Mesoscopic speckle

Sheng Zhang,[1,2] Yitzchak Lockerman,[1] and Azriel Z. Genack[1]
[1]*Department of Physics, Queens College, The City University of New York, Flushing, New York 11365, USA*
[2]*ChiralPhotonics Inc., Pine Brook, New Jersey 07058, USA*

(Received 13 April 2009; revised manuscript received 1 September 2010; published 10 November 2010)

We have measured first and second order statistics of the velocity of phase singularities v in evolving speckle patterns of microwave radiation transmitted through random quasi-1D samples as the frequency is swept and relate these to global statistics of speckle evolution. When v is normalized by the standard deviation of the fractional intensity change, the probability distribution and correlation function of v approach those for random Gaussian fields even for localized waves. Analogous results are found for transmitted intensity normalized by the total transmission. This provides a unified framework for the statistics of speckle evolution and intensity.

DOI: 10.1103/PhysRevE.82.051114 PACS number(s): 42.25.Dd, 42.25.Bs, 42.30.Ms

The study of enhanced mesoscopic fluctuations [1–3] has unified the study of transport and provided precise indicators of Anderson localization. In one respect, however, it misses the forest for the trees. Studies of first and second order statistics of transmitted flux density or intensity [2,4] for monochromatic waves and their consequences for global fluctuations [3] have largely been explored without reference to the structure of the speckle pattern [5,6], which, is built upon a network of phase singularities [7–9]. These are located at points of vanishing intensity at which the phase cannot be defined since the in- and out-of-phase components of the field both vanish. The phase changes by 2π around these singularities. The intensity and phase variation in a typical speckle pattern is shown in Fig. 1. The displacement of singularities is a reliable measure of speckle change and a sensitive probe of material defects and deformation [10]. The structure of the speckle pattern at a given frequency is generic [7–9]. However, various statistical measures of changes in the field speckle pattern with frequency shift are indicators of mesoscopic correlation of multiply scattered waves within random media [11–13]. A striking example of the mesoscopic nature of changes in the speckle pattern with frequency is the observation that the probability distribution of the average of velocity of phase singularities in a given speckle pattern as the frequency is tuned have precisely the same form of the distribution of total transmission and depend upon a single parameter, the variance of the corresponding distribution [13]. Other measures of the overall change of speckle patterns with frequency shift relating to the changing phase and intensity in the pattern have the same statistics [13].

The similarity in global statistics of speckle change and transmission is puzzling since these spectra are qualitatively different. The speckle pattern is relatively quiescent when tuning through peaks in total transmission. Thus the values of parameters reflecting the change in the speckle pattern are low when the transmission reaches its maximum. The structure of the speckle pattern is relatively stable when the frequency is tuned near resonance with quasimodes of the medium since it is then dominated by the pattern for a single mode of the medium. In contrast, speckle patterns changes rapidly between resonances as the relative contributions of different modes changes. Similarities between the statistics of intensity and singularity velocity are further surprising since intensity is a bounded continuous variable over space in contrast to singularity velocity which is defined only at singular points and diverges when singularities are created or annihilated in pairs [7,8].

In order to understand the similar statistics of total transmission and speckle evolution, we consider here the statistics of the corresponding local variables. We present the first measurements of the first and second order statistics of the velocity of phase singularities as the frequency of the incident wave is tuned, $v = dr_s/d\nu$, and compare these to corresponding statistics of the intensity, I. Microwave measurements are made in quasi-1D samples for diffusive and localized waves. The cumulant spatial correlation function of velocity, $C_v(\Delta r) = \langle v(r)v(r+\Delta r)\rangle / \langle v\rangle^2 - 1$ is decomposed into short and long range components in analogy with the cumulant intensity correlation function, $C_I(\Delta r)$. The long range correlator κ_v is nearly equal to the variance of the standard deviation of fractional intensity change within the speckle pattern, η, which characterizes the change of the speckle pattern as a whole. This is similar to the near equality of the degree of intensity correlation κ_I to the variance of total transmission. However, due to the small number of singularities in the transmitted field, the variance of the average velocity, $\mathrm{var}(v_a)$ can be significantly larger than κ_v. When v is normalized by η, and I is normalized by the spatial average of the intensity I_a, for an incident transverse mode or source

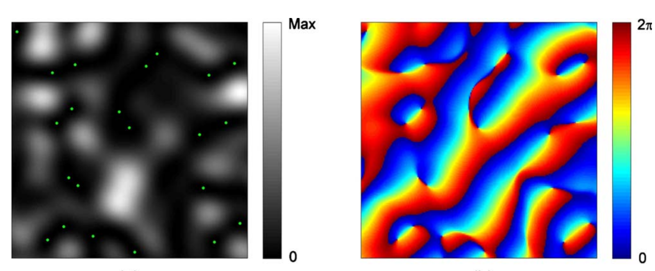

FIG. 1. (Color online) (a) Example of the intensity pattern of a random wave field. The dots mark the location of the nulls of intensity, i.e., the phase singularities. (b) The corresponding phase pattern for the same field, where different colors (or grayscales) represent different phase.

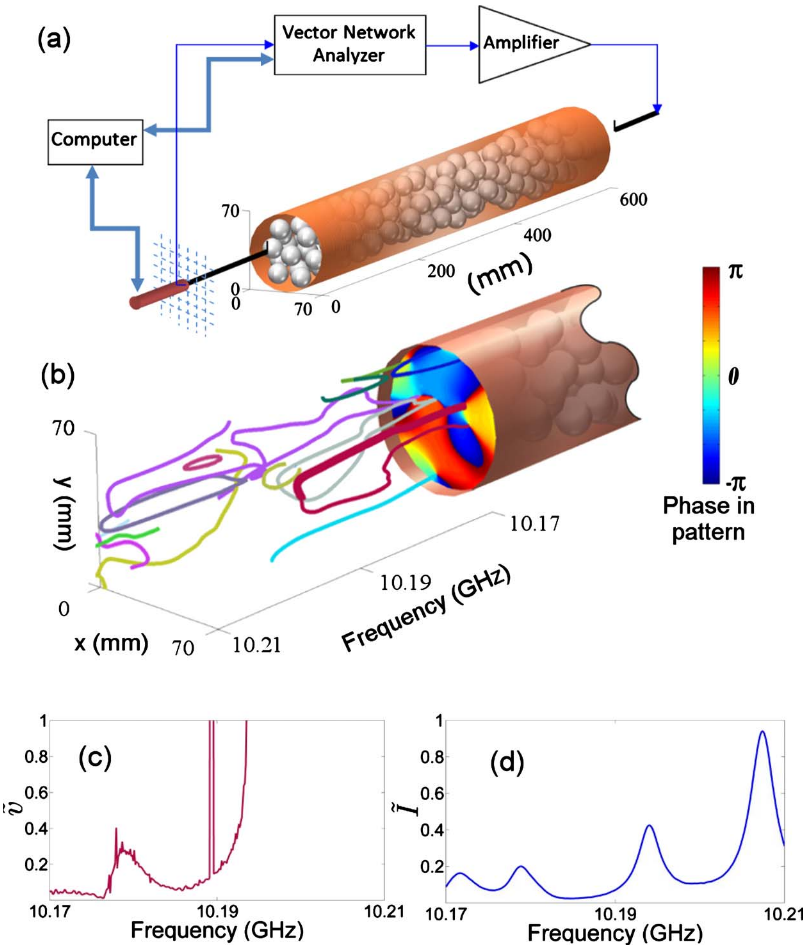

FIG. 2. (Color online) (a) Schematic of the experiment setup. (b) Example of a phase pattern in the field at the output of the random quasi-1D sample. The trajectories of singularities as the frequency is tuned are shown in 3D. (c) Velocity spectrum associated with the thick red singularity line. (d) Spectrum of intensity at the point in the center of the output plane.

position a, their corresponding statistics closely match those for Gaussian random wave fields. These considerations provide a unified description of the statistics of transmission and speckle evolution and elucidate the statistics in both realms.

We measured the microwave field transmitted through samples of alumina spheres contained in a 61-cm-long copper tube with a diameter of 7.0 cm. The sample is composed of 0.95-cm-diameter alumina spheres with refractive index 3.14 embedded in Styrofoam shells of refractive index 1.04 to produce an alumina volume fraction of 0.068. The intensity and phase of the polarization component of the transmitted field along a 4-mm-long wire antenna are measured with use of a vector network analyzer. The spatial distribution of the transmitted field over a range of frequencies is obtained by measuring field spectra at each point on a 1-mm-square grid over the output surface of the sample. In Fig. 2(a), we show a schematic of the sample and the measured phase pattern at the output of the sample at 10.085 GHz for one configuration of disorder. Measurements are made over the frequency ranges 14.7–15.7 GHz and 10–10.24 GHz, in which waves are diffusive and localized, respectively. Frequency steps are chosen to be approximately 1/7 of the field correlation frequency. Measurements are made in 40 and 71 different configurations for diffusive and localized waves, respectively. In order to accurately determine the positions of phase singularities, the 2D sampling theorem is applied to the data to reconstruct the speckle patterns on a 50×50 μm^2 grid. The sampling theorem is also used to interpolate in the frequency domain, so that spectra with 120 kHz and 250 kHz frequency steps are obtained for localized and diffusive waves, respectively. Singularities in phase are seen at points surrounded by phase change over a full range of 2π. The trajectories of phase singularities in the 3D space of two spatial dimensions and one frequency dimension is shown in Fig. 2(a). Spectra of the speed of one of the singularities $\tilde{v}=v/\langle v\rangle$ and of the intensity $\tilde{I}=I/\langle I\rangle$ at the point in the center of the pattern are shown in Figs. 2(b) and 2(c), respectively. We will use tilde to indicate the quantity that is normalized by its ensemble average throughout the paper. The velocity diverges when singularity lines are parallel to the transverse plane. In the 2D space of the output plane, this corresponds to points, at which a pair of singularities are either created or annihilated [8].

Probability distributions of velocity, $P(\tilde{v})$, are shown in Fig. 3(a) for diffusive and localized waves. These results are compared to simulations for Gaussian random waves generated by the superposition of 300 randomly phased plane waves, $E(x,y,z)=\Sigma_i A_i \exp[i(k_x x+k_y y+k_z z)]$. Each of the components of the k-vector and the amplitude A_i are drawn from a Gaussian distribution. The singularity velocity in the x-y plane is tracked as z increases. The simulated distribution is close to the measured $P(\tilde{v})$ for diffusive waves, but are not in good agreement with the theoretical formula, $P(\tilde{v}) = \frac{8\pi^2\tilde{v}}{(\pi^2\tilde{v}^2+4)^2}$, derived from Gaussian random waves in [8]. $P(\tilde{v})$

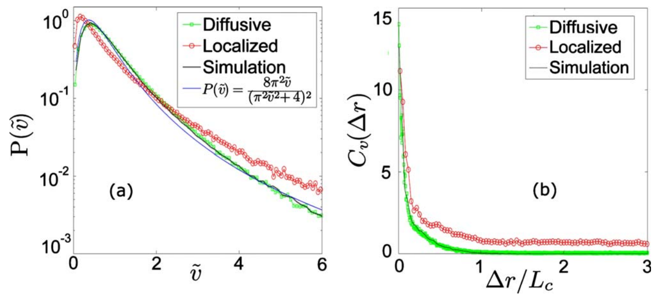

FIG. 3. (Color online) (a) The probability distributions and (b) cumulant spatial correlation functions of $\tilde{v}$, for diffusive and localized waves.

is noticeably broader for localized waves. Measurements of $C_v(\Delta r)$ for diffusive and localized waves are shown in Fig. 3(b). Corresponding theoretical expressions have not been reported to our knowledge. The high value of $C_v(0)$ is consistent with the divergence of the velocity of singularities as $\Delta r \to 0$ when singularities are created or annihilated [8]. $C_v(\Delta r)$ falls rapidly with Δr and reaches a constant value denote by κ_v, which is 0.039 for diffusive and 0.648 for localized waves, respectively. Thus the correlation function can be expressed as the sum of short-range and a constant terms: $C_v(\Delta r)=C_{v,short}(\Delta r)+\kappa_v$.

The statistics of singularity velocity can be compared to first and second order statistics of polarized intensity which are plotted in Figs. 4(a) and 4(b). Fluctuations of I are greatly enhanced for localized waves. Similar to $C_v(\Delta r)$, the cumulant correlation function for intensity, $C_I(\Delta r)$, which is plotted in Fig. 4(b) using the same data used in Fig. 3, may be expressed as, $C_I(\Delta r)=C_{I,short}(\Delta r)+\kappa_I$. Here κ_I is the degree

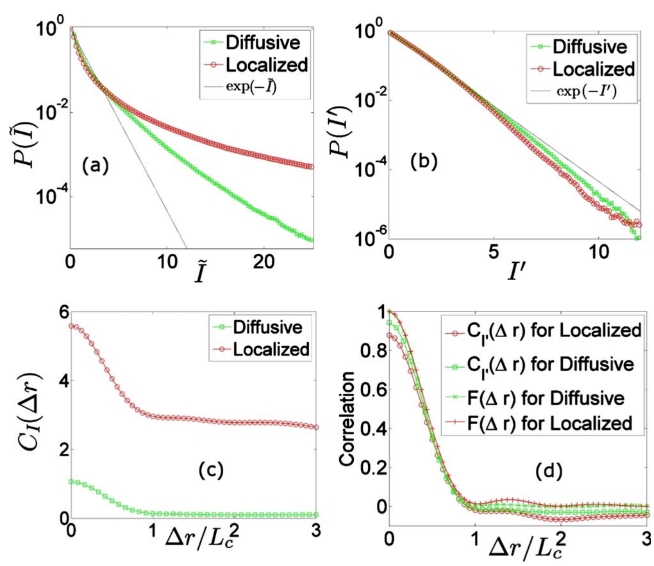

FIG. 4. (Color online) First and second order statistics of $\tilde{I}$ and I' for diffusive and localized waves. (a) Probability distributions of $\tilde{I}$. (b) Probability distributions of I'. (c) Cumulant correlation functions of $\tilde{I}$. The separations Δr are normalized by the corresponding correlation lengths L_c, which are the first zeros of the real part of the field correlation functions. (d) Cumulant correlation functions of I' and their comparison to the square of the field correlation functions.

TABLE I. Comparison of κ_I and κ_v to variance of global measures of intensity and speckle changes.

	κ_I	$\mathrm{var}(\tilde{I}_a)$	κ_v	$\mathrm{var}(\tilde{v}_a)$	$\mathrm{var}(\tilde{\sigma}_{\Delta\varphi})$	$\mathrm{var}(\tilde{\sigma}_{\Delta I^*})$	$\mathrm{var}(\tilde{v}'_a)$
Diff	0.12	0.14	0.039	0.193	0.087	0.045	0.131
Loc	3.0	3.3	0.648	1.240	0.743	0.586	0.267

of correlation, which is the value of $C_I(\Delta r)$ at points at which the field correlation functions vanishes [14]. $\kappa_I=0.12$ for the diffusive and 3.0 for localized waves. In quasi-1D samples with a large number of transverse modes, the field in individual speckle patterns can be assumed to be a Gaussian random variable. Thus the probability distribution of polarized intensity normalized by the average intensity within the speckle pattern, $I'(r)=I(r)/I_a$, is expected to be an negative exponential distribution, i.e., $P(I')=\exp(-I')$, and should be statistically independent of the total transmission [3]. The measured $P(I')$, however, can be seen in Fig. 4(c) to deviate slightly from this prediction, because the number of transverse waveguide modes is small; approximately 30 at 10 GHz and 50 at 15 GHz. Agreement with Gaussian statistics is better for diffusive waves since the number of modes is larger. Based on the assumption that $\tilde{I}'$ and I_a are statistically independent, we have $\langle I'(r)I'(r+\Delta r)I_a^2\rangle=\langle I'(r)I'(r+\Delta r)\rangle\langle I_a^2\rangle$. The cumulant intensity correlation function, $C_I=\Gamma_I(\Delta r)-\langle I\rangle^2$, can then be expressed as

$$C_I(\Delta r)=[C_{I'}(\Delta r)+1][\mathrm{var}(\tilde{I}_a)+1]-1 \tag{1}$$

$C_{I'}(\Delta r)$ and the square of the corresponding field correlation function, $F(\Delta r)$, are calculated for diffusive and localized waves and seen to be similar in Fig. 4(d). Since $C_{I'}(\Delta r)\to 0$ for large Δr, Eq. (1) gives, $\kappa_I=\mathrm{var}(\tilde{I}_a)$. This is roughly consistent with the measured values of $\mathrm{var}(\tilde{I}_a)$ of 0.14 for diffusive and 3.3 for localized waves (see Table I). $C_{I'}(\Delta r)$ is closer to $F(\Delta r)$ for diffusive waves than for localized waves because field statistics are closer to Gaussian when the number of transverse modes is larger.

The character of speckle pattern change can be traced to the combined factors of long-range correlation in speckle change and the statistical independence of local and global fluctuations as is the case for intensity. We consider the motion of phase singularities and assume that the global change of speckle patterns, denoted by η, is statistically independent of the velocity of individual singularities normalized by this change, $v'=v/\eta$. $C_v(\Delta r)$ can then be expressed as

$$C_v(\Delta r)=[C_{v'}(\Delta r)+1][\mathrm{var}(\tilde{\eta})+1]-1 \tag{2}$$

If the local changes in the speckle patterns were a Gaussian random process, we would expect that $C_{v'}(\Delta r)\to 0$ for large Δr, since the fields in distant regions would not be correlated. In the limit of large Δr, this would give $\kappa_v=\mathrm{var}(\tilde{\eta})$.

Among the parameters, η, which can quantify the change of the speckle pattern as a whole are: the average velocity of phase singularities v_a; the standard deviation of phase changes $\sigma_{\Delta\varphi}$; and the standard deviation of fractional intensity change, $\sigma_{\Delta I^*}$, where $\Delta\varphi=\varphi(\nu+\Delta\nu)-\varphi(\nu)$, $\Delta I^*=[I(\nu$

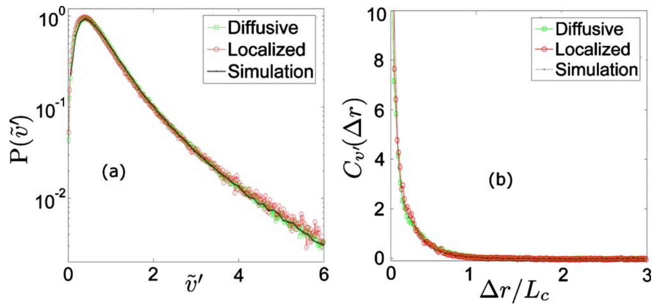

FIG. 5. (Color online) Probability distributions (a) and the cumulant spatial correlation functions (b) of the normalized velocity of phase singularities $v' = v/\sigma_{\Delta I^*}$ for diffusive and localized waves and the comparison to simulations for Gaussian random waves.

$+\Delta\nu) - I(\nu)]/[I(\nu+\Delta\nu)+I(\nu)]$. The standard deviation is defined over all measured points on the output surface. Spectra of these quantities in one sample configuration are similar for localized waves and their probability distributions have the same functional form as the probability distribution of total transmission, though the variances of the distributions differ [13]. The measured variances of v_a, $\sigma_{\Delta\varphi}$ and $\sigma_{\Delta I^*}$ together with κ_v for both diffusive and localized waves are given in Table I. We find that $\mathrm{var}(\tilde{\sigma}_{\Delta I^*})$ is closest to κ_v in both cases, while $\mathrm{var}(\tilde{v}_a)$ and $\mathrm{var}(\tilde{\sigma}_{\Delta\varphi})$ are higher. This is also reflected in the smoother spectra of $\tilde{\sigma}_{\Delta I^*}$ than for $\tilde{v}_a$ and $\tilde{\sigma}_{\Delta\varphi}$ [13]. In the limit, $\Delta\nu \to 0$, $\Delta I^*/\Delta\nu$ does not diverge near singularities as does $\Delta\varphi/\Delta\nu$ [8]. Thus, $\mathrm{var}(\sigma_{\Delta I^*})$ more reliably reflects the change of the speckle pattern as a whole, while $\sigma_{\Delta\varphi}$ is strongly affected by the immediate region around singularities and v_a obviously depends only on the small numbers of singularities. This suggests that $\eta = \sigma_{\Delta I^*}$ is a more practical choice as an indicator of global speckle change.

When v is normalized by $\eta = \sigma_{\Delta I^*}$, first and second order velocity statistics for diffusive and localized waves collapse to the results found in simulations for Gaussian waves (Fig. 5), just as was found for statistics of the intensity normalized by I_a, I'. Despite many similarities, there are key differences in the relationships between global and local statistics for v and I. Unlike intensity, which is defined at all points, there are only a small number of singularities in the speckle pattern; on average 15 for diffusive and 10 for localized waves. This difference leads to a further difference between the statistics of v and I, that $\mathrm{var}(\tilde{v}_a)$ is significantly greater than κ_v whereas $\mathrm{var}(\tilde{I}_a)$ is only slightly larger than κ_I as can be seen Table I. The source of this difference can be seen by first considering the relationship of $\mathrm{var}(\tilde{I}_a)$ and κ_I. The variance of total transmission can be expressed in terms of the spatial correlation function of intensity,

$$\mathrm{var}(\tilde{I}_a) = \frac{1}{A}\int_A C_I(\Delta r) d\Delta r^2 = \frac{1}{A}\int_{A,short} C_{I,short}(\Delta r) d\Delta r^2 + \kappa_I = \Gamma_{I,short} + \kappa_I, \quad (3)$$

where, A is the total area of the output surface. The small excess of $\mathrm{var}(\tilde{I}_a)$ over κ_I, indicates that the assumptions made in Eq. (1) are not strictly valid. A quantitative measure of the breakdown of independence of I' and I_a is the relative magnitudes of the contributions to $\mathrm{var}(\tilde{I}_a)$ by the integral of the short-range correlation function $C_{I,short}(\Delta r)$ over the output surface and k_I. Since $C_{I,short}(\Delta r)$ falls rapidly to 0 for $\Delta r > L_C$ and the correlation length L_C is much smaller than the diameter of the sample cross section, the integral over A, giving $\Gamma_{I,short} = 0.015$ for diffusive and 0.088 for localized waves, is significantly smaller than the corresponding values of κ_I. Thus the assumptions made are approximately valid and $\mathrm{var}(\tilde{I}_a) \approx \kappa_I$, as expected from Eq. (1). Using Eq. (1), we can approximate $\Gamma_{I,short}$ as

$$\Gamma_{I,short} \approx (1+\kappa_I)\frac{1}{A}\int_A C_{I',short}(\Delta r) d^2(\Delta r), \quad (4)$$

in which, $\frac{1}{A}\int_A C_{I',short}(\Delta r) d^2(\Delta r)$ corresponds to purely Gaussian random fluctuation.

Equation (3) cannot be applied directly to v because the singularities do not exist at every point as does the intensity. Finding singularities separated by Δr must be described as a correlated random process with a probability which is not uniform in Δr. Thus $\mathrm{var}(\tilde{v}_a)$ cannot be expressed simply as a two-dimensional integral of $C_v(\Delta r)$ as was the case for intensity. However, the short-range contribution to $\mathrm{var}(\tilde{v}_a)$ can be evaluated using the measured values of $\mathrm{var}(\tilde{v}'_a)$, which corresponds to fluctuations for Gaussian waves, multiplied by the mesoscopic enhancement factor $(1+\kappa_v)$ as in Eq. (4) for the intensity. Here v'_a is the average of the normalized velocity v' over a full speckle pattern. We then expect that

$$\mathrm{var}(\tilde{v}_a) \approx (1+\kappa_v)\mathrm{var}(\tilde{v}'_a) + \kappa_v. \quad (5)$$

In the limit $\kappa_v \to 0$, $\mathrm{var}(\tilde{v}_a)$ reduces to the Gaussian term $\mathrm{var}(\tilde{v}'_a)$. Using values of $\mathrm{var}(\tilde{v}'_a)$ in Table I, Eq. (5) gives $\mathrm{var}(\tilde{v}_a) \approx 0.175$ and 1.09, for diffusive and localized waves, which are in reasonable agreement with the corresponding measured values, 0.193 and 1.24.

In conclusion, we have constructed a unified framework for the statistics of transmission and speckle change. When the local variable is normalized by a global variable reflecting the speckle pattern as a whole, mesoscopic fluctuations disappear. In the limit in which a large number of modes contribute to the field, first and second order statistics of the normalized local variable approach the statistics of a Gaussian random process. We expect that the statistics of change in the speckle pattern, which may arise from internal motion of the sample, temperature change, time delay following pulsed excitation or by non-monochromatic excitation can be described within this framework.

We thank Bing Hu for contributions to the experiment and data analysis. This work was supported by the NSF under Grant No. DMR-0907285.

[1] *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier Science, Amsterdam, 1991); R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985); B. L. Altshuler and D. E. Khmelnitskii, JETP Lett. **42**, 359 (1985); P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).

[2] M. J. Stephen and G. Cwilich, Phys. Rev. Lett. **59**, 285 (1987); P. A. Mello, E. Akkermans, and B. Shapiro, *ibid.* **61**, 459 (1988); S. Feng, C. Kane, P. A. Lee, and A. D. Stone, *ibid.* **61**, 834 (1988); A. Z. Genack, N. Garcia, and W. Polkosnik, *ibid.* **65**, 2129 (1990); H. Hu, A. Strybulevych, J. H. Page, S. E. Skipetrov, and B. A. van Tiggelen, Nat. Phys. **4**, 945 (2008).

[3] M. P. van Albada, J. F. de Boer, and A. Lagendijk Phys. Rev. Lett. **64**, 2787 (1990); M. Stoytchev and A. Z. Genack, *ibid.* **79**, 309 (1997); E. Kogan and M. Kaveh, Phys. Rev. B **52**, R3813 (1995); Th. M. Nieuwenhuizen and M. C. W. van Rossum, Phys. Rev. Lett. **74**, 2674 (1995); M. C. W. van Rossum and Th. M. Nieuwenhuizen, Rev. Mod. Phys. **71**, 313 (1999); A. A. Chabanov, M. Stoytchev, and A. Z. Genack, Nature (London) **404**, 850 (2000).

[4] J. C. Dainty, J. Mod. Opt. **17**, 761 (1970).

[5] J. W. Goodman, *Statistical Properties of Laser Speckle Patterns* (Springer, 1975).

[6] J. W. Goodman, J. Opt. Soc. Am. **66**, 1145 (1976).

[7] J. F. Nye and M. V. Berry, Proc. R. Soc. London, Ser. A **336**, 165 (1974); M. V. Berry, J. Phys. A **11**, 27 (1978).

[8] M. V. Berry and M. R. Dennis, Proc. R. Soc. London, Ser. A **456**, 2059 (2000).

[9] I. Freund and N. Shvartsman, Phys. Rev. A **50**, 5164 (1994); W. Wang, S. G. Hanson, Y. Miyamoto, and M. Takeda, Phys. Rev. Lett. **94**, 103902 (2005); S. Zhang and A. Z. Genack, *ibid.* **99**, 203901 (2007).

[10] S. Sirohi, *Speckle Metrology* (Marcel Dekker, New York, 1993); W. Wang *et al.*, Opt. Commun. **248**, 59 (2004).

[11] A. Z. Genack, Phys. Rev. Lett. **58**, 2043 (1987).

[12] A. Z. Genack and J. M. Drake, EPL **11**, 331 (1990).

[13] S. Zhang, B. Hu, P. Sebbah, and A. Z. Genack, Phys. Rev. Lett. **99**, 063902 (2007).

[14] P. Sebbah, B. Hu, A. Z. Genack, R. Pnini, and B. Shapiro, Phys. Rev. Lett. **88**, 123901 (2002).