k^2/m_e\omega \gtrsim 1$. However, in this case one must note that the Fermi speed is close to or can even be larger than the speed of light in vacuum and so, nonrelativistic quantum fluid model may no longer be appropriate. The quantum parameter H has no significant role for the regime considered here, as can be seen that $M(\equiv V_g)$ mainly dominates in the term $-M^2 + H^2$ [Eq. (29)] because of large group velocity ($\approx c$). In order that H can be comparable to M, one might have to consider relatively higher densities ($> 10^{36}\ \mathrm{m}^{-3}$) and

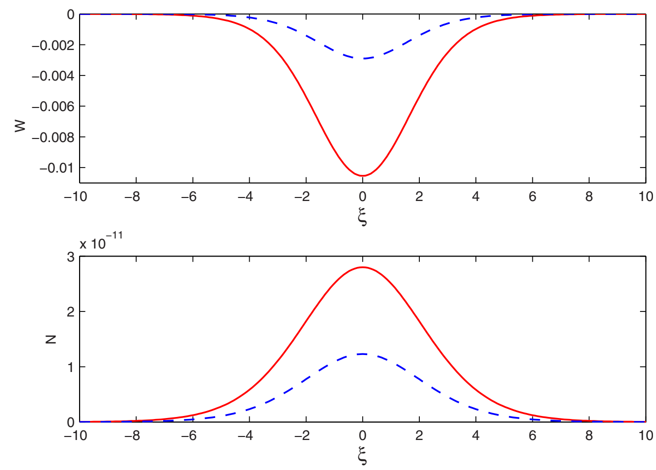

FIG. 2. (Color online) Whistler solitary solution of Eqs. (28) and (29) with associated electric field W (upper panel) and density perturbation N (lower panel) for $B_0 = 5.8 \times 10^8$ T (solid line) and 5.9×10^8 T (dashed line). The other parameter values are $n_0 = 10^{34}\ \mathrm{m}^{-3}$, $\Theta = 0.8$, $\omega = 0.4$. The corresponding M values are $M = 234.88$ (solid line) and 234.91 (dashed line).

weakly magnetized ($\ll 10^8$ T) plasmas. However, in this case the coefficient $\lambda_1 + \lambda_2 M(\sim 10^5)$ will be much larger than the other coefficients, which might prevent any hope for localized solution. As shown in Fig. 4, one can excite a nondiverging whistler with a positive group dispersion in other regime, e.g., $\omega = 0.189$, $\Theta = 0.7$, $n_0 \sim 7 \times 10^{36}\ \mathrm{m}^{-3}$, and $B_0 \sim 5 \times 10^8$ T for which $V_g' = 0.262$, $H = 0.11$, $v_g = 2.37 \times 10^8$ m/s, $M = 45.24$, $V_{Fe} = 2.25 \times 10^8$ m/s, $c_s = 5.24 \times 10^6$ m/s. This basically corresponds to oscillatory pulse associated with a field-aligned density hump ($N \sim 10^{-10}$).

IV. GROWTH RATE OF INSTABILITY

Nonlinear interaction of the hf pump EM whistlers (ω, k) with lf electrostatic field-aligned perturbations (Ω, K) gives

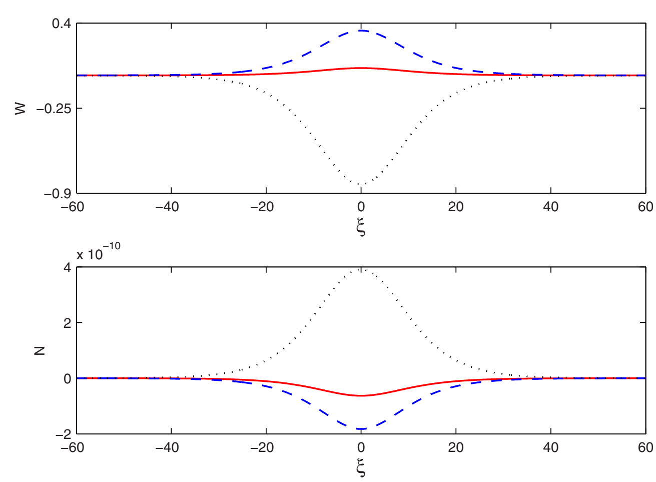

FIG. 3. (Color online) Whistler solitary solution of Eqs. (28) and (29) with associated electric field W (upper panel) and density perturbation N (lower panel) for $\omega = 0.4$ (solid line) and 0.38 (dashed and dotted line). The other parameter values are $B_0 = 5 \times 10^8$ T, $n_0 = 10^{36}\ \mathrm{m}^{-3}$ (for solid and dashed line) and $2.1 \times 10^{36}\ \mathrm{m}^{-3}$ (for dotted line), $\Theta = 0.2$. The values of M are $M = 35.38$ (for solid and dashed line) and 22.23 (for dotted line).

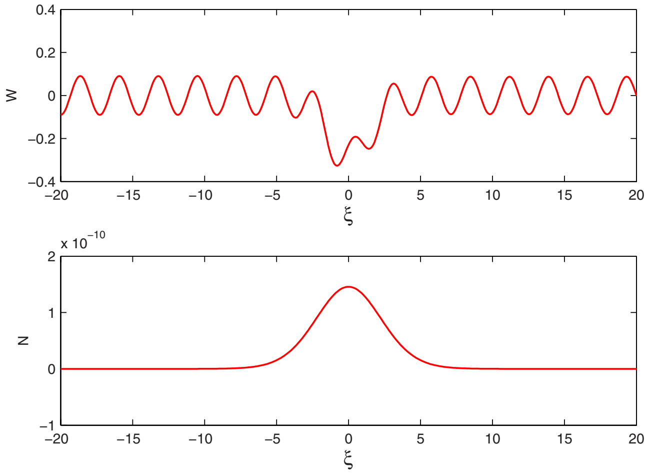

FIG. 4. (Color online) Ducted whistler obtained as solution of Eqs. (28) and (29) with associated electric field W (upper panel) and density perturbation N (lower panel) for $\omega=0.189$, $B_0=5\times10^8$ T, $n_0=7\times10^{35}$ m^{-3}, $\Theta=0.7$. The other parameters are $M=45.24$, $V_g'=0.262$, $H=0.11$, $v_g=2.37\times10^8$ m/s.

rise upper and lower side bands with frequency and wave numbers, respectively, $(\omega+\Omega,k+K)$ and $(\omega-\Omega,k-K)$. The latter interacts with the pump and thus produces a lf ponderomotive force which eventually reinforces the lf electrostatic oscillations. When all the perturbations are aligned along the external magnetic field, the parametric interactions of EM waves can be described from Eqs. (19)–(21) by the following dispersion relation:

$$\Lambda K^2 V_g'[K^2(1+H^2K^2)-(1+K^2)\Omega^2] = 4V_g E_0^2(1-\Omega^2)(\lambda_1 K+\lambda_2\Omega)(K\Gamma_1+\zeta\Omega), \quad (30)$$

where $\zeta=k\omega\omega_c^3/\lambda_{De}(\omega-1)^2$ and ω, k have been normalized by ω_c and λ_{De}^{-1}, respectively. Some simplification can be in order. Note that under the quasineutrality assumption, the coefficient of Ω^4, Ω^3, and the term $\propto K^4$ as well as the term $\propto\lambda_1$ in the coefficient of Ω^2 will not appear. Also, for lf propagation of whistlers $(\omega<\omega_c)$, λ_2 is smaller and thus being neglected. Moreover, the ratio of the term $\propto\zeta$ in the coefficient of Ω (which appears due to the parallel electron streaming v_{ez}) and the constant term $\propto\Gamma_1$ scales as $(k/K)\omega(m_e\Omega\omega_c/\hbar k^2)$ and we need $(k/K)\omega(m_e\Omega\omega_c/\hbar k^2)\ll1$ for spin effects to be dominant. Thus, in this case the dispersion relation reduces to

$$\Omega^2\approx K^2(1+H^2K^2)-E_0^2\eta^2, \quad (31)$$

where $\eta^2=\pm4V_g\lambda_1\Gamma_1/\omega_c^2\Lambda V_g'$ in which ω,k etc. are being normalized. Clearly, MI sets in for modulation wave numbers satisfying $K\sqrt{1+H^2K^2}<E_0\eta$, or $K<K_c\approx E_0\eta$ for highly dense medium and small K. The growth rate of instability $(\Omega=i\gamma)$ is then given by

$$\gamma\approx\sqrt{E_0^2\eta^2-K^2(1+H^2K^2)}. \quad (32)$$

Hence, in the long-wavelength limit $(K\to0)$ maximum growth rate of instability can be achieved, and is roughly proportional to the pump wave electric field E_0 and η. For parameters as in Fig. 1, we obtain $\gamma\approx2.77$. It basically restricts the characteristic length-scale to a certain value for the formation of envelope solitons through MI.

V. DISCUSSION AND CONCLUSION

In the present investigation focusing on whistler waves we point out that the spin contribution is substantial when $\hbar k^2/m_e\omega\gg1$, i.e., when $\hbar\omega_c/m_ec^2>1$ and $\omega^2\ll c^2k^2$. This corresponds to the case in which the magnetic field strength, $B_0\gtrsim B_Q$ and the particle density is very high, i.e., $n_0\gtrsim10^{36}$ m^{-3} for which the magnetic field is nonquantizing and does not affect the thermodynamic properties of electrons. However, in such regimes, the Fermi velocity may approach or exceed the whistler group velocity (close to c in the present study), and so the nonrelativistic quantum fluid model may no longer be appropriate to consider. In the present work, we have considered $B_0\sim10^8$ T and the density to vary in the range $10^{34}\lesssim n_0\lesssim10^{36}$ m^{-3} in order that the nonrelativistic fluid model is valid to some extent. Moreover that $\omega_{pe}\gtrsim\omega_c$ and the terms due to spin magnetization current together with the SPF are comparable to the classical counter parts. Furthermore, in this regime the velocity of electrons remains much smaller than the whistler group velocity $(\sim c)$.

Since the whistler group speed is much higher than the IA speed, whistler solitons are not significantly affected by the particle dispersion associated with the Bohm potential as well as the Fermi-Dirac pressure, though the length scale of excitation is of the order of the Compton wavelength. However, those effects reduce the plasma characteristic wavelength of excitation. Such effects can be more significant in some other regimes when $M\lesssim1$ and/or for possible excitation of the ion wakefields at nanoscales. Note that since degenerate electrons follow the Fermi-Dirac pressure law (where the Fermi temperature is density dependent), the cold plasma limit cannot be recovered from the present study unless one consideres, e.g., isothermal equation of state to be relevant for low or moderate density plasmas. Furthermore, $H\to0$ means that one approaches the higher density regimes and $H=0$ is the case when one simply disregards the quantum tunneling effect.

The parameter regimes considered here can be achievable in the magnetized white dwarfs $(\sim10^{36}$ m$^{-3})$ as well as in the next generation intense laser-solid density plasma experiments $(\sim10^{34}$ m$^{-3})$, in x-ray free electron lasers, and in plasmonic devices. One can, in principle, go beyond the parameter regimes considered here (since there is no specific theoretical limit for the density), however, we have to be careful about those parameter values for the excitation of localized whistlers and for spin-ponderomotive force to have a role. The latter may dominantly accelerate the ions by separating the electric charges and building up a high electric field. However, plasma can sustain such high electric fields, and so it remains an attractive medium for particle acceleration, which is still a most important area of research works in both laboratory and astrophysical plasmas.

In conclusion, we have presented a new set of nonlinear equations which governs the dynamics of modulated whistlers interacting with the field-aligned electrostatic lf density perturbations due to IA fluctuation, in a magnetized spin quantum plasma. Both the classical as well as the spin-induced ponderomotive force has been considered to modify the local plasma density in a self-consistent manner. Numerical simulation of the governing equations in the stationary frame [Eqs. (28) and (29)] reveals the existence of supersonic stationary envelope solitons characterized by a single or double-hump whistler wave electric fields that are trapped in a self-created density cavity. This happens for wave frequency satisfying $\omega < \omega_c/2$ and when the whistler has negative group dispersion. When the whistler frequency is smaller than $\omega_c/4$ and the group dispersion is positive at higher densities, one can excite a nondiverging whistler wave, i.e., a ducted whistler. The latter corresponds to a field-aligned density hump with $N \sim 10^{-10}$. Furthermore, the whistler solitons with density dips and humps can occur depending on the consideration of the frequency regime as well as the magnetic field strength and/or the particle density.

We ought to mention that our present investigation on the nonlinear propagation of EM whistlers might play an important role in studies of beat-wave particle accelerators [43] as well as in the problem of radio-frequency electron-cyclotron-resonance heating [44] of plasmas where the driver, instead of being a laser, is a whistler wave.

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[1] W. H. Preece, Nature (London) **49**, 554 (1894).
[2] A. P. Misra, G. Brodin, M. Marklund, and P. K. Shukla, J. Plasma Phys. **76**, 857 (2010).
[3] R. L. Stenzel, Geophys. Res. Lett. **3**, 61 (1976).
[4] P. K. Shukla and L. Stenflo, Phys. Rev. A **30**, 2110 (1984).
[5] V. I. Karpman and H. Washimi, J. Plasma Phys. **18**, 173 (1977).
[6] P. K. Shukla, Nature (London) **274**, 874 (1978).
[7] N. N. Rao, Phys. Rev. A **37**, 4846 (1988).
[8] K. H. Spatschek, P. K. Shukla, M. Y. Yu, and V. I. Karpman, Phys. Fluids **22**, 576 (1979).
[9] V. I. Karpman, F. R. Hansen, T. Huld, J. P. Lynov, H. L. Pécseli, and J. J. Rasmussen, Phys. Rev. Lett. **64**, 890 (1990).
[10] J. S. Zhao, J. Y. Lu, and D. J. Wu, Astrophys. J. **714**, 138 (2010).
[11] D. W. Forslund, J. M. Kindel, K. Lee, and E. L. Lindmann, Phys. Rev. Lett. **36**, 35 (1976); J. L. Bobin, Phys. Rep. **122**, 173 (1985).
[12] C. F. Kennel, G. Schmidt, and T. Wilcox, Phys. Rev. Lett. **31**, 1364 (1973); C. F. Kennel and R. Pellat, J. Plasma Phys. **15**, 335 (1976).
[13] C. T. Russell, D. D. Childers, and P. J. Coleman, J. Geophys. Res. **76**, 845 (1971).
[14] A. V. Kostrov, M. E. Gushchin, S. V. Korobkov, and A. V. Strikovskii, JETP Lett. **78**, 538 (2003).
[15] G. L. Huang, D. Y. Wang, and Q. W. Song, J. Geophys. Res. **109**, A02307 (2004).
[16] C. Cattell, J. R. Wygant, K. Goetz *et al.*, Geophys. Res. Lett. **35**, L01105 (2008).
[17] O. Santolik, D. A. Gurnett, J. S. Pickett, M. Parrot, and N. Cornilleau-Wehrlin, J. Geophys. Res. **108**, 1278 (2003).
[18] C. M. Culley, J. W. Bonnell, and R. E. Ergun, Geophys. Res. Lett. **35**, L17S16 (2008).
[19] R. L. Stenzel, J. M. Urrutia, and K. D. Strohmaier, Phys. Rev. Lett. **96**, 095004 (2006).
[20] B. Eliasson and P. K. Shukla, Phys. Rev. Lett. **99**, 205005 (2007).
[21] P. M. Nilson, L. Willingale, M. C. Kaluza *et al.*, Phys. Rev. Lett. **97**, 255001 (2006).
[22] R. L. Stenzel, Phys. Fluids **19**, 857 (1976).
[23] G. Manfredi and P.-A. Hervieux, Appl. Phys. Lett. **91**, 061108 (2007).
[24] M. P. Robinson, B. Laburthe Tolra, M. W. Noel, T. F. Gallagher, and P. Pillet, Phys. Rev. Lett. **85**, 4466 (2000).
[25] S. A. Wolf, D. D. Awschalom, R. A. Buhrman *et al.*, Science **294**, 1488 (2001).
[26] H. A. Atwater, Sci. Am. **296**, 56 (2007).
[27] V. S. Beskin, A. V. Gurevich, and Ya. N. Istomin, *Physics of the Pulsar Magnetosphere* (Cambridge University Press, Cambridge, England, 1993).
[28] A. K. Harding and D. Lai, Phys. Rep. **69**, 2631 (2006).
[29] J. J. Fortney, S. H. Glenzer, M. Koenig, B. Militzer, D. Saumon, and D. Valencia, Phys. Plasmas **16**, 041003 (2009).
[30] S. Eliezer, P. Norreys, J. T. Mendonça, and K. Lancaster, Phys. Plasmas **12**, 052115 (2005).
[31] S. H. Glenzer, O. L. Landen, P. Neumayer *et al.*, Phys. Rev. Lett. **98**, 065002 (2007).
[32] V. M. Malkin, N. J. Fisch, and J. S. Wurtele, Phys. Rev. E **75**, 026404 (2007).
[33] C. L. Gardner and C. Ringhofer, Phys. Rev. E **53**, 157 (1996).
[34] G. Manfredi and F. Haas, Phys. Rev. B **64**, 075316 (2001).
[35] P. K. Shukla, Phys. Lett. A **352**, 242 (2006).
[36] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007); G. Brodin and M. Marklund, New J. Phys. **9**, 277 (2007); Phys. Rev. E **76**, 055403 (2007).
[37] V. N. Oraevsky and V. B. Semikoz, Phys. At. Nucl. **66**, 466 (2003).
[38] G. Brodin, M. Marklund, J. Zamanian, A. Ericsson, and P. L. Mana, Phys. Rev. Lett. **101**, 245002 (2008); J. Zamanian, M. Marklund, and G. Brodin, New J. Phys. **12**, 043019 (2010).
[39] P. K. Shukla, Nat. Phys. **5**, 92 (2009).
[40] G. Brodin, A. P. Misra, and M. Marklund, Phys. Rev. Lett. **105**, 105004 (2010).
[41] M. Marklund and G. Brodin, in *New Aspects of Plasma Physics*, Proceedings of the 2007 ICTP Summer College on Plasma Physics, edited by P. K. Shukla, L. Stenflo, and B. Eliasson

(AIP; World Scientific, London, 2008).
[42] G. Manfredi, Fields Inst. Commun. **46**, 263 (2003).
[43] T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983); P. Chen, F.-Y. Chang, G.-L. Lin, R. J. Noble, and R. Sydora, Plasma Phys. Controlled Fusion **51**, 024012 (2009).
[44] V. I. Berezhiani and D. D. Tskhakaya, Fiz. Plazmy **7**, 675 (1981); [Sov. J. Plasma Phys. **7**, 369 (1981)]; R. L. Stenzel, J. M. Urrutia, and K. D. Strohmaier, Plasma Phys. Controlled Fusion **49**, A17 (2007).