Magnetoelectric birefringence as a unique effect in isotropic media

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Aspects of magnetoelectric birefringence phenomena are investigated in the context of electromagnetic wave propagation in isotropic nonlinear media in the eikonal approximation. It is shown that these phenomena can be produced as a unique effect in isotropic systems in the presence of external electric and magnetic fields, provided specific dielectric properties are present.

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I. INTRODUCTION

In nonlinear media electromagnetic wave propagation is governed by nonlinear equations [1]. Nonlinearities are introduced by means of constitutive relations linking external and induced fields. In isotropic media these relations can be presented as $\vec{D}=\varepsilon(E,B)\vec{E}$ and $\vec{H}=\mu^{-1}(E,B)\vec{B}$ where ε and μ describe the permittivity and the permeability of the medium, respectively. Several effects concerning light propagation can appear in such media depending on its specific dielectric properties and the applied external fields. Of particular interest in this paper is birefringence (or double refraction), which appears when the speed of the wave takes a distinct value for each propagating mode [2] in a given direction. These distinct modes correspond to ordinary and extraordinary rays. The former propagates isotropically while the latter depends on direction. Birefringence is found not only in nonlinear media but also in the context of nonlinear electrodynamics, as is predicted to occur in quantum electrodynamics [3,4]. This effect is used widely in optical devices technology [5], as well as a technique for investigating properties of several physical systems, including astrophysical systems [6,7].

Magnetoelectric birefringence is a birefringence effect whose difference between the refractive indexes associated with the propagation of ordinary and extraordinary rays is proportional to the product of the applied electric and magnetic fields. It was long ago reported in the literature [8–11] but it was only recently measured [12,13]. One of the difficulties reported [12] in its measurement is the presence of other standard birefringence, as Kerr [14] and Cotton-Mouton [15] effects, whose magnitude is usually far greater than the magnetoelectric one. Magnetoelectric birefringence is also predicted to occur in nonlinear electrodynamics in the context of quantum electrodynamics [16]. A theoretical description of magnetoelectric birefringence in isotropic media was recently considered [17] in the context of the eikonal approximation, where the possibility to produce it as a unique effect in isotropic nonlinear media was suggested.

In this paper monochromatic waves of circular frequency ω and propagation vector $\vec{q}$ incident on an isotropic nonlinear medium are considered in the regime of the eikonal approximation. In such a medium, the electromagnetic fields are governed by nonlinear equations. Dispersive effects are neglected and the speed of the waves is defined by ω/q. We examine two circumstances where magnetoelectric birefringence as a unique effect could be implemented and eventually measured. These two circumstances occur when light is propagating in a nonlinear medium with dielectric coefficients satisfying $\varepsilon=\varepsilon(B)$ and $\mu=\mu_c$ or, symmetrically, $\varepsilon=\varepsilon_c$ and $\mu=\mu(E)$. In both cases the presence of electric and magnetic external fields (those not associated with the incident wave) are fundamental. With only one externally applied field the effect disappears.

In the next section, the eigenvalue problem is stated in terms of the dielectric coefficients. In Sec. III, two cases where magnetoelectric birefringence appears as a unique effect in isotropic nonlinear media are shown. Final remarks are presented in Sec. IV. In all sections the units are such that $c=1$.

II. FRESNEL EQUATION

In the absence of sources and currents, the electrodynamics in a continuum medium at rest is completely determined by the Maxwell equations

$$\vec{\nabla}\cdot\vec{D}=0, \tag{1}$$

$$\vec{\nabla}\cdot\vec{B}=0, \tag{2}$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{3}$$

$$\vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}, \tag{4}$$

together with the constitutive relations $\vec{D}=\vec{\vec{\varepsilon}}\cdot\vec{E}$ and $\vec{H}=\vec{\vec{\mu}}\cdot\vec{B}$. The dielectric tensors $\vec{\vec{\varepsilon}}$ and $\vec{\vec{\mu}}$ are usually known as the permittivity and permeability tensors, respectively, and they encompass all information about the electromagnetic properties of the medium.

For the cases of our interest the constitutive relations will be simply $\vec{D}=\varepsilon(B)\vec{E}$ and $\vec{H}=\mu^{-1}(E)\vec{B}$. Particularly, the cases

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with $\varepsilon(B)$ and constant μ or $\mu(E)$ and constant ε will be considered separately in the next section.

In order to describe the propagation of the electromagnetic waves within the eikonal approximation [2] the method of field disturbances [1,18] is here employed. Let Σ, defined by $\mathcal{Z}(t,\vec{x})=0$, be a smooth (differentiable of class $\mathcal{C}^n, n>2$) hypersurface. The function $\mathcal{Z}$ is understood to be a real-valued smooth function of the coordinates $(t,\vec{x})$ and regular in the neighborhood U of Σ. The spacetime is divided by Σ in two disjoint regions U^-, for $\mathcal{Z}(t,\vec{x})<0$, and U^+, for $\mathcal{Z}(t,\vec{x})>0$. The discontinuity of an arbitrary function $f(t,\vec{x})$ (supposed to be a smooth function in the interior of $U^{\pm}$) on Σ is a smooth function in U, and is given by

$$[f(t,\vec{x})]_\Sigma \doteq \lim_{\{P^{\pm}\}\to P} [f(P^+)-f(P^-)], \tag{5}$$

with P^+,P^- and P belonging to U^+,U^- and Σ, respectively. The electromagnetic fields are smooth functions in the interior of U^+ and U^- and continuous across Σ. However they have a discontinuity in their first derivatives, behaving as [18]

$$[\vec{E}]_\Sigma = 0, \quad [\vec{B}]_\Sigma = 0, \tag{6}$$

$$[\partial_t \vec{E}]_\Sigma = \omega \vec{e}, \quad [\partial_t \vec{B}]_\Sigma = \omega \vec{b}, \tag{7}$$

$$[\vec{\nabla}\cdot\vec{E}]_\Sigma = \vec{q}\cdot\vec{e}, \quad [\vec{\nabla}\cdot\vec{B}]_\Sigma = \vec{q}\cdot\vec{b}, \tag{8}$$

$$[\vec{\nabla}\times\vec{E}]_\Sigma = \vec{q}\times\vec{e}, \quad [\vec{\nabla}\times\vec{B}]_\Sigma = \vec{q}\times\vec{b}, \tag{9}$$

where $\vec{e}$ and $\vec{b}$ are related to the derivatives of the electric and magnetic fields on Σ as $\vec{e}\doteq[\partial\vec{E}/\partial\mathcal{Z}]_\Sigma$ and $\vec{b}\doteq[\partial\vec{B}/\partial\mathcal{Z}]_\Sigma$, and they correspond to the polarization vectors of the propagating waves [19].

Applying these boundary conditions to the Maxwell equations we obtain the eigenvalue equation [20]

$$\sum_{j=1}^{3} Z_{ij} e_j = 0, \tag{10}$$

where the Fresnel matrix Z_{ij} is given by

$$Z_{ij} = \left(\varepsilon - \frac{q^2}{\mu\omega^2}\right)\delta_{ij} + \frac{1}{\mu\omega^2} q_i q_j - \frac{\dot{\varepsilon}}{\omega}(\vec{q}\times\vec{B})_j E_i - \frac{\mu'}{\omega\mu^2}(\vec{q}\times\vec{B})_i E_j \tag{11}$$

with $\dot{\varepsilon}\doteq(1/B)\partial\varepsilon/\partial B$ and $\mu'\doteq(1/E)\partial\mu/\partial E$. Further, for any vector $\vec{y}$ we denote its ith component as y_i.

Non trivial solutions of the eigenvalue problem stated by Eq. (10) can be obtained if, and only if,

$$\det|Z_{ij}| = 0. \tag{12}$$

The above equation is known as the generalized Fresnel equation and its solutions are identified as the dispersion relations, which describe the propagation of electromagnetic waves in the nonlinear medium characterized by the coefficients ε and μ under the action of external applied fields $\vec{E}$ and $\vec{B}$.

III. MAGNETOELECTRIC BIREFRINGENCE

Magnetoelectric birefringence has been investigated since long ago [8–11] but only recently has it been measured [12,13] in the laboratory. This effect is difficult to measure because its magnitude is small compared to the standard birefringence effects (Kerr and Cotton-Mouton birefringence), which are claimed to always appear together with the magnetoelectric birefringence.

Within the eikonal approximation it was suggested [17] the possibility of producing a kind of linear (in the product of the electric and magnetic fields) magnetoelectric birefringence as a unique effect in isotropic nonlinear media. In this section two possible configurations where this effect can be implemented are examined.

The standard method used to solve the eigenvalue equation [Eq. (10)] consists in expanding the corresponding eigenvector $\vec{e}$ in a particular basis of vectors in the three-dimensional space [3,17,21]. We adopt an alternative method which consists of deriving the formulas for the traces of linear operators [22]. Following this method, the determinant of the Fresnel matrix yields

$$\det|Z_{ij}| = -\frac{1}{6}(Z_1)^3 + \frac{1}{2}Z_1 Z_2 - \frac{1}{3}Z_3 = 0, \tag{13}$$

where we defined the traces

$$Z_1 \doteq \sum_{i=1}^{3} Z_{ii}, \tag{14}$$

$$Z_2 \doteq \sum_{i,j=1}^{3} Z_{ij} Z_{ji}, \tag{15}$$

$$Z_3 \doteq \sum_{i,j,l=1}^{3} Z_{ij} Z_{jl} Z_{li}. \tag{16}$$

A. Magnetoelectric birefringence in dielectric media with $\varepsilon=\varepsilon(B)$ and constant $\mu=\mu_c$

For this class of nonlinear media, Eq. (11) takes the form

$$Z_{ij} = \left(\varepsilon - \frac{q^2}{\mu\omega^2}\right)\delta_{ij} + \frac{1}{\mu\omega^2} q_i q_j - \frac{\dot{\varepsilon}}{\omega}(\vec{q}\times\vec{B})_j E_i \tag{17}$$

leading to the following traces:

$$Z_1 = 3\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right) + \frac{q^2}{\mu_c\omega^2} + \frac{\dot{\varepsilon}}{\omega}\vec{q}\cdot\vec{E}\times\vec{B}, \tag{18}$$

$$Z_2 = 3\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)^2 + \frac{q^4}{\mu_c^2\omega^4} + \frac{\dot{\varepsilon}^2}{\omega^2}(\vec{q}\cdot\vec{E}\times\vec{B})^2 + 2\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)\left(\frac{q^2}{\mu_c\omega^2} + \frac{\dot{\varepsilon}}{\omega}\vec{q}\cdot\vec{E}\times\vec{B}\right), \tag{19}$$

$$Z_3 = 3\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)^3 + \frac{q^6}{\mu_c^3\omega^6} + \frac{\dot{\varepsilon}^3}{\omega^3}(\vec{q}\cdot\vec{E}\times\vec{B})^3 + 3\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)\left(\frac{q^4}{\mu_c^2\omega^4} + \frac{\dot{\varepsilon}^2}{\omega^2}(\vec{q}\cdot\vec{E}\times\vec{B})^2\right) + 3\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)^2\left(\frac{q^2}{\mu_c\omega^2} + \frac{\dot{\varepsilon}}{\omega}\vec{q}\cdot\vec{E}\times\vec{B}\right). \quad (20)$$

Now, introducing Eqs. (18)–(20) in Eq. (13), yields

$$\varepsilon\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right)\left(\varepsilon - \frac{q^2}{\mu_c\omega^2} + \frac{\dot{\varepsilon}}{\omega}\vec{q}\cdot\vec{E}\times\vec{B}\right) = 0. \quad (21)$$

It is worth to mention that the above result can also be obtained from the general dispersion relation presented before [17], where the expansion in a particular basis of vectors was considered.

The above equation presents two distinct solutions. The first one does not depend on the direction of propagation of the wave and is recognized as the dispersion relation for the ordinary ray. The corresponding speed ω/q is given by

$$v_o = \pm\frac{1}{\sqrt{\mu_c\varepsilon}}. \quad (22)$$

The second solution presented in Eq. (21) corresponds to the dispersion relation associated with the extraordinary ray and it depends on the direction of propagation of the wave $\hat{q} = \vec{q}/q$. Its corresponding speed is

$$v_e^{\pm} = -\frac{\dot{\varepsilon}\hat{q}\cdot\vec{E}\times\vec{B}}{2\varepsilon} \pm \sqrt{\frac{\dot{\varepsilon}^2(\hat{q}\cdot\vec{E}\times\vec{B})^2}{4\varepsilon^2} + \frac{1}{\mu_c\varepsilon}}. \quad (23)$$

In this paper the indexes o and e denote quantities associated with the ordinary and the extraordinary rays, respectively.

Let us assume the expansion of the permittivity coefficient ε as

$$\varepsilon = \varepsilon_c + \varepsilon_2 B^2. \quad (24)$$

For all cases it is assumed that $\varepsilon_2 B^2 \ll \varepsilon_c$. Thus, the speeds of ordinary and extraordinary rays reduce to

$$v_o = \pm\frac{1}{\sqrt{\mu_c\varepsilon_c}}\left(1 - \frac{\varepsilon_2}{2\varepsilon_c}B^2\right), \quad (25)$$

$$v_e^{\pm} = -\frac{\varepsilon_2}{\varepsilon_c}\hat{q}\cdot\vec{E}\times\vec{B} \pm \frac{1}{\sqrt{\mu_c\varepsilon_c}}\left(1 - \frac{\varepsilon_2}{2\varepsilon_c}B^2\right). \quad (26)$$

We notice that birefringence does not occur if either of the following cases occurs: (i) direction of propagation belonging to the plane containing the external fields, i.e., $\hat{q}\cdot(\hat{E}\times\hat{B})=0$; (ii) parallel electric and magnetic fields $\hat{E}\cdot\hat{B}=1$. In these cases $v_e = v_o$ as expected.

With respect to the direction of wave propagation, the difference between the values of ordinary and extraordinary speeds achieves a maximum when we set $\hat{q} = \pm(\vec{E}\times\vec{B})/\|\vec{E}\times\vec{B}\|$. In this case $\hat{q}\cdot\vec{E}\times\vec{B} = EB\sin\theta$ and the magnitude of the birefringence effect is found to be

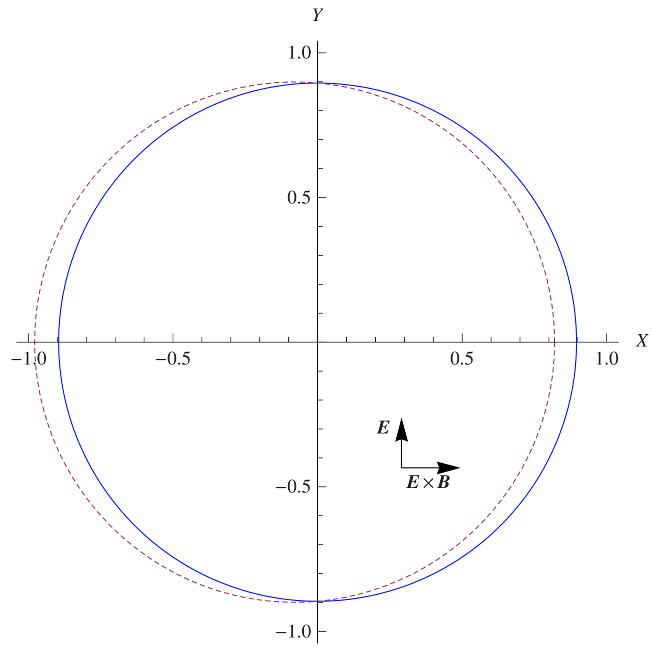

FIG. 1. (Color online) Normal surfaces associated with ordinary (circular solid line) and extraordinary (dashed line) rays propagating in an isotropic nonlinear medium with dielectric coefficients given by $\varepsilon = \varepsilon_c + \varepsilon_2 B^2$ and constant μ. The plot is based on Eqs. (22)–(24) with $\hat{E}=\hat{y}$, $\hat{B}=\hat{z}$ and $\hat{q}\cdot\hat{B}=0$. The numerical values were taken such that $(\varepsilon_c\mu_c)^{-1/2}=0.9$, $\varepsilon_2/\varepsilon_c=0.001$, and $v_o \approx 0.896$.

$$|n_\parallel - n_\perp| \cong \mu_c\varepsilon_2 EB\sin\theta, \quad (27)$$

where $n_\parallel \doteq (1/v_e)_\parallel = 1/v_o$ and $n_\perp \doteq (1/v_e)_\perp$ are the refractive indexes in the directions parallel and perpendicular to the direction of the external electric field, respectively. As it can be inferred from Eq. (27), the effect achieve its maximum value when the external fields are crossed ($\theta=\pi/2$). As anticipated, for the case of parallel external fields ($\theta=0$) birefringence disappear.

In Fig. 1 the normal surfaces associated with ordinary (circular solid line) and extraordinary (dashed line) rays propagating in an isotropic nonlinear medium with permittivity ε given by Eq. (24) are presented. It is assumed that the waves were produced in a given earlier time at the origin. The plot is based on Eqs. (22)–(24) with $\hat{E}=\hat{y}$, $\hat{B}=\hat{z}$ and $\hat{q}\cdot\hat{B}=0$. The choice of crossed fields ($\theta=\pi/2$) was done in order to produce a stronger effect, but the same qualitative behavior occur with $\theta \neq \pi/2$. The direction of propagation is determined by the angle φ given by $\hat{q}\cdot\hat{E}\times\hat{B}=\cos\varphi$. The maximum birefringence effect occurs when φ is equal to 0 or π, corresponding to Eq. (27) with $\theta=\pi/2$. As shown in the figure, the speed of extraordinary ray is smaller than the speed of ordinary ray in the interval $-\pi/2 < \varphi < \pi/2$. The opposite occurs if $\pi/2 < \varphi < 3\pi/2$ and they are the same in the case of φ being equal to $\pi/2$ or $3\pi/2$, which appears as nonbirefringent directions.

Summing up, in the presence of external electric and magnetic fields, electromagnetic waves propagating in isotropic media whose dielectric properties are described by Eq. (24) present double refraction, whose magnitude is measured by means of Eq. (27). This corresponds to a kind of magnetoelectric birefringence appearing as a unique effect, i.e., without any other standard accompanying effects.

The polarization modes described by $\vec{e}$ can be obtained for the ordinary and extraordinary rays. They correspond to the eigenvectors of Eq. (10) with Z_{ij} given by Eq. (17). Let us consider an expansion of $\vec{e}$ in terms of a convenient basis of the three-dimensional space, which can be chosen as the external fields $\vec{E}$ and $\vec{B}$ and the wave vector $\vec{q}$. Thus,

$$\vec{e} = a\vec{E} + b\vec{B} + c\vec{q}. \tag{28}$$

Thus, introducing this equation in Eq. (10) and using the assumptions set for the nonlinear medium, we obtain

$$a\left(\varepsilon - \frac{q^2}{\mu_c\omega^2} + \frac{\dot{\varepsilon}}{\omega}\vec{q}\cdot\vec{E}\times\vec{B}\right) = 0, \tag{29}$$

$$b\left(\varepsilon - \frac{q^2}{\mu_c\omega^2}\right) = 0, \tag{30}$$

$$a\left(\frac{\vec{q}\cdot\vec{E}}{\mu_c\omega^2}\right) + b\left(\frac{\vec{q}\cdot\vec{B}}{\mu_c\omega^2}\right) + c\varepsilon = 0. \tag{31}$$

The solution of the above system leads to the general polarization vectors,

$$\hat{e}_o = a_1\left(\vec{B} - \frac{\vec{q}\cdot\vec{B}}{\varepsilon\mu_c\omega^2}\vec{q}\right), \tag{32}$$

$$\hat{e}_e = a_2\left(\vec{E} - \frac{\vec{q}\cdot\vec{E}}{\varepsilon\mu_c\omega^2}\vec{q}\right), \tag{33}$$

where a_1 and a_2 are normalization factors ($\hat{e}_o\cdot\hat{e}_o=\hat{e}_e\cdot\hat{e}_e=1$). We notice that when the external fields tend to be parallel (no birefringence situation) the polarization vectors tend to be the same, as expected. One can see that if the propagation occurs perpendicularly to the plane containing the external fields, the polarization vectors reduce to $\hat{e}_o=\hat{B}$ and $\hat{e}_e=\hat{E}$. It corresponds to the birefringence effect described by Eq. (27), which achieves its maximum value with $\theta=\pi/2$. For the case presented in Fig. 1 the propagation vector associated with the ordinary ray reduces to $\hat{e}_o=\hat{B}$. In these two cases $\hat{e}_e\cdot\hat{e}_o=\hat{E}\cdot\hat{B}=\cos\theta$.

It should be remarked that the polarization vectors can also be derived by using other basis of the vector space, as the Cartesian basis $\hat{x},\hat{y},\hat{z}$, for instance. The obtained results does not depend on the particular choice of basis.

B. Magnetoelectric birefringence in dielectric media with $\mu=\mu(E)$ and constant $\varepsilon=\varepsilon_c$

In this case Eq. (11) reduces to

$$Z_{ij} = \left(\varepsilon - \frac{q^2}{\mu\omega^2}\right)\delta_{ij} + \frac{1}{\mu\omega^2}q_iq_j - \frac{\mu'}{\omega\mu^2}(\vec{q}\times\vec{B})_iE_j. \tag{34}$$

The traces of Z_{ij} can be derived in a similar way as done in last section. However, for this symmetric case these quantities can be obtained directly from Eqs. (18)–(20) by identifying $\{\varepsilon\to\varepsilon_c,\ \mu_c\to\mu,\ \dot{\varepsilon}\to\mu'/\mu^2\}$. Thus, using Eq. (13) we obtain,

$$\varepsilon_c\left(\varepsilon_c - \frac{q^2}{\mu\omega^2}\right)\left(\varepsilon_c - \frac{q^2}{\mu\omega^2} + \frac{\mu'}{\omega\mu^2}\vec{q}\cdot\vec{E}\times\vec{B}\right) = 0. \tag{35}$$

As before, the dispersion relations for ordinary and extraordinary rays can be obtained from the above expression, leading to the corresponding speeds. Assuming the expansion of the permeability coefficient μ as $\mu=\mu_c+\mu_2E^2$, with $\mu_2E^2\ll\mu_c$, we obtain

$$v_o = \pm\frac{1}{\sqrt{\mu_c\varepsilon_c}}\left(1 - \frac{\mu_2}{2\mu_c}E^2\right), \tag{36}$$

$$v_e^{\pm} = -\frac{\mu_2}{\varepsilon_c\mu_c^2}\hat{q}\cdot\vec{E}\times\vec{B} \pm \frac{1}{\sqrt{\mu_c\varepsilon_c}}\left(1 - \frac{\mu_2}{2\mu_c}E^2\right). \tag{37}$$

Again, the speeds of ordinary and extraordinary rays coincide (no birefringence) when propagation occurs in the plane containing the external fields (electric and magnetic) or even when these fields are parallel. The qualitative behavior of the normal waves corresponding to both rays is the same as found in the case studied in Sec. III A (see Fig. 1).

With respect to the direction of propagation, the maximum difference between ordinary and extraordinary speeds occurs when $\hat{q}=\pm(\vec{E}\times\vec{B})/\|\vec{E}\times\vec{B}\|$ and yields

$$|n_\parallel - n_\perp| \cong \frac{\mu_2}{\mu_c}EB\sin\theta, \tag{38}$$

which consists in the measurable quantity corresponding to the magnetoelectric birefringence phenomenon. Further, the effect will be maximized in the case of crossed fields ($\theta=\pi/2$) and disappear if the external fields are parallel ($\theta=0$). As before, the magnetoelectric birefringence appears as a unique effect, disappearing if any external field is turned off.

Finally, the normalized polarization vectors can be obtained in the same lines as done in Sec. III A.

IV. CONCLUDING REMARKS

The propagation of monochromatic electromagnetic waves in isotropic media was investigated in the limit of geometrical optics. The corresponding eigenvector problem was presented and solved for systems characterized by dielectric coefficients $\varepsilon(B)$ and constant μ, and also for $\mu(E)$ and constant ε. Magnetoelectric birefringence was examined for each situation and the polarization vectors were presented. For certain class of isotropic nonlinear media, as liquids, satisfying the above mentioned conditions, a kind of magnetoelectric birefringence occurs as a unique effect, that is, without the appearance of the standard birefringence phenomena (as Kerr and Cotton-Mouton effects). Since the magnitude of the standard birefringence is far greater than the magnetoelectric one, producing the latter as a unique effect may result in an experimental enhancement in its measurement. In both cases, the presence of external electric and magnetic fields are necessary to produce birefringence. Further, setting the external fields to be parallel, the effect disappears. The magnitude of this phenomenon achieves its maximum value for crossed external fields, with propagation occurring perpendicularly to them.

Closing, magnetoelectric birefringence was recently measured in pure molecular liquids [13]. In the case of a sample of methylcyclopentadienyl-Mn-tricarbonyl the effect (in this case not as a unique effect) was measured with $K_{ME}=51 \times 10^{-12}$ V^{-1} T^{-1}, where the parameter K_{ME} gives an idea of the magnitude of the birefringence effect. For instance, in Sec. III A $K_{ME}=\mu_c \varepsilon_2/\lambda$ where λ denotes the wavelength of the light propagating in the medium. Generally, the effect depends on the properties of the medium and also on the magnitude of external fields. Finally, in order to produce it as a unique effect, the requirements stated in Secs. III A or III B should be provided.

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