Thermal-inertial ratchet effects: Negative mobility, resonant activation, noise-enhanced stability, and noise-weakened stability

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Transport properties of a Brownian particle in thermal-inertial ratchets subject to an external time-oscillatory drive and a constant bias force are investigated. Since the phenomena of negative mobility, resonant activation and noise-enhance stability were reported before, in the present paper, we report some additional aspects of negative mobility, resonant activation and noise-enhance stability, such as the ingredients for the appearances of these phenomena, multiple resonant activation peaks, current reversals, noise-weakened stability, and so on.

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I. INTRODUCTION

Recently, there has been a growing interest in the study of the transport in the *asymmetric* spatially periodic potentials driven by *zero-mean forces* [1]. This phenomenon is often called "ratchet effect." Now the ratchet effect has been investigated in the context of molecular motors [2,3], colloidal matter [4], transport of atoms in optical traps [5], granular matter [6], electron transport in asymmetric geometries [7], vortex transport and manipulation in type-II superconductors [8], and transport in Josephson junctions [9,10].

In the present paper, we will investigate a thermal-inertial ratchet system subject to an external time-oscillatory drive of zero mean and a constant bias force. Here the word "thermal" means that we consider the internal thermal fluctuation in the system, which is modeled by a Gaussian white noise whose strength is proportional to the temperature of the system, and the word "inertial" means that the particle is underdamped. We will report some additional aspects of negative mobility, resonant activation, and noise-enhance stability, such as the ingredients for the appearances of these phenomena, multiple resonant activation peaks, current reversals, noise-weakened stability, and so on.

II. MODEL AND AVERAGED VELOCITY

The system is modeled by a Langevin equation in the dimensionless form

$$\ddot{x} + \gamma\dot{x} = -J_1 \sin(\pi x) - J_2 \sin[2\pi(x + 1/4)] + a\cos(\omega t) + F + \xi(t), \tag{1}$$

where γ is the friction coefficient, J_1 and J_2 are two positive constants, F is the constant bias force (which can be negative, zero or positive), a and ω are the amplitude and frequency of the time-oscillatory drive $a\cos(\omega t)$, respectively, and $\xi(t)$ is a thermal Gaussian white noise with zero mean and correlation function $\langle\xi(t)\xi(t')\rangle = 2D\delta(t-t')$ with D depending linearly on the temperature. The detailed transformations of the position, the time and the other parameters to the dimensionless forms and vice versa have been presented in Ref. [11].

Equation (1) can provide a mathematical background underlying a great number of applications, such as mobility in super-ionic conductors, net voltage in systems consisting of Josephson junctions, and dynamics of charge-density waves, in ring laser gyroscopes and for phase-locking phenomena. In addition, the ratchet potential as in Eq. (1) has been a subject of recent extensive studies [12].

The averaged velocity $\langle v\rangle$ is obtained for the averages of the velocity v over a large number of the various initial conditions (for every initial condition, we use a different realization of thermal noise) and over long enough times well after the transient process of $x(t)$. Since the movement determined by Eq. (1) may be chaotic noisy [13], the time evolution can be very sensitive to the initial conditions (which is one of the characteristic features for the appearance of chaos [14]). Thus, the number of initial conditions, over which the average will be made, must be large enough, so that the averaged velocity is independent of the number of initial conditions selected by us.

III. NEGATIVE MOBILITY

The phenomenon of negative (differential) mobility is well-known in many branches of physics [15,16]. It appears when the velocity v depends on the constant force F in the form $\mu = dv/dF < 0$ where μ is the mobility coefficient. In this section, we try to find the phenomenon of negative mobility of the averaged velocity.

In Fig. 1, we depict the dependence of the averaged velocity $\langle v\rangle$ of the Brownian particle versus the constant force F for some selected values of the noise strength D (D $=0.001$, 0.003, 0.007, 0.02, and 0.05, respectively) and with the other parameters $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $\omega=5$, and $a=5$. In this figure, every point is calculated by taking the average of $M=2000$ initial conditions (for every initial condition, we use a different Gaussian white noise) and $N=10^6$ different discrete times (the time step is taken as $\Delta t=0.001$). Figure 1 can show a clear emergence of the negative mobility (NM) phenomenon of the averaged velocity versus the constant bias force (see the negative slopes). In the absence of the thermal noise (i.e., $D=0$), for our ratchet system, there is still the NM phenomenon to appear (see Fig. 2). In Fig. 2 (the insert figure is the enlargement for the corresponding part in Fig. 2), we depict the averaged velocity as a function of the constant force with $D=0$ (the other parameters are same as

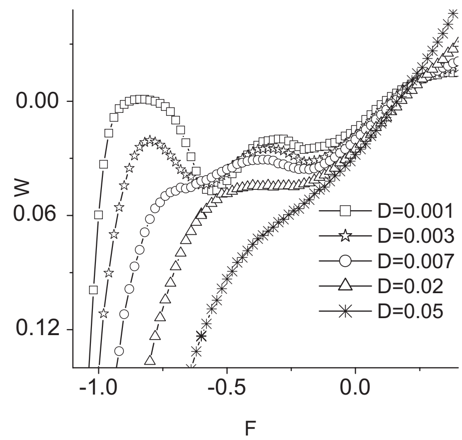

FIG. 1. $\langle v \rangle$ versus F for different values of the noise strength ($D=0.001$, 0.003, 0.007, 0.02, and 0.05, respectively) with the other parameters $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $\omega=5$, and $a=5$.

the ones in Fig. 1). From Figs. 1 and 2, we can notice that the NM phenomenon can emerge in the case of weak thermal noise and in the absence of the thermal noise. We have calculated a great number of cases for the averaged velocity versus F with the values of D varying from $D=0$ to $D=0.5$ (such as $D=0$, $D=0.0003$, $D=0.0007$, $D=0.001$, $D=0.002$, $D=0.003$, $D=0.004$, $D=0.005$, $D=0.006$, $D=0.007$, $D=0.009$, $D=0.01$, $D=0.013$, $D=0.016$, $D=0.02$, $D=0.025$, $D=0.03,\ldots$) and the same other parameters as in Fig. 1. We find that it is certain that the NM phenomenon only appears in the weak thermal noise (for the same other parameters as in Fig. 1, the region of D for the appearance of NM is about from $D=0$ to $D\approx 0.02$). Since the strength of the thermal Gaussian white noise is proportional to the temperature of the system, we can conclude that, only in the lower temperature does the NM phenomenon appear for our ratchet systems; if the temperature is not low enough, we cannot get the NM phenomenon for our ratchet system. In addition, we know that, in the absence of the time-oscillatory drive or under the overdamped case, no NM phenomenon emerges, see Refs. [17,18]. So, now the inertia of the particle, the time-oscillatory drive, and the constant bias force (since the negative mobility refers to the averaged velocity having negative slopes as a function of the constant bias force) are the ingredients for the appearance of the NM phenomenon. Now, the nonlinear cooperation of the mass of the particle, the external time-oscillatory drive and the constant bias force results in the emergence of the NM phenomenon for the averaged velocity.

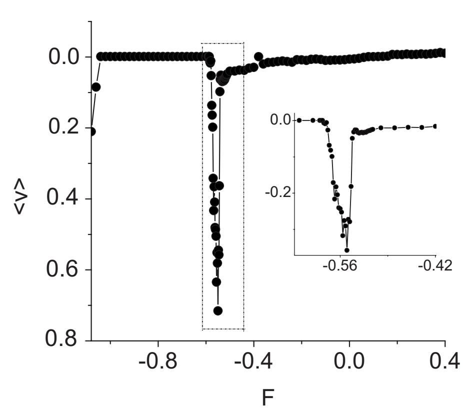

FIG. 2. $\langle v \rangle$ versus F for $D=0$ with the same other parameters as in Fig. 1. The insert figure: the enlargement for the corresponding part in this figure.

In a recent paper [19], the effect of the correlation time of a colored noise on the appearance of NM of the averaged velocity was considered for a spatially periodic *symmetric* system. It remains to be investigated how this colored noise affects the appearance of NM of the averaged velocity for the ratchet system (i.e., the spatially periodic *asymmetric* system).

IV. RESONANT ACTIVATION OF AVERAGED VELOCITY

Resonant activation (RA) is a phenomenon for which the mean first passage time for the particle to escape over the fluctuating potential barrier can exhibit a minimum as a function of the flipping rate of the fluctuating potential barrier or as a function of the frequency of the time-oscillatory drive [20–22]. Since the RA phenomenon was first identified by Doering and Gadoua [20], this phenomenon has been further studied by a great number of other authors [21,22]. Now, the RA phenomenon has been reported to appear in chemistry, biology, physics, and other science theoretically [21] and experimentally [22].

All of the above work [20–22] for the RA phenomenon has been focused on the mean first passage time for the particle to escape over the fluctuating potential barrier. In Ref. [23], Gommers *et al.* investigated the averaged velocity of the atoms in a nonadiabatically driven optical lattice system subject to a phase modulation of the lattice beams. This modulation is an external time-oscillatory drive as the one in Eq. (1). They theoretically and experimentally found the RA phenomenon of the averaged velocity of the atoms as a function of the frequency of the modulation. Similarly, here what we are interested in is the RA phenomenon for the averaged velocity of the particles of the thermal-inertial ratchet system whose stochastic differential equation is Eq. (1) as a function of the frequency of the external time-oscillatory drive.

Through the same numerical simulations as in the Sec. III, we have calculated the averaged velocity as a function of the frequency of the time-oscillatory drive for different values of the noise strength. Some results of our numerical simulations are depicted in Fig. 3 with $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $a=5$, and $F=0.4$ for different values of the noise strength ($D=0.003$, 0.01, 0.02, and 0.03, respectively). This figure shows a clear appearance of resonant activation of the averaged velocity (or current of the particles) as a function of the ln (natural logarithm) of the frequency of the time-oscillatory drive. This figure also represents that when the noise strength is increased, the range of resonant activation can become wider and wider but its main peak can become lower and lower and the second peak can become higher and higher. Here, it should be mentioned that in the absence of the noise, there is still resonant activation of the flux (or average velocity) to appear, which is caused by the interplay between the oscillatory drive and the constant bias force (see the insert figure in Fig. 3).

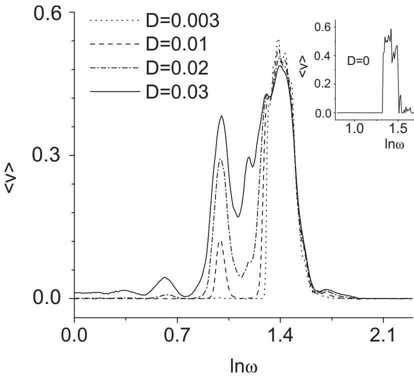

FIG. 3. $\langle v\rangle$ versus $\ln \omega$ with $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $a=5$ and $F=0.4$ for different values of the noise strength ($D=0.003$, 0.01, 0.02, and 0.03 respectively). The insert figure: $\langle v\rangle$ versus $\ln \omega$ for the case of $D=0$ with the same other parameters as in this figure.

Moreover, by the calculation of the averaged velocity as a function of the driving frequency for different values of the constant bias force, we found that resonant activation also dramatically depends on the constant bias force. In Fig. 4, for different values of the constant bias force ($F=0.4$, 0.3, 0.2, -0.2, -0.4, and -0.6, respectively), we plot the curves for the averaged velocity as a function of the ln of the driving frequency, with the other parameters $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $a=5$, and $D=0.001$. This figure clearly shows that the appearance of resonant activation relies on the values of the constant force dramatically. As the noise's effect on resonant activation of the averaged velocity (or current), in the absence of the constant force (i.e., $F=0$), there is still resonant activation of the current to emerge, which is induced by the interplay between the oscillatory drive and the noise. This case is represented in the insert figure of Fig. 4. In addition, from the insert figure in Fig. 4, we can observe that, by controlling the drive frequency, the phenomenon of *multiple reversals* of current [24] can occur.

Here, we can conclude that the ingredients for the appearance of resonant activation of the averaged velocity are the time-oscillatory drive (since RA in the present paper refers to

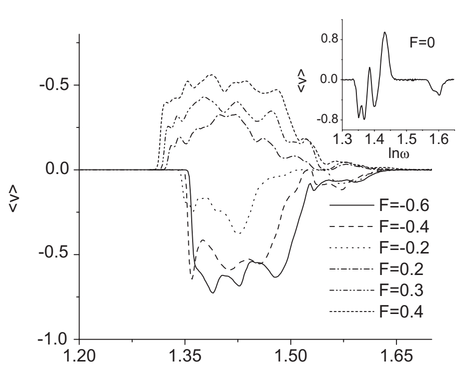

FIG. 4. $\langle v\rangle$ versus $\ln \omega$ for different values of the constant bias force ($F=0.4$, 0.3, 0.2, -0.2, -0.4 and -0.6, respectively), with the other parameters $J_1=\pi$, $J_2=2\pi$, $\gamma=1$, $a=5$, and $D=0.001$. The insert figure: $\langle v\rangle$ versus $\ln \omega$ for the case of $F=0$ with the same other parameters as in this figure.

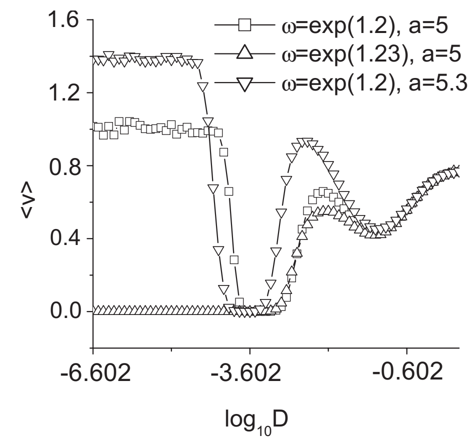

FIG. 5. $\langle v\rangle$ versus $\log_{10} D$ for $\omega=\exp(1.2)$ and $a=5$, $\omega=\exp(1.23)$ and $a=5$, and $\omega=\exp(1.2)$ and $a=5.3$, respectively, with the other parameters $J_1=3.14159$, $J_2=2\times 3.14159$, $\gamma=1$, and $F=0.4$.

the averaged velocity having absolute peaks versus the oscillatory drive frequency), the thermal noise (see Fig. 3), and the constant bias force (see Fig. 4). Now the inertia of the particle is not the necessary ingredient for the emergence of resonant activation. When the inertial term in Eq. (1) is absent (i.e., the particle is overdamped), there is still resonant activation of the averaged velocity to appear (see Ref. [25]).

We noted that in Ref. [26], Luchinsky *et al.* presented a result (i.e., Fig. 4(a) in Ref. [26]) for the averaged velocity as a function of the drive frequency, which could show a peak. However, they studied a system without the constant bias force.

V. Noise-enhanced stability and noise-weakened stability of averaged velocity

The phenomenon of noise-enhanced stability (NES) was first found numerically by Dayan *et al.* [27] and observed experimentally by Mantegna and Spagnolo [28]. For this phenomenon, the mean first passage time (MFPT) for the particle to escape over the fluctuating potential barrier can be prolonged by the increase of the noise strength. Now the NES phenomenon has been investigated extensively [29].

By using the same numerical simulations as in the Sec. III and Sec. IV, we have calculated the averaged velocity as a function of the noise strength for some selected values of the frequency and the amplitude of the time-oscillatory drive. Some results are plotted in Fig. 5 for the averaged velocity versus the $\log_{10}$ (common logarithm) of the noise strength for $\omega=\exp(1.2)$ and $a=5$, $\omega=\exp(1.23)$ and $a=5$, and $\omega=\exp(1.2)$ and $a=5.3$, respectively, with the other parameters $J_1=3.14159$, $J_2=2\times 3.14159$, $\gamma=1$, and $F=0.4$. From this figure, we can see that the averaged velocity (or current) has a minimum as a function of the noise strength. In the vicinity of the left-hand side of the minimum, with increasing the noise strength, the averaged velocity can be slowed down, which just corresponds to the NES phenomenon that the MFPT over the fluctuating potential barrier can be prolonged with the increase of the noise strength. But, in the vicinity of the right-hand side of the minimum, with the increase of the noise strength, the averaged velocity can be speeded up. Here, we call the phenomenon that increasing the noise strength leads to the slowing-down of the averaged velocity appearing in Fig. 5 "*NES of averaged velocity*," and the one that the increase of the noise strength leads to the speeding-up of the averaged velocity "*noise weakened stability (NWS) of averaged velocity*."

If the system is overdamped [i.e., the inertial term in Eq. (1) is neglected], the NES and NWS phenomena also appear for the averaged velocity (see the negative slopes in Fig. 2 of Ref. [25]). In addition, it is clear that, in the absence of the noise, no NES and NWS phenomena for the averaged velocity emerge (since now the noise strength is zero and the NES and NWS phenomena for the averaged velocity refer to the averaged velocity having a minimum as a function of the noise strength); and the NES and NWS phenomena for the averaged velocity only occur for some certain values of the frequency and the amplitude of the time-periodic drive (see Fig. 5). Thus, the thermal fluctuation and the time-oscillatory drive are the ingredients for the appearance of the NES and NWS phenomena of the averaged velocity for our ratchets model. The NES and NWS phenomena are similar to the phenomena that the averaged velocity can show the NM phenomenon (see Sec. III) and that the averaged velocity can represent the RA phenomenon (see Sect. IV). Here, these phenomena are all the results of the nonlinear dependence of the averaged velocity on the external (or internal) forces parameters, such as the values of the constant bias force, the driving frequency, the noise strength, and so on. Moreover, in Refs. [25,30], we presented some figures (i.e., Fig. 4 in Ref. [25] and Fig. 3(a) in Ref. [30]), which could show a minimum for the averaged velocity versus the noise strength. However, these systems studied in [25,30] are different from the one in the present paper. In Refs. [25,30], the potential of the systems are symmetric (i.e., not ratchets); while in the present paper, the potential of the system is asymmetric (i.e., ratchets).

In conclusion, we have reported some additional aspects of negative mobility, resonant activation and noise-enhance stability, such as the ingredients for the appearances of these phenomena, multiple resonant activation peaks, current reversals, noise-weakened stability, and so on, for the averaged velocity of a thermal-inertial ratchets system with a periodic signal and a constant bias force in the underdamped case. It remains to be studied whether our results exit in the other thermal-inertial ratchet systems.

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