When does noise destroy or enhance synchronous behavior in two mutually coupled light-controlled oscillators?

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We study the influence of white Gaussian noise in a system of two mutually coupled light-controlled oscillators (LCOs). We show that under certain noise intensity conditions, noise can destroy or enhance synchronization. We build some Arnold tonguelike structures in order to explain the effects due to noise. It is remarkable that noise-enhanced synchronization is possible only when the variances of the noise acting on each of the LCOs are different.

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Synchronization is a ubiquitous phenomenon present in natural systems and in manmade devices, which can be defined as an adjustment of rhythms in self-sustained oscillators due to their interaction [1]. Noise effects have been extensively studied and one of the more basic observations is that noise disturbs the coherent behavior of a system [2]. Furthermore, new insights about noise effects have been provided, such as stochastic resonance, in which, under certain circumstances, noise can help rather than hinder the performance of some devices [3], and it is possible to find its manifestations in a wide variety of systems such as optical [4], electronic [5], and biological [6] ones, especially in neuronal systems [7]. On the other hand, stochastic synchronization addresses the phenomenon of irregular phase locking between two noisy nonlinear oscillators or between a nonlinear oscillator and an external driving force [8]. The concepts of stochastic resonance and stochastic synchronization are closely related. A strong link between synchronization and noise has been found in the so-called noise-induced [9] and noise-enhanced [10] synchronization. The competition between noise and coupling in the induction of synchronization is discussed in [11]. A review of noise effects on chaotic systems is presented in [12].

In this work, we analyze the case in which the oscillators are influenced by δ-correlated Gaussian noise characterized by their variances σ_i^2. The choice of noncommon noises is related to the fact that in some cases the subsystems have internal complex mechanisms with elements which are potentially sensitive to noise. Independent electronic devices are affected internally by noise whether in their active (e.g., power supply and diodes) or passive (e.g., resistors and capacitors) components. Our interesting result concerns the fact that in a system of two nonidentical pulse-coupled oscillators, the variances of the noise acting on each oscillator must be different in order to observe noise-enhanced synchronization.

Light-controlled oscillators (LCOs) are simple electronic devices that can be used to study realistic systems such as neurons, cardiac cells, and fireflies, among others, whose individual behavior follows an integrate-and-fire model. Our system is composed of two noisy nonidentical mutually coupled LCOs that are relaxation oscillators whose (noise-free) natural periods T_{0i} are given by the sum of the durations of their charging $t_{\lambda 0i}$ and discharging $t_{\gamma 0i}$ stages. These time intervals are determined by the values of the electronic components: $t_{\lambda 0i}=(R_{\lambda i}+R_{\gamma i})C_i \ln 2=\ln 2/\lambda_i$ and $t_{\gamma 0i}=R_{\gamma i}C_i \ln 2=\ln 2/\gamma_i$ [13]. Each LCO uses a voltage source V_{Mi} and is made up of photosensors and infrared light-emitting diodes, optoelectronic components that allow the LCO to interact with other LCOs by means of light pulses characterized by the coupling strength β_{ij} [14]. Additionally, an LM555 timer chip provides the LCO with the ability to oscillate, since it controls the charge and the discharge between an upper threshold $(2V_{Mi}/3)$ and a lower threshold $(V_{Mi}/3)$. Our scheme corresponds to parametric noise acting on the source voltage V_{Mi}, causing changes to the LCOs' amplitude signal that remains constant for non-noisy oscillators. The equation that describes the model for N LCOs is

$$\frac{dV_i(t)}{dt} = \lambda_i\{[V_{Mi}+\xi_i(t)\sqrt{D}] - V_i(t)\}\epsilon_i(t) - \gamma_i V_i(t)[1-\epsilon_i(t)] + \sum_{i,j}^{N} \beta_{ij}\delta_{ij}[1-\epsilon_j(t)], \qquad (1)$$

where $\xi_i(t)$ represents a random number chosen from a normal distribution with mean zero and variance σ_i^2. The noisy term $\xi_i(t)\sqrt{D}$ is determined by the noise variance σ_i^2, and δ_{ij} indicates whether or not there is an interaction between LCO$_i$ and LCO$_j$, such that $\delta_{ij}=1$ when there is an interaction, $\delta_{ij}=0$ when there is no interaction, and $\delta_{ii}=0$ always, to denote no self-interaction. In Eq. (1), the oscillator state $\epsilon_i(t)$ is defined by

$$\epsilon_i(t)=1 \quad \text{extinguished LCO} \quad \text{(charge)},$$

$$\epsilon_i(t)=0 \quad \text{fired LCO} \quad \text{(discharge)},$$

and the timer chip LM555 acts on the LCOs' states as

$$\text{If} \quad V_i(t)=\tfrac{1}{3}[V_{Mi}+\xi_i(t)\sqrt{D}] \text{ and } \epsilon_i(t)=0 \quad \text{then} \quad \epsilon_i(t_+)=1.$$

$$\text{If } V_i(t) = \tfrac{2}{3}[V_{Mi} + \xi_i(t)\sqrt{D}] \text{ and } \epsilon_i(t) = 1 \quad \text{then } \epsilon_i(t_+) = 0. \tag{2}$$

The usual values taken for the LCOs give a ratio of around 2% between the discharging time and the charging one. This small value for the ratio indicates that we have two time scales, a typical characteristic of relaxation oscillators. In order to study the synchronization features of our system, we define the instantaneous phase of an LCO (with label i) in accordance with the Poincaré map method [1,15] as $\phi_i(t) = 2\pi(t-t_i^{(k)})/(t_i^{(k+1)}-t_i^{(k)})+2\pi k$; with this, we can compute the instantaneous linear phase difference (LPD) with stroboscopic observation between the LCOs as

$$\Delta\Phi_{12}^{\text{linear}} = \phi_2(t_1^{(k+1)}) - \phi_1(t_1^{(k+1)}) = 2\pi \frac{t_2^{(k+1)} - t_1^{(k+1)}}{t_2^{(k+1)} - t_2^{(k)}}, \tag{3}$$

where $t_i^{(k+1)}$ is the time at which the $(k+1)$th firing event of LCO_i takes place. The above expression gives the appropriate result in the case of LCOs whether the oscillations are merely periodic or if they are disturbed by noise, since we can consider the beginning of the flashing events as the points lying on the Poincaré section in the phase space. The normalization of the phase differences in the circle [0:1] allows us to obtain the cyclic phase difference (CPD) defined as

$$\Delta\phi_{12}^{\text{cyclic}} = \frac{1}{2\pi}[\Delta\Phi_{12}^{\text{linear}} \bmod 2\pi]. \tag{4}$$

Using Eqs. (1) and (3) we have studied the influence of δ-correlated Gaussian noise on two mutually coupled LCOs. In Eq. (1), we can associate $\sqrt{D}$ to the noise intensity that we have varied in the interval [0.0, 2.0]. In the case of two mutually coupled LCOs, the width of the synchronization region is directly proportional to the coupling strength for 1:1 synchronization [13], and we expect that in this region the mean frequencies of the LCOs in the presence of coupling are roughly the same, i.e., $\langle\Omega_1/\Omega_2\rangle \approx 1$ [16]. Synchronization domains can be determined, as shown in Fig. 1, using statistical criteria such as the LPD mean value ($\langle\text{LPD}\rangle$) or the CPD variance (var_{CPD}) [17].

First, we consider two nonidentical LCOs with parameter values $T_{01}=34.0$ ms and $T_{02}=32.5$ ms, which—in the absence of noise—synchronize for $\beta \gtrapprox 400$; this result has been obtained by computing the 1:1 Arnold tongue for this system. When Gaussian noise acts on the LCOs, we observe that the system desynchronizes for determined values of noise intensity (Fig. 1).

In Fig. 1(a) the $\langle\text{LPD}\rangle$ as a function of noise intensity is shown for three values of coupling strength β and when the LCO's noise variances are such that $\sigma_1^2 < \sigma_2^2$. We see that the noise destroys the synchronous regime ($\langle\text{LPD}\rangle \approx 0$) for increasing noise intensities. On the other hand, this effect is less important when the coupling strength increases. Using var_{CPD}, it is possible to determine the synchronization region as well [Fig. 1(b)]; in this case, $\text{var}_{\text{CPD}} \approx 0$ in the synchronous regime and $\text{var}_{\text{CPD}} \approx 0.083$ when the system is not synchronized. Taking a noise intensity value included in the synchronization region for a specific coupling strength and representing the LPD evolution and the CPD probability distribution, we verify that the LPD remains almost constant [Fig. 1(c)], and the CPD distribution has a well-defined maximum [Fig. 1(f)]. For a greater noise intensity value, the system leaves the synchronous regime, the LPD drops via several phase slips [Fig. 1(d)], and the CPD distribution splits with a tendency toward a uniform distribution [Fig. 1(g)]. If the noise intensity value is still greater, we find that the LPD evolution drops [Fig. 1(e)] as in the case of nonidentical uncoupled LCOs with $T_{01} > T_{02}$ [17], and the CDP distribution becomes almost uniform [Fig. 1(h)] showing the signature of an asynchronous regime. Finally, Figs. 1(i) and 1(j) show the synchronization regions when $\sigma_1^2 > \sigma_2^2$. In this case, noise does not affect the system considerably, i.e., the system remains synchronous for a wide interval of noise intensity values.

Only high noise intensity values produce LCOs synchrony loss via phase slips [Fig. 1(l)]; when the synchrony is totally lost, the LPD grows in time as in the case of uncoupled LCOs with $T_{01} < T_{02}$ [17] [Fig. 1(m)]. This means that noise induces a decreasing period in LCO_1, a situation that can be easily understood since noise is high and, consequently, the thresholds will be more rapidly reached. The var_{CPD} criterion shows some regions in which $\text{var}_{\text{CPD}} \not\approx 0$ [Fig. 1(j)]. Nevertheless, the corresponding LPD evolution and the CPD distribution show that synchronization is still present [Figs. 1(k) and 1(n)].

Determining the synchronization regions as above, we can construct Arnold tonguelike structures (Fig. 2). If the noise variance of the LCO with lower period (LCO_2) is greater or equal to that of the LCO with higher period (LCO_1), then the synchronous regime is disturbed by noise and, obviously, the higher the noise intensity, the greater the noise influence over the system, and this fact can drive the system to leave the synchronous state (Fig. 2). Moreover, the noise influence is more important when the difference between noise variances is larger and the coupling strength is less [Figs. 2(a) and 2(c)]. The relationship between the noise intensity ($\sqrt{D}$) and the coupling strength (β) computed at the boundary line separating synchronous and asynchronous regions is almost linear. On the other hand, when the noise variance of the LCO with lower period (LCO_2) is less than that of the LCO_1, the synchronous regime is only destroyed for high values of noise intensity [Figs. 2(b) and 2(d)]. One of the differences with respect to the case in which $\sigma_1^2 \le \sigma_2^2$ is the fact that the slope of the boundary line separating synchronous and asynchronous regions is considerably lower for $\sigma_1^2 > \sigma_2^2$ [Figs. 2(b) and 2(d)] where the slope of the boundary line is almost null. For equal noise variances ($\sigma_1^2 = \sigma_2^2$), we see that the synchronous regime withstands better the noise effects with respect to the case in which $\sigma_1^2 < \sigma_2^2$, in the sense that the synchronization regions are wider [Figs. 2(a) and 2(c)], despite the fact that σ_1^2 is greater.

Until now, we have considered the noise effects tending to desynchronize the system. Nevertheless, noise may also enhance synchrony when the LCOs do not synchronize in noise-free situations. As was shown above, it is possible to determine synchronization regions using $\langle\text{LPD}\rangle$. Now, if we consider the same system of two mutually coupled LCOs but in a situation in which the LCOs do not synchronize

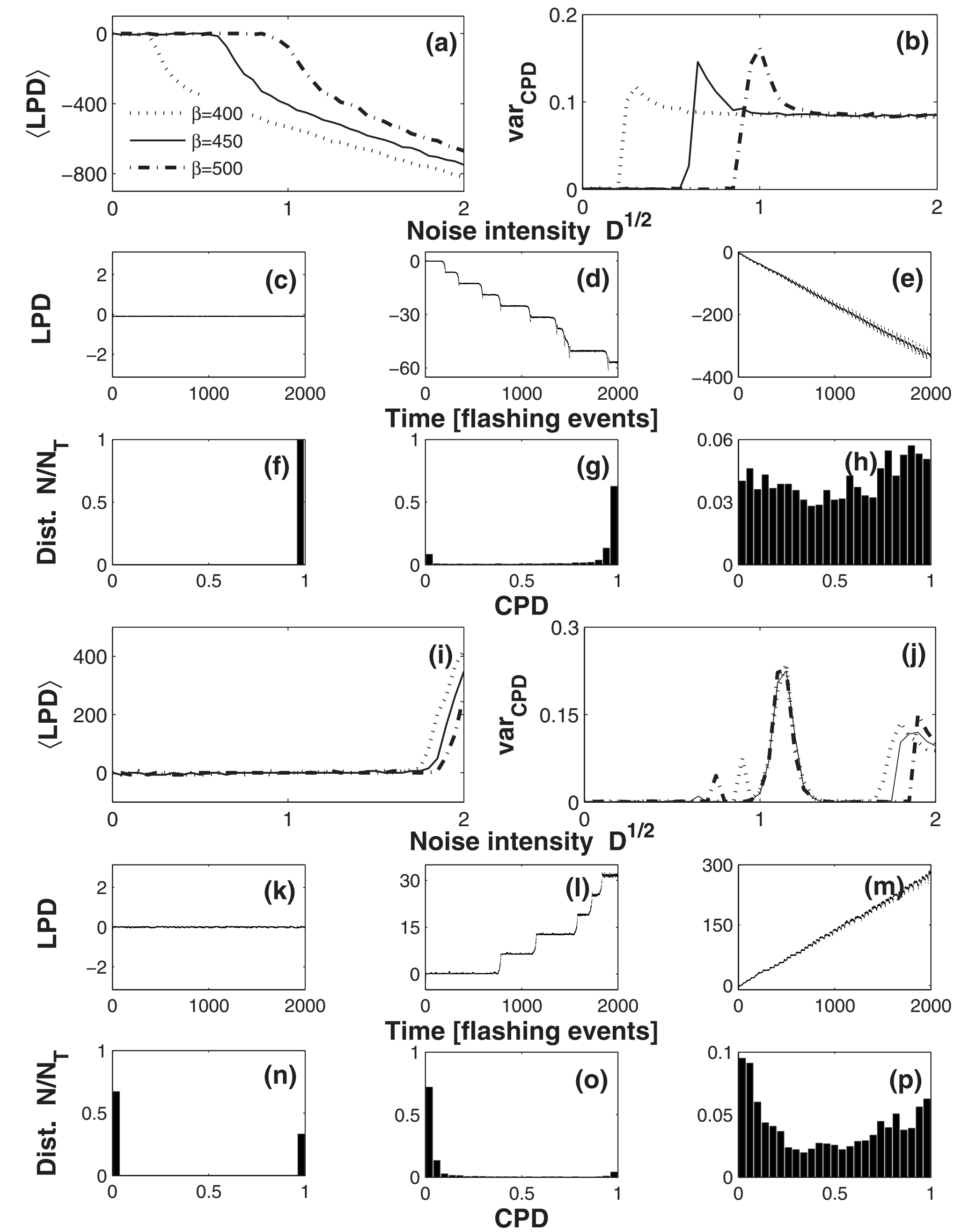

FIG. 1. Determining synchronization regions for two nonidentical noisy LCOs and different coupling strengths using as criterion [(a) and (i)] the LPD mean value and [(b) and (j)] the CPD variance as a function of the noise intensity. The LCOs' noise variances are (a)–(h) $\sigma_1^2=0.0005$ and $\sigma_2^2=0.001$, and (i)–(p) $\sigma_1^2=0.003$ and $\sigma_2^2=0.0003$. For a determined coupling strength ($\beta=450$), the corresponding LPD evolution and the CPD distribution are represented for different noise intensities: (c) and (f) $\sqrt{D}=0.20$, (d) and (g) $\sqrt{D}=0.65$, (e) and (h) $\sqrt{D}=1.00$, (k) and (n) $\sqrt{D}=1.15$, (l) and (o) $\sqrt{D}=1.80$, and (m) and (p) $\sqrt{D}=2.00$.

($\beta \lesssim 400$), we note that the presence of independent noises can drive the system to a statistical synchronous state for certain noise intensity values (Fig. 3). For defined noise variances, the synchronization region enlarges with the increasing of β [Figs. 3(a) and 3(b)]. From Fig. 3(c) we observe that for a determined value of β, the noise variances modify the synchronization region in such a way that for increasing values of σ_2^2, the beginning of the synchronization region shifts toward greater values of noise intensity and the region becomes larger. Nevertheless, Fig. 3(d) shows that as σ_2^2 tends toward σ_1^2, the synchronization region vanishes. Hence, the condition $\sigma_1^2 > \sigma_2^2$ must be fulfilled in order to enhance synchronization.

Depending on the noise intensity values, the system remains in an asynchronous regime or it achieves a synchronous one. However, if the noise intensity is still greater, the system desynchronizes again but with the characteristic that the LCOs' periods behave in such a way that $T_1 < T_2$.

The shape of the curves in Figs. 3(a) and 3(c) exhibits a great resemblance to the typical curves obtained from fre-

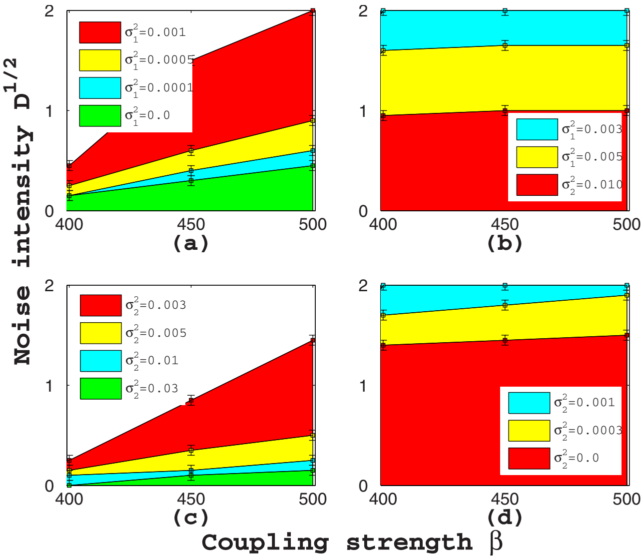

FIG. 2. (Color online) Patches representing synchronization regions for different noise variances. (a) $\sigma_2^2 \geq \sigma_1^2$, (b) $\sigma_2^2 < \sigma_1^2$, with σ_2^2=0.001; (c) $\sigma_1^2 \leq \sigma_2^2$, (d) $\sigma_1^2 > \sigma_2^2$, with σ_1^2=0.003.

quency difference ($\Delta\Omega = \Omega_2 - \Omega_1$) vs detuning ($\Delta\omega = \omega_2 - \omega_1$) plots in 1:1 synchronization, with Ω being the frequency in the presence of coupling and ω the natural frequency [1,18]. In $\Delta\Omega$ vs $\Delta\omega$ plots, the middle of the synchronization domain represents the situation in which the frequency mismatch is null ($\Delta\omega = 0$). Following this, we can say that in the middle of the synchronization domains shown in Figs. 3(a) and 3(b) the noise acts on the system in such a way that the LCOs' frequencies are roughly the same. Moreover, in Fig. 4 we can observe that in the synchronization region ($\langle \mathrm{LPD} \rangle \approx 0$), as was expected, the mean frequency

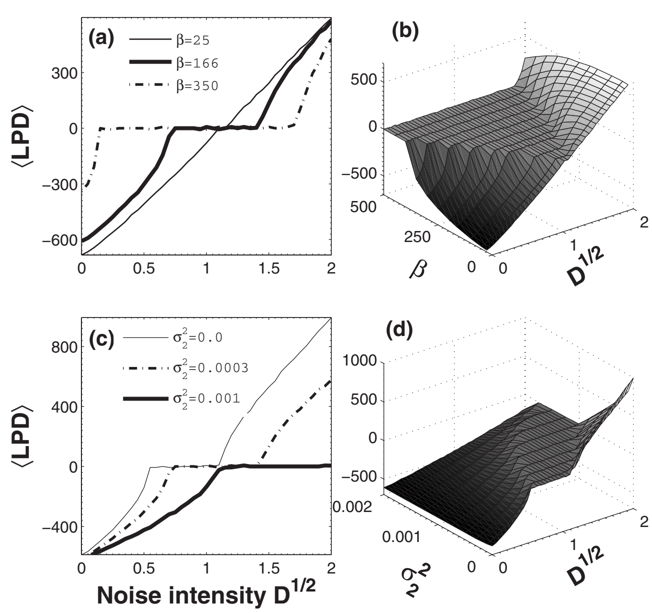

FIG. 3. (a) Synchronization regions for specific values of β with σ_1^2=0.003 and σ_2^2=0.0003 and (b) its generalization in a surface. (c) Synchronization regions for different LCO_2 noise variances σ_2^2 with σ_1^2=0.003 and β=166 and (d) its generalization in a surface.

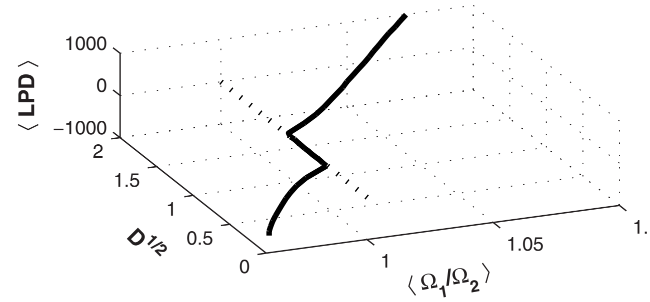

FIG. 4. $\langle \mathrm{LPD} \rangle$ as a function of noise intensity $\sqrt{D}$ and the mean coupling frequency ratio $\langle \Omega_1/\Omega_2 \rangle$. The values for noise variances are σ_1^2=0.003 and σ_2^2=0.0003, and for the coupling strength β=166.

ratio is $\langle \Omega_1/\Omega_2 \rangle \approx 1$, confirming again that this region corresponds to a synchronous one.

As stated above, we can construct Arnold tonguelike structures to denote the synchronization regions for different coupling strengths β and show how the noise enhances synchronization (Fig. 5). The underlying mechanism that permits noise-enhanced synchronization is related to the fact that for strong noise intensities there is a considerable contraction in the phase space, and also the amplitude of one of the LCOs decreases.

In conclusion, we have shown that we can determine the synchronization regions of coupled LCOs using statistical parameters linked to the LPD, in particular the mean value as a function of the noise intensity exhibits the same shape that is found in frequency difference vs frequency mismatch plots. As was expected, inside the synchronization regions, the LPD remains almost constant in time and the corresponding probability distribution has a well-defined peak. Moreover, $\langle \mathrm{LPD} \rangle \approx 0$ is associated to $\langle \Omega_1/\Omega_2 \rangle \approx 1$. δ-correlated Gaussian noise can destroy synchronous regimes, and this

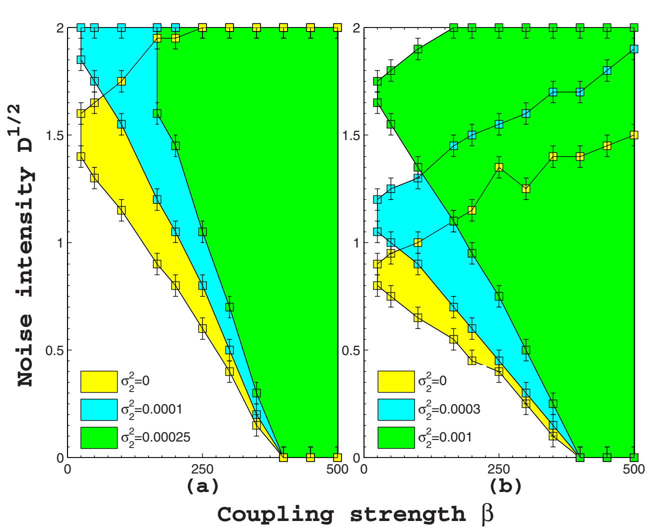

FIG. 5. (Color online) Patches representing synchronization regions for different noise variances. (a) σ_1^2=0.001 and (b) σ_1^2=0.003.

effect is more significant if the noise variance of the lower period LCO is greater than that corresponding to the other LCO. Noise-enhanced synchronous regimes are possible when the noise variance of the higher-period LCO is greater than that of the other LCO. The synchronization regions can be characterized by means of Arnold tonguelike structures onto the β-$\sqrt{D}$ plane. The last allows us to link the concepts of frequency and noise intensity, i.e., noise acts principally at the level of frequency modification permitting the change of regime (synchronous-asynchronous and vice versa) in the system of coupled LCOs.

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