Morphology of technological levels in an innovation propagation model

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We study the dynamical properties of the propagation of innovation on a two-dimensional lattice, random network, scale-free network, and Cayley tree. In order to investigate the diversity of technological level, we study the scaling property of width, $W(N,t)$, which represents the root mean square of the technological level of agents. Here, N is the total number of agents. From the numerical simulations, we find that the steady-state value of $W(N,t)$, $W_{sat}(N)$, scales as $W_{sat}(N) \sim N^{-1/2}$ when the system is in a *flat ordered phase* for $d \geq 2$. In the flat ordered phase, most of the agents have the same technological level. On the other hand, when the system is in a *smooth disordered phase*, the value of $W_{sat}(N)$ does not depend on N. These behaviors are completely different from those observed on a one-dimensional (1D) lattice. By considering the effect of the underlying topology on the propagation dynamics for $d \geq 2$, we also provide a mean-field analysis for $W_{sat}(N)$, which agrees very well with the observed behaviors of $W_{sat}(N)$. This directly shows that the morphological properties in order-disorder transition on a 1D lattice is completely different from that on higher dimensions. It also provides an evidence that the upper critical dimension for the roughening transition of the propagation of innovation is $d_u=2$.

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I. INTRODUCTION

In the past few years, there have been many theoretical attempts to understand the complex phenomena in various fields, ranging from physics to biological, social, and economical sciences [1,2]. In particular, concepts and methods developed in nonequilibrium statistical physics have popularly used to explain the observed complex phenomena [3]. Among these studies, the complex behavior of adoption dynamics has been one of the most interesting topics because it is easily found in various systems. This adoption dynamics usually exhibits a dynamical behavior which is similar to the punctuated equilibrium phenomena. The punctuated equilibrium behavior is characterized by the intermittent bursts separating relatively long periods of quiescence [4]. For example, in biological systems, a new phenotype or genotype with high fitness emerges and lurks in the background for a long time. Then, it suddenly spreads over the whole ecosystem. Similar behavior is also observed in socioeconomic systems. In socioeconomic systems, a new technology such as cellular phone is invented and sneaks in the background for a long time. Then, it suddenly explodes into mass use. It also has long been recognized that the invention and spreading of a new opinion or paradigm shows the similar dynamical properties as the technological developments [5,6]. In our study, innovations of technologies are regarded as a broad sense and stand for not only technological devices or tools but also ideas in social systems, phenotype or genotype in biological systems, etc.

Recently, a very simple model for innovation propagation dynamics in a socioeconomic system through the interaction between agents was studied [7–11]. In this simple model, once a new technology appears, the agents should decide whether they adopt it or not. The adoption causes a cost, C, but it also improves the business performance, levels off the life quality of each individual, or leads more robust biological species. In this approach, the technological evolution has two main mechanisms: (1) *innovation*—a new technology with high fitness emerges as a result of invention; (2) *propagation*—under certain conditions, the new technology gets adopted and spreads over the entire system, resulting in an overall technological progress. A single tunable parameter, C, which is fixed and the same for all agents, determines the dynamical properties for the propagation. Earlier studies showed that there exist two different stable phases on a one-dimensional (1D) lattice: an ordered and a disordered phase [9]. When C is less than $C_{th} \simeq 1.0$, the driving process easily produces avalanches. These avalanches lead to an ordered phase in which most of the agents have the same technological level, and the order parameter becomes greater than 0 when $C < C_{th}$. On the other hand, when $C > C_{th}$ there are almost no avalanches and the increases of technological levels are mostly caused by random growth, and each agent has different random technological levels. As a result, the model is in a disordered phase when $C > C_{th}$, where the order parameter becomes 0. The morphological changes of the model on a 1D lattice were also studied by Llas *et al.* [9] using the concepts of the kinetic surface roughening phenomena [12]. From the numerical simulations, they showed that the morphology of the technological level for $C < C_{th}$ becomes smooth; i.e., the steady-state value of the width does not depend on the system size (see Sec. III B for more details). On the other hand, the morphology of the technological level is rough when $C > C_{th}$. Therefore, the model exhibits a transition from a *smooth ordered phase* to a *rough disordered phase* as C increases.

The early studies on the interplay between the underlying topology and the dynamics of the model have been generally focused on the optimal advance rate of technological level [10]. The dynamical property of optimal advance rate was shown to undergo a crossover to a fully connected model (or mean-field model) when the shortcut density becomes larger

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than $\mathcal{O}(1/N)$. Here, N is the number of agents [10]. However, studies on the morphological properties are mostly restricted to only on a 1D lattice. As shown in [9], the concepts of the kinetic surface roughening phenomena provide much richer information on the morphology of the technological level. For example, the observed roughening transition indicates that although the system is in the ordered phase, there is still some diversity among agents on 1D lattice. Moreover, it is now very well known that the structure of interaction between each agent generally produces a complex network and such underlying topology crucially affects the dynamical properties as mentioned in [10]. Thus, it is very important and interesting to understand what is the effect of the underlying topology on the morphological properties of the technological level in an innovation-propagation model. Therefore, in this paper, we study the model of innovation on a two-dimensional (2D) square lattice and two networks, random network and scale-free (SF) network. We also compare the obtained results with those on Cayley tree. From the numerical simulations, we find that there still exists an order-disorder phase transition even for $d \geq 2$. However, we show that the kinetic roughening transition is not observed when $d \geq 2$. This indicates that the resulting morphological properties of the technological level in two dimensions and in networks are completely different from that in one dimension. The observed behavior can be explained by mean-field argument. Combining the results, we conclude that the upper critical dimension for roughening transition is $d_u = 2$. Since the real structure of the interaction between agents involved in innovation propagation is not the same as a 1D lattice structure, understanding the dynamical properties of the innovation propagation in $d \geq 2$ and networks is more important than those in $d = 1$. Our conclusion provides evidence that the difference of technological levels caused by the propagation of innovation is not fluctuation dominant in real world and can be understood by a mean-field argument.

II. MODEL

A. Innovation propagation

Recently, a model for diffusion of technological innovation on 1D lattice was studied [8–11]. In the simplest version of the model, a population of N agents lie at each site (or node) of a 1D lattice with a periodic boundary condition. In our study we use the 2D lattice and networks for the underlying structure. Each agent i is characterized by a real variable h_i which stands for the fitness or the technological level of the agent i. The payoff that an agent receives from possessing a certain technological level is assumed to be proportional to h_i. The technological level evolves by the following dynamical rules. (i) Innovation process—at each time step t, a randomly chosen agent i updates its technological level by

$$h_i \to h_i + \Delta_i, \tag{1}$$

where Δ_i is a random variable. This driving process accounts for the external pressure that may lead to a spontaneous invention of a new technology. (ii) Upgrading process—all agents $j \in \Gamma_i$, Γ_i being the set of nearest neighbors of agent i, upgrade their technological level by imitating i's technological level $(h_j = h_i)$ if $h_i - h_j \geq C$. Here, C is a constant parameter that stands for the cost of an agent j to upgrade their technology as well as their personal resistance to the change. (iii) Avalanche process—if any j has decided to upgrade its level, then let neighbors of j also decide whether to upgrade or not. This procedure is repeated until no one else wants to upgrade and results in an avalanche of imitation events. In the following simulations we use the uniform distribution of Δ_i in the interval $\Delta_i \in [0,1]$. For other distributions of Δ_i, such as the Poisson distribution, our main results are not changed.

B. Underlying networks

For the underlying topologies to represent the interaction between a pair of agents, we consider two types of networks: random network and SF network. We also consider a 2D square lattice. For the construction of random network, we use the Erdös-Rényi (ER) network model. The ER network is simply generated by connecting each pair of nodes with probability p [13]. The degree distribution of ER network is known to satisfy the Poisson distribution which indicates that the degree distribution is homogeneous. In contrast to ER network, SF networks show high heterogeneity in the degree distribution. The degree distribution of the SF network satisfies a power law, $P(k) \sim k^{-\gamma}$. In many systems, such as the Ising model, the critical behaviors are crucially affected by the topological heterogeneity [14]. In order to generate the SF networks with tunable degree exponent, γ, we use the static model suggested by Goh *et al.* [15]. In this model, a weight $w_i = i^{-\alpha}$ is assigned to each node i $(i = 1, 2, \ldots, N)$, where $0 \leq \alpha < 1$. By adding a link between unconnected nodes i and j with probability $w_i w_j / (\sum_{n=1}^{N} w_n)^2$, one can obtain a network whose degree distribution satisfies a power law, $P(k) \sim k^{-\gamma}$, and γ is related to α as $\gamma = (1+\alpha)/\alpha$. Thus, by adjusting α we easily obtain a network with any γ (>2).

III. SIMULATION RESULTS

A. Order-disorder transition

In order to study the properties of phase transition, we use the conventional order parameter defined as

$$M = S_{\max}/N, \tag{2}$$

where $S_{\max}$ is the size of the largest cluster in which all agents have the same h_i and N is the number of nodes (or sites). Figures 1(a)–1(c) show the changes of stationary value of M, M_{stat} against C on each topologies. As shown in Fig. 1, regardless of the underlying topologies, the model shows the order-disorder transition as observed on 1D lattices [9]. However, from the data we find that $C_{th} > 4$ for 2D square lattice, ER networks, and SF networks. The obtained values of C_{th}'s on 2D lattices or on networks are larger than that measured on 1D lattices $C_{th} \simeq 1$ [9].

B. Diversity of technological level

Since the technological level of each agent can be mapped into the height of interface, in order to measure the diversity

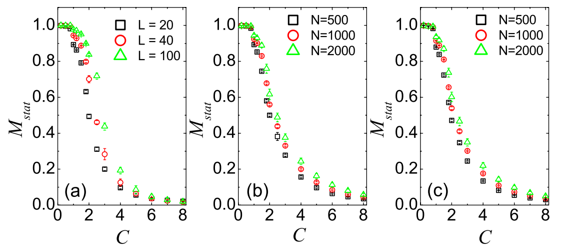

FIG. 1. (Color online) Plot of M_{stat} against C on (a) 2D lattices, (b) ER networks, and (c) SF networks.

of technological level we use the concepts developed in the studies on kinetic surface roughening [12]. The roughness of the interface is generally described by the width, $W(N,t)$, defined as

$$W(N,t) = \left(\frac{1}{N} \sum_{i=1}^{N} [h_i(t) - \bar{h}(t)]^2 \right)^{1/2}. \quad (3)$$

Here, $\bar{h}(t)$ stands for the average technological level at time t,

$$\bar{h}(t) = \frac{1}{N} \sum_{i=1}^{N} h_i(t). \quad (4)$$

When the interface is self-affine, $W(N,t)$ satisfies the finite-size scaling ansatz [16,17]

$$W(N,t) \sim N^{\alpha} f\left(\frac{t}{N^z}\right), \quad (5)$$

where the function $f(x)$ scales as $f(x) \sim x^{\beta}$ for $x \ll 1$ and $f(x) \to \text{const}$, for $x \gg 1$. The dynamic exponent z satisfies the relation $z = \alpha/\beta$.

On 1D lattices, the value of $W(N,t\to\infty) \equiv W_{sat}(N)$ of the technological level is known to increase as N increases when the system is in the rough disordered phase. The diverging behavior of $W_{sat}(N)$ as N increases indicates that the morphology of technological level for $C > C_{th}$ is rough. On the other hand, $W_{sat}(N)$ for $C < C_{th}$ goes to a constant value which does not depend on N when the system is in the smooth ordered phase [9]. The N independent value of W_{sat} is a generic feature of smooth morphology in roughening transition. Thus, the transition observed in 1D lattices is known to be related to the morphological changes from the smooth ordered phase to the rough disordered phase as C increases [12].

Figures 2(a)–2(c) shows the measured $W(N,t)$ on 2D square lattices, ER networks, and SF networks when the system is in the ordered phase. For comparison we also display $W(N,t)$ measured on Cayley tree in Fig. 2(d) when $C < C_{th}$. In contrast to the smooth ordered phase in 1D systems, $W(N,t)$ decreases as N increases on 2D square lattice, ER network, SF network, and Cayley tree. The data in Figs. 2(e)–2(h) show the behavior of $W_{sat}(N)$ obtained from the

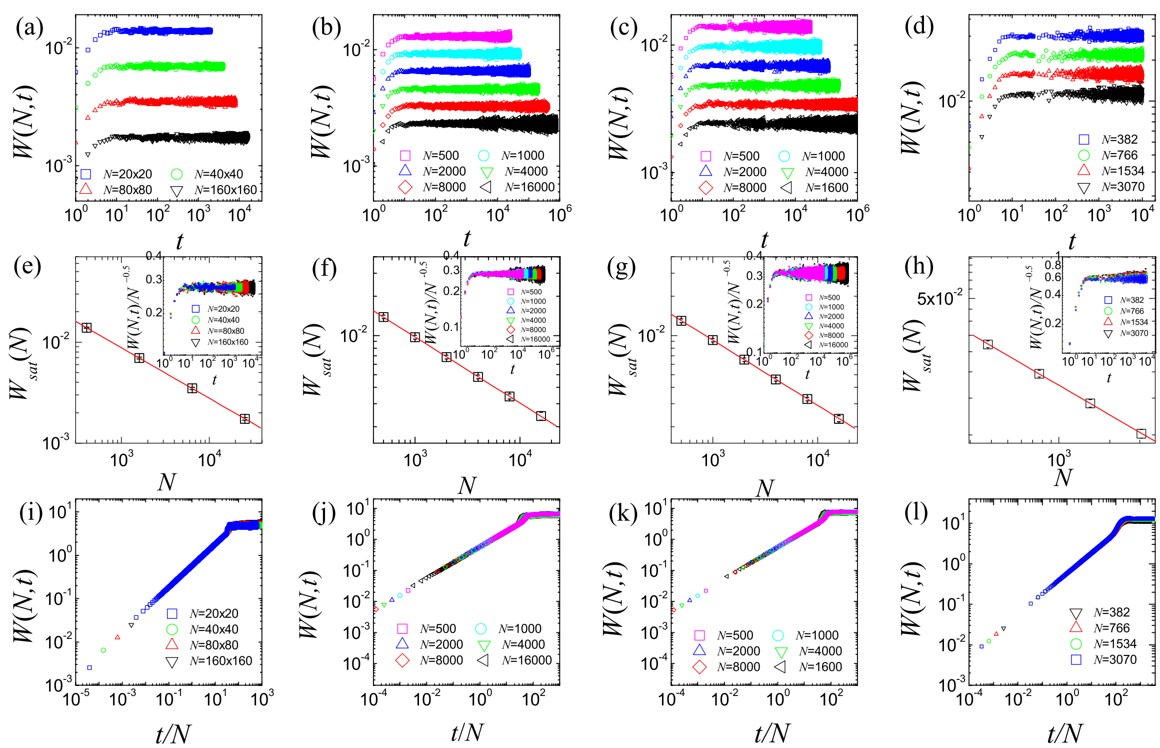

FIG. 2. (Color online) Plot of $W(N,t)$ for various N on (a) 2D square lattices, (b) ER networks, (c) SF networks with γ=2.7, and (d) Cayley tree when C=0.5. Plot of $W_{sat}(N) \equiv W(N,t\to\infty)$ against N on (e) 2D square lattice, (f) ER networks, (g) SF networks with γ=2.7, and (h) Cayley tree when C=0.5. The solid line represents the relation $W(N,t\to\infty) \sim N^{\alpha}$ with α=−0.5. Insets in (e)–(h): scaling plot of $W(N,t)$ measured on each underlying structure in order to show the scaling behavior $W(N,t) = N^{-0.5} f(t)$. Plot of $W(N,t)$ against t/N on (i) 2D square lattice when C=18, (j) ER networks when C=18, (k) SF networks with γ=2.7, and (l) Cayley tree when C=20. The data in (i)–(l) clearly show that $W(N,t)$'s for $C > C_{th}$ scale as $W(N,t) = f(t/N)$.

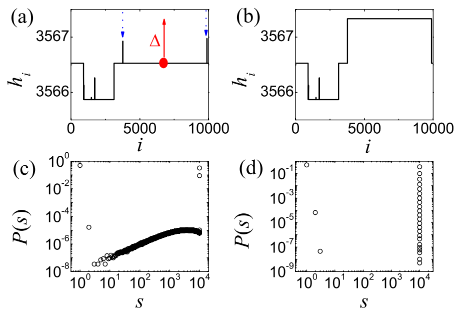

FIG. 3. (Color online) (a) Morphology of the 2D innovation model before the update ($N=1000$). The (red) dot represents a randomly selected site. The sites located under the dashed arrows play a role of barrier. (b) Morphology after the growth of selected site and avalanche when the system is in the smooth ordered phase. (c) Plot of $P(s)$ on 1D lattice for $N=10\,000$ when the system is in the smooth ordered phase ($C=0.5$). (d) Plot of $P(s)$ measured on the 2D square lattice when the system is in the flat ordered phase ($C=0.5$) for $N=10\,000$.

data in Figs. 2(a)–2(d). Using the least-squares fit of the data to Eq. (5), we obtain $\alpha=-0.50\pm0.01$ for 2D square lattices, $\alpha=-0.49\pm0.01$ for ER networks, $\alpha=-0.50\pm0.02$ for SF networks, and $\alpha=0.48\pm0.02$ for Cayley trees. In the ordered phase on 2D lattices and complex networks, $W(N,t)$ scales as

$$W(N,t) = N^{\alpha}f(t) \quad (\alpha = -0.5 < 0) \tag{6}$$

instead of scaling ansatz (5) with $\alpha>0$ [see also the insets in Figs. 2(e)–2(h)]. The negative value of α indicates that the difference of the technological levels between agents decreases and the morphology becomes (completely) flat in the limit $N\to\infty$. Thus, the ordered phase on a 2D lattice, networks, or Cayley tree is the *flat ordered phase*. If the system is in the disordered phase, then $W(N,t)$ saturates to a constant value when $t\to\infty$ [see Figs. 2(i)–2(l)] and satisfies the scaling $W(N,t)=f(t/N)$. Thus, in contrast to the results in 1D lattices, the morphological transition observed in Fig. 2 undergoes from the flat ordered phase to the *smooth disordered phase* on $d=2$ lattice, networks, and Cayley tree.

C. Origin of flattening

The behavior of $W(N,t)$ for $d\geq2$ and networks can be qualitatively understood by the comparison of morphology and avalanche size distribution. The avalanche size, s, is defined by the number of total updated nodes during an avalanche process. Figure 3(a) shows a morphology before the update of the selected site (red dot) on the 1D lattice. When the site is selected to increase the technological level by Δ, the successive increases of the technological level of neighboring sites occur until the avalanche reaches the sites j's marked by the dashed arrows. The technological levels of the sites j's do not satisfy the condition $h_j-h_{j\pm1}>C$. Thus, the avalanche stops at the site j [see Fig. 3(b)]. As a result, the sites marked by the dashed arrow play a role of barrier for the spreading of new technology on one-dimensional structure, which causes many avalanches of moderate sizes as shown in Fig. 3(c). On the other hand, for 2D square lattices, there exist many routes to bypass such barriers. Therefore, as shown in Fig. 3(d), the probability of small avalanches rapidly decreases and most of the avalanches have the size of the systems for $d\geq2$, which causes the flat ordered phase in $d\geq2$ instead of the smooth ordered phase in $d=1$ when $C<C_{th}$. Nearly the same mechanism causes the smooth disordered phase in $d\geq2$ instead of the rough disordered phase in $d=1$ when $C>C_{th}$.

D. Mean-field derivation of $W_{sat}(N)$ for $d\geq2$

Since the diameter of random networks and SF networks scales as $\ln N$ [18], the random networks and SF networks are generally regarded as an infinite-dimensional object [19]. By combining the results obtained from 2D square lattices, ER networks, SF networks, and Cayley trees, we conclude that the upper critical dimension of the innovation propagation model is 2 (i.e., $d_u=2$). Moreover, the observed behavior of $W_{sat}(N)$ for $d\geq2$ can be explained by the mean-field-like argument based on the results in Figs. 1 and 3. When the system is in the flat ordered phase, the existence of detouring paths for $d\geq2$ and the systemwide avalanches leads $M\to1$. In order to satisfy $M\to1$ in the limit $N\to\infty$, most of the sites have the same technological level $h(t)$ at time t when the system is in a steady state. Only a small finite number of nodes have different technological levels (see Fig. 1). For this case, let m be the average number of agents who have heights $h(t)+\bar{\Delta}$ due to the spontaneous innovation, and the technological levels of other $(N-m)$ nodes be $h(t)$. Here, $\bar{\Delta}$ represents the mean additional height of m nodes. With simple algebra we obtain

$$\bar{h}(t) = h(t) + \frac{m}{N}\bar{\Delta}, \tag{7}$$

and $W_{sat}(N)$ becomes

$$W_{sat}(N) = \bar{\Delta}\left(\frac{m}{N}\right)^{1/2} + O\left[\left(\frac{m}{N}\right)^{2/3}\right]. \tag{8}$$

Therefore, for large N, $W_{sat}(N)$ scales as $W_{sat}(N)\sim N^{-1/2}$ in the flat ordered phase, which agrees with the results shown in Figs. 2(e)–2(h). On the other hand, when $C>C_{th}$, each node has a different level of technology to satisfy $M\to0$. Thus, the fluctuation of $h_i(t)$ becomes relatively larger than that for $C<C_{th}$ but still bounded by C. In this case, on the average $h_i-\bar{h}$ becomes order of $\bar{\Delta}$ for all i. Thus, when the system is in the smooth disordered phase, we obtain

$$W_{sat}(N) \approx A\bar{\Delta}, \tag{9}$$

where A is a constant. Equation (9) agrees very well with the results shown in Figs. 2(i)–2(l).

IV. SUMMARY AND DISCUSSION

In summary we investigate the dynamical properties of the innovation propagation model for $d\geq2$. From the measurement of M we show that the model for the propagation of innovation shows an order-disorder transition like on 1D lattices. However, by measuring $W(N,t)$ we find that there exists the transition from the flat ordered phase to the smooth disordered phase for $d \geq 2$ due to the existence of many detouring paths. More specifically, for $d \geq 2$ we find that $W_{sat}(N)$ scales as $W_{sat}(N) \sim N^{-1/2}$ when $C < C_{th}$ and $W_{sat}(N)$ saturates to a constant value which does not depend on N. By combining the measurement of M and the underlying topologies, we provide a mean-field analysis for $W_{sat}(N)$ on higher dimensions and conclude that the upper critical dimension of the roughening transition of the innovation model is $d_u = 2$. This result indicates that the morphological transition in the model depends only on the dimension of interaction topology. Since the interaction topology of real world is known to be complex networks, which is generally regarded as infinite-dimensional topology, the propagation of innovation in real world is expected to follow the mean-field dynamics. In addition, note that the qualitative behavior of M in $d \geq 2$ is not different from that on 1D lattices. However, the measurement of $W(N,t)$ can distinguish the morphological differences between $d=1$ and $d \geq 2$. Therefore, $W(N,t)$ is more proper quantity than M to study the diversity of the technological level in real world.

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