Proton stopping using a full conserving dielectric function in plasmas at any degeneracy

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In this work, we present a dielectric function including the three conservation laws (density, momentum and energy) when we take into account electron-electron collisions in a plasma at any degeneracy. This full conserving dielectric function (FCDF) reproduces the random phase approximation (RPA) and Mermin ones, which confirms this outcome. The FCDF is applied to the determination of the proton stopping power. Differences among diverse dielectric functions in the proton stopping calculation are minimal if the plasma electron collision frequency is not high enough. These discrepancies can rise up to 2% between RPA values and the FCDF ones, and to 8% between the Mermin ones and FCDF ones. The similarity between RPA and FCDF results is not surprising, as all conservation laws are also considered in RPA dielectric function. Even for plasmas with low collision frequencies, those discrepancies follow the same behavior as for plasmas with higher frequencies. Then, discrepancies do not depend on the plasma degeneracy but essentially do on the value of the plasma collision frequency.

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I. INTRODUCTION

The stopping power of plasmas, when a charged particle traverses it, is a topic of long-standing theoretical and experimental interest. A comprehensive treatment of the stopping has important applications in astrophysics [1,2], solid-state physics [3–6], and energy deposition of a particle driven in the inertial confinement fusion [7,8]. Dielectric formalism has become one of the most used methods to describe this stopping power. The use of this formalism was introduced by Fermi [9]. Subsequent developments made it possible to extend the dielectric formalism to provide a more comprehensive description of the stopping of ions in matter [10,11]. For dilute plasmas, the dielectric formulation of the energy-loss rate was first studied by Pines and Bohm [12], Akhiezer and Sitenko [13], and other scientists. Large number of calculations of the stopping of ions and electrons in plasmas has been carried out since then using the classical linear response function in the random phase approximation (RPA) (see [14] for a complete list). This approximation consists of considering the effect of the particle as a perturbation, so that the energy loss is proportional to the square of the particle charge. Then the theory of slowing down is reduced to a treatment of the properties of the medium only, and a linear description of these properties may be applied. The linear properties of an infinite gas of free electrons can be described by its dielectric function.

Theoretical studies of the RPA dielectric function are usually focused on two main domains of plasma physics. (a) Dense plasmas at low temperatures, usually described with degenerate electron gas models and the use of quantum mechanical methods, as plasmas of interest for inertial confinement fusion (ICF), see [14]. (b) Dilute plasmas at high temperatures, usually described with nondegenerate electron gas models and the use of a classical description; it includes the case of plasmas of interest for magnetic confinement fusion (MCF), see [14]. The transition from nondegenerate to degenerate plasmas in the range of high densities ($n_e \simeq 10^{23}-10^{27}$ $e\text{cm}^{-3}$) is a subject of much interest for current studies of ICF. The approach to those extreme conditions is being tested nowadays using ion beams generated by lasers [15–21]. Then next works have extended the calculations to consider an electron gas of any degeneracy. In the past, Skupsky [22], Arista and Brandt [23], and Maynard and Deutsch [24] have considered the calculation of the energy loss in a quantum mechanical plasma of arbitrary degeneracy but without including plasma electron collisions.

The RPA is usually valid for high-velocity projectiles and when plasma electron-electron collisions are not considered. Nevertheless, for real plasmas, RPA is not sufficient and these collisions must be taken into account. Mermin [25] and later Das [26] derived an expression for the dielectric function caring for the plasma electron collisions and also preserving the local particle density. Mermin dielectric function has been successfully applied to solids (dense degenerate electron gas) [27], for classical plasmas (nondegenerate electron gas) [28,29] and also for partially degenerate plasmas [30]. For solids, Mermin dielectric function was used obtaining the electron collision frequency from experiments [31–33], but this frequency must be calculated *a priori* for plasmas. Many works have been devoted to calculate this frequency [34–36], others treat it as a free parameter [37–40], but in the present investigation this value is taken from a previous work [41].

The aim of this work is to study the influence of plasma electron-electron collisions on the stopping power of a plasma at any degeneracy when protons move trough it via dielectric formalism. For this, the paper is divided into three main sections. In Sec. II, the RPA and Mermin dielectric functions of plasmas at any degeneracy are calculated, but, as it is said before, Mermin function only obeys the density conservation law. Then in Sec. III, a new dielectric function is established where the electron collision events are constrained by all the conservation laws: the full conserving dielectric function. Finally in Sec. IV, we use latter dielectric function to calculate proton stopping power of plasmas at any degeneracy.

II. RPA AND MERMIN DIELECTRIC FUNCTIONS AT ANY DEGENERACY

The RPA dielectric function is developed in terms of the wave number k and of the frequency ω provided by a consistent quantum mechanical analysis. We use atomic units (a.u.), $e=\hbar=m_e=1$, through all the paper.

The RPA analysis yields to the expression [10]

$$\epsilon(k,\omega)=1+\frac{1}{\pi^2k^2}\int d^3k'\,\frac{f(\vec{k}+\vec{k}')-f(\vec{k}')}{\omega+i\upsilon-(E_{\vec{k}+\vec{k}'}-E_{\vec{k}'})}, \qquad (1)$$

where $E_{\vec{k}}=k^2/2$. The temperature dependence is included through the Fermi-Dirac function

$$f(\vec{k})=\frac{1}{1+\exp[\beta(E_k-\mu)]}, \qquad (2)$$

being $\beta=1/k_BT$ and μ the chemical potential of the plasma with electron density n_e and temperature T. In this part of the analysis, we assume the absence of collisions so that the damping constant approaches zero, $\upsilon\to 0$.

Analytic RPA dielectric function (DF) for plasmas at any degeneracy can be obtained directly from Eq. (1) [42]

$$\epsilon_{\mathrm{RPA}}(k,\omega)=1+\frac{1}{4z^3\pi k_{\mathrm{F}}}[g(u+z)-g(u-z)], \qquad (3)$$

where $g(x)$ corresponds to

$$g(x)=\int_0^\infty\frac{ydy}{\exp(E_{\mathrm{F}}\beta y^2-\beta\mu)+1}\ln\left(\frac{x+y}{x-y}\right), \qquad (4)$$

$u=\omega/k\upsilon_{\mathrm{F}}$ and $z=k/2k_{\mathrm{F}}$ are the common dimensionless variables [10]. $D=E_{\mathrm{F}}\beta$ is the degeneracy parameter and $\upsilon_{\mathrm{F}}=k_{\mathrm{F}}=\sqrt{2E_{\mathrm{F}}}$ is Fermi velocity in a.u.

As mentioned in the introduction, the RPA is not sufficient for partially coupled plasmas and the target electron interactions have to be taken into account. The first corrective effect taken to rectify this situation was carried out by Mermin [25], who was able to derive a DF which conserved electron number during collisions

$$\epsilon_{\mathrm{M}}(k,\omega)=1+\frac{(\omega+i\upsilon)[\epsilon_{\mathrm{RPA}}(k,\omega+i\upsilon)-1]}{\omega+i\upsilon[\epsilon_{\mathrm{RPA}}(k,\omega+i\upsilon)-1]/[\epsilon_{\mathrm{RPA}}(k,0)-1]}, \qquad (5)$$

where the RPA dielectric function is taken from Eq. (1). Electron collisions are considered through their collision frequency, υ. It is easy to see that when $\upsilon\to 0$, the Mermin function reproduces the RPA one.

III. FULL CONSERVING DIELECTRIC FUNCTION

Mermin dielectric function violates the two remaining conservation laws, momentum, and energy, thus we need to introduce a model: the one-component system of electrons whereby electrons are only scattered by other electrons. Consequently the dynamics of such scattering events are constrained by all the conservation laws. The one-component model has the additional virtue of allowing us to calculate dynamical local field corrections of the dielectric function arising entirely from electron-electron correlation effects [43]. Here, the expression for the FCDF is obtained by an extension of the relaxation-time approximation [44]

$$\epsilon_{\mathrm{FCDF}}(k,\omega)=1+V(k)\frac{C_0+E}{1+F}, \qquad (6)$$

where

$$E=\left(\frac{C_2}{\omega i/\upsilon-1}\right)\frac{C_2B_0-C_0B_2}{D_4B_0-D_2B_2}$$

and

$$F=\frac{i\upsilon}{\omega+i\upsilon}\left[\frac{D_2C_2-D_4C_0-\frac{i\omega\upsilon C_2}{k^2n_e}(C_2B_0-C_0B_2)}{D_4B_0-D_2B_2}-1\right]+\frac{i\omega\upsilon C_0}{k^2n_e} \qquad (7)$$

are the conserving damping corrections.

B_n is the nth momentum of the integrand of the static Lindhard polarizability function,

$$B_n(k)=\frac{2}{(2\pi)^3}\int d^3p|p|^n\frac{f(\vec{k}+\vec{k}')-f(\vec{k}')}{E_{\vec{k}+\vec{k}'}-E_{\vec{k}'}}$$

and related dynamic functions

$$C_n(k,\omega)=\frac{2}{(2\pi)^3}\int d^3p|p|^n\frac{f(\vec{k}+\vec{k}')-f(\vec{k}')}{\omega+i\upsilon-(E_{\vec{k}+\vec{k}'}-E_{\vec{k}'})}$$

and

$$D_n(k,\omega)=\frac{i\upsilon C_n-\omega B_n}{\omega+i\upsilon}.$$

From the general form of Eq. (6) we can obtain the other models revised in this work. The RPA dielectric function, Eq. (1), corresponds to the choices $E=0$, $F=0$ and $\upsilon\to 0$; when υ is not zero we get the damped RPA one. The Mermin dielectric function, Eq. (5), is retrieved for $E=0$, nonzero υ and

$$F=\frac{-i\upsilon}{\omega+i\upsilon}\left(1+\frac{C_0}{B_0}\right).$$

Finally the FCDF is given by Eq. (6) with nonzero υ.

To check the reliability of our model at any degeneracy, we can calculate the real and imaginary parts of the dielectric functions for a plasma at any degeneracy. For example we choose $T=10$ eV and $n_e=10^{23}$ cm^{-3}, i.e., with degeneracy parameter $D=0.785$, see Fig. 1. Solid lines represent RPA dielectric function from Eq. (3). To include electron-electron collisions in the calculations, we need the exact relaxation frequency, $\upsilon=0.252\ \omega_p$, where $\omega_p=\sqrt{4\pi n_e}$ is the plasma frequency. This value is obtained from [41] regarding only electron-electron collisions. When we consider the electron collisions in the relaxation-time approximation (RTA) dielectric function, the real and imaginary values are damped, but we do not recover the same RPA results in real case in the static limit, $\omega\to 0$. To resolve this issue, we can use the

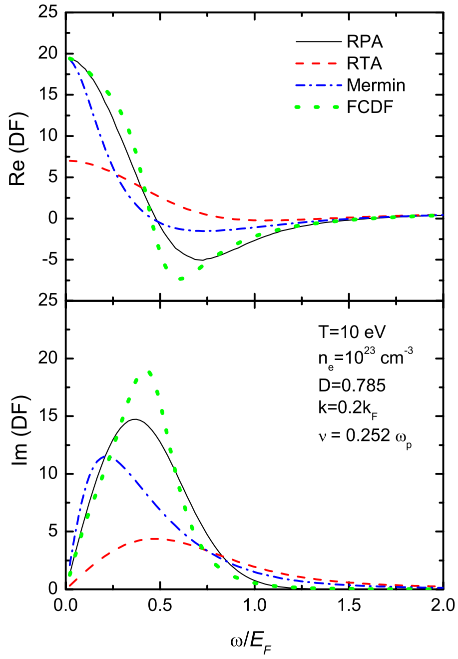

FIG. 1. (Color online) Real and imaginary parts of different dielectric functions (DF) as a function of ω/E_F for a partially degenerate plasma, $T=10$ eV and $n_e=10^{23}$ cm^{-3} ($D=0.785$). The wave vector is $k/k_F=0.2$ and the finite relaxation frequency is $\upsilon=0.252\ \omega_p$.

Mermin DF. In this case the values are less damped but we recover same results as in the RPA case for the static limit. But we know that the Mermin DF only conserves the number density violating the two remaining conservation laws. If we consider three conservation laws, we expect an important variation of all values approaching the RPA values. It is not surprising that as we include more conservation laws the behavior of the DFs resembles more closely to the RPA, a model where all the conservation laws are enforced.

IV. PROTON STOPPING POWER

For proton energy-loss calculations, it is worth defining the energy-loss function (ELF)

$$Im\left(\frac{-1}{\epsilon_x(k,\omega)}\right), \tag{8}$$

where $\epsilon_x(k,\omega)$ is any of the dielectric functions stated before.

Once we have calculated the plasma energy-loss function, which includes its electron-electron collisions, we can estimate the energy loss by a proton that traverses our plasma. This energy loss will be mostly due to proton interaction with the plasma electrons. To calculate this electronic energy loss we use dielectric formalism. In the dielectric formalism, we can determine the energy loss by the electronic stopping, defined as the electronic energy loss per path unit, $S_e=dE/dx$. The formula to calculate the electronic stopping for a pointlike ion with charge Z traveling with constant velocity v through a plasma is very well known [9,10]

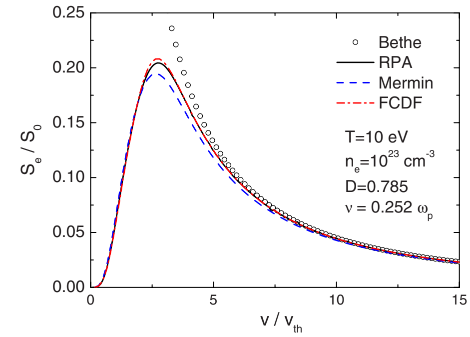

FIG. 2. (Color online) Proton electronic stopping, as a function of its velocity, normalized to $S_0=(Zk_F)^2$. The plasma target is the same as in Fig. 1. Solid line corresponds to the result with RPA DF, dashed line is the one with Mermin DF and dotted line is the one with FCDF. Symbols feature Bethe formula at high velocities.

$$S_e(v)=\frac{2Z^2}{\pi v^2}\int_0^{\infty}\frac{dk}{k}\int_0^{kv}d\omega\omega\, Im\left(\frac{-1}{\epsilon_x(k,\omega)}\right), \tag{9}$$

which depends on the plasma only through its Elf. Then, we can compare the electronic stopping that results from the different DLFC functions for plasmas at any degeneracy.

Next figures represent proton electronic stopping for different plasma degeneracies and different dielectric functions, normalized to $S_0=(Zk_F)^2$, as a function of its velocity, normalized to the plasma electron thermal velocity, $v_{th}=\sqrt{k_BT}$. All stopping calculations are contrasted with Bethe formula at high velocities to check them.

The first case analyzed is a plasma with the same temperature and electronic density values as in Fig. 1, these features correspond to a partially degenerate plasma, $D=0.785$, see Fig. 2. Solid line corresponds to the calculation with the RPA dielectric function, i.e., not considering target electron-electron collisions, Eq. (3). Dashed line is the result considering the electron collisions through the Mermin dielectric function, Eq. (5) and dotted line refers to the result considering the electron collisions through the full conserving dielectric function, Eq. (6). This plasma has a large enough collision frequency, $\upsilon=0.252\ \omega_p$, to discriminate between the various dielectric functions. When target electron collisions are taken into account the stopping values decrease a great deal. Then if we include momentum and energy conservation laws in the dielectric function, FCDF, the result becomes similar, but a bit larger, than in the RPA model, where every conservation law is enforced.

To check the reliability of our model at any degeneracy, we can repeat the calculation of the stopping of the former dielectric functions for other plasma parameters. First, we examine a degenerate plasma with $T=0.056$ eV and $n_e=6\times10^{22}$ cm^{-3}, i.e., with degeneracy parameter $D=99.727$, see Fig. 3. As we see all results look very similar, this is due to a rather small relaxation frequency, $\upsilon=0.039\ \omega_p$. Then, there are no large discrepancies among dielectric functions. But the behavior is the same as in the partially degenerate case; when we care for collisions with the Mermin dielectric function, the stopping values are slightly damped. On the other hand, when momentum and energy conservation laws are

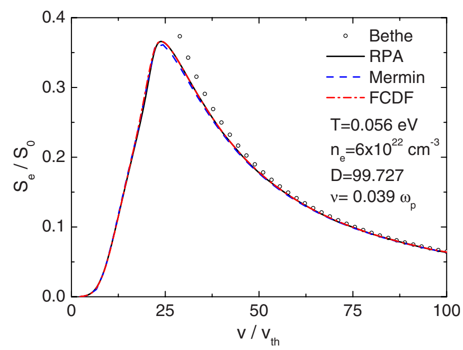

FIG. 3. (Color online) The same as Fig. 2 but for a degenerate plasma, $T=0.056$ eV and $n_e=6\times10^{22}$ cm^{-3} ($D=99.727$). The relaxation frequency is $\upsilon=0.039\ \omega_p$.

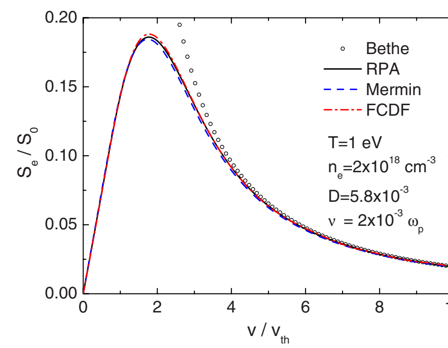

FIG. 4. (Color online) The same as Fig. 2 but for a nondegenerate plasma, $T=1$ eV and $n_e=2\times10^{18}$ cm^{-3} ($D=5.8\times10^{-3}$). The relaxation frequency is $\upsilon=2\times10^{-3}\ \omega_p$.

included in the full conserving dielectric function these values feature the RPA ones as in the partially degenerate case.

Finally, we study the stopping using the same dielectric functions as before but this time for a nondegenerate plasma, see Fig. 4. The plasma parameters are $T=1$ eV and $n_e=2\times10^{18}$ cm^{-3}, with degeneracy parameter $D=5.8\times10^{-3}$. In this case the relaxation frequency is even small than for the degenerate case, $\upsilon=2\times10^{-3}\ \omega_p$, so we expect minimal discrepancies among the calculations with different dielectric functions. Using Mermin dielectric function results in a remote relaxation of the stopping values while using the FCDF results in a similar, or a little bit higher, values than in the RPA case.

V. CONCLUSION

In conclusion, the first thing we can say it is that we have been able to calculate a dielectric function which includes the three conservation laws (density, momentum, and energy) when we take into account plasma electron-electron collisions for plasmas at any degeneracy. This full conserving dielectric function reproduces the former very well known dielectric functions stated in the bibliography, the RPA, and Mermin ones, which confirms our outcome.

Then we have applied this full conserving dielectric function to the determination of the proton stopping power in plasmas at any degeneracy. This estimation has been compared with the same calculation derived from other dielectric functions. Discrepancies in the proton stopping power calculation are not very relevant if the plasma collision frequency is not high enough. We have seen that only in the partially degenerate plasma, $D=0.785$, the collision frequency is sufficiently large to produce important variations in the stopping calculation. These variations are around 10% between RPA values and the Mermin ones, and around by 2% between the RPA ones and FCDF ones at maximum stopping value. It is not surprising that as we consider more conservation laws the behavior of the dielectric functions yields back the RPA, a model with every conservation law enforced. Even though discrepancies for degenerate and nondegenerate cases are not very relevant, they follow the same behavior as for the partially degenerate case. Then we can assert that variations do not depend on the plasma degeneracy; but they essentially rely on value of the plasma collision frequency.

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