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Synchronization of quasiperiodic oscillations to a periodic force studied with semiconductor lasers

A. Loose, H.-J. Wünsche, and F. Henneberger
AG Photonik, Humboldt-Universität zu Berlin, Newtonstr. 15, 12489 Berlin, Germany
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We experimentally study synchronization processes in a system of two different multisection semiconductor lasers. Periodic self-pulsations of laser 1 are injected into laser 2, which is operating in a regime with two-frequency quasiperiodic self-pulsations. The experimental system demonstrates the new type of transitions to synchrony between three frequencies which has been recently revealed using generic coupled phase and van der Pol oscillator models. In particular, resonances of quasiperiodic oscillations at integer winding numbers three and five are shown to break up before locking to the injected periodic signal. Moreover, carefully determining the coherence of the noisy oscillations, we reveal so far unexplored processes of coherence transfer to nonsynchronized oscillations.

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Synchronization processes play an important role in nature and technology, as oscillatory behavior is ubiquitous [3,4]. For the limiting cases of both periodic motions of nearly identical oscillators and chaotic systems, synchronization is well understood and sophisticated mathematical methods exist for its description, e.g. [5–7]. Surprisingly, the synchronization of quasiperiodic oscillations, which are of intermediate complexity, is less well explored [8–10]. However, important phenomena in different fields have more than one fundamental frequency, as e.g., planetary motion in astronomy [11], cardiorespiratory synchronization in biology [12], and coupled periodic processes are highly relevant in climate research and chemical engineering, e.g. [13]. Recent theoretical studies of Anishchenko *et al.* using phase and van der Pol oscillator models indicate an intricate scenario already for the simplest possible case of quasiperiodic motion [9,14]. The synchronization of two mutually locked oscillators to an external frequency decisively differs from classical synchronization of a limit cycle. It sensitively depends on the relation between external and internal forces as well as on the resonance type. In this Rapid Communication, we present an experimental study on the synchronization of quasiperiodic motions in the field-carrier dynamics of semiconductor lasers. We find that the main characteristics of the oscillator models persist in this more complex system, demonstrating their generic nature. Besides locking to the external frequency, we also address the influence of the external signal on the coherence of the oscillations. We reveal processes of coherence transfer to nonsynchronized oscillations, which are unknown so far.

Two multisection semiconductor lasers, fabricated by the Fraunhofer Heinrich-Hertz-Institut, are used. As has been demonstrated previously [15,16], well-defined dynamical regimes of operation can be prepared in such lasers by adjusting the injection currents appropriately. In the present context, the emergence of intensity self-pulsations is of relevance. Pulsations of frequency f_{MB} related to the beating of two cavity modes (MB) are born in a Hopf bifurcation. Undamping of the relaxation oscillations (RO) in a torus bifurcation leads to a second pulsation of frequency $f_{\text{RO}} < f_{\text{MB}}$. The superposition of these two oscillations of different physical origin provides the quasiperiodic dynamics we are interested in. An essential feature in our study is that f_{RO} and f_{MB} can be tuned in a sufficiently wide range by the pump currents which allows us to arrange certain conditions [15].

The scheme of the experimental setup is shown in Fig. 1. The slave laser is running in the quasiperiodic regime with both types of pulsations, while the master laser serving as external force is adjusted so as to exhibit mode-beating pulsations only. For distinction with the slave laser, we denote their frequency here by f_{ext}. The external coupling strength can be set by an attenuator element controlling the master intensity that reaches the slave. The signal from the slave laser is analyzed by an optical spectrum analyzer with a wavelength resolution of 0.1 nm as well as an rf-spectrum analyzer with 40 GHz bandwidth. The intensity autocorrelation function $C(\tau)$ is constructed by Fourier transformation of the rf-spectra. An appropriate frequency window containing only the peak under consideration is evaluated and averages over typical 10 measurements are used. The coherence

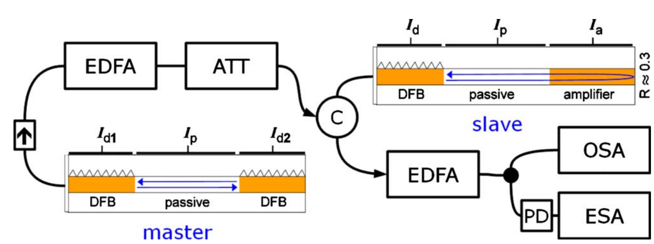

FIG. 1. (Color online) Sketch of the experimental setup and schematic side view of the multisection devices, for device technical details see, e.g. [15]. Distributed feedback (DFB) laser current I_d, phase current I_p, and amplifier current I_a are supplied with an accuracy of ± 0.05 mA. The temperature is stabilized at 20.01 ± 0.01 °C in both lasers. Light emitted from the antireflection coated DFB facet is coupled into a single mode fiber. Arrow, optical isolator. C, optical circulator. ATT, attenuator and powermeter (Eigenlight 420 WDM). EDFA, erbium doped fiber amplifier. OSA, optical spectrum analyzer (HP 71451-B). PD, u2t ultrafast photodiode. ESA, electrical spectrum analyzer (Rohde & Schwarz FSP 9).

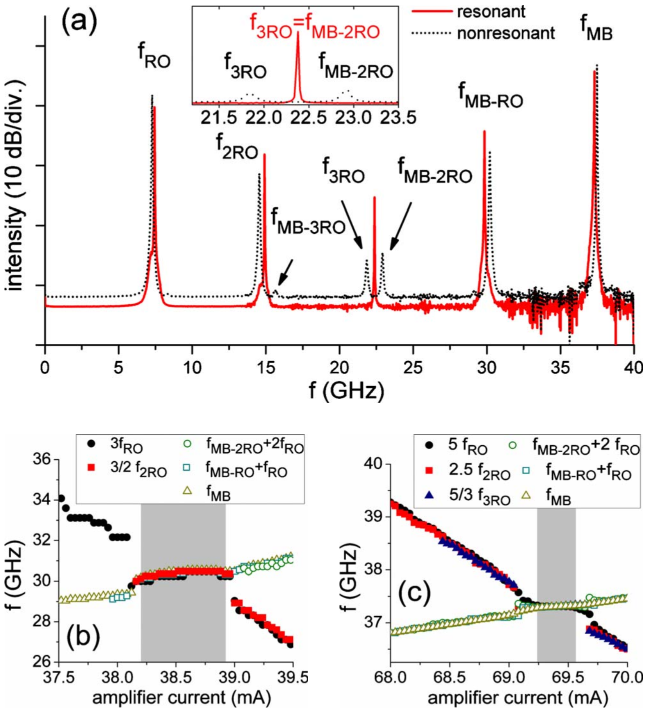

FIG. 2. (Color online) Exemplary power spectra of a quasiperiodic oscillation in the free running slave. I_d=89.97 mA, I_p=42.95 mA. (a) Red (gray) solid line: internally synchronized, I_a=69.43 mA. Black dotted line: no internal synchronization, I_a=70.20 mA. Inset: details at mid frequency. Note the increased peak amplitude and decreased peak width for internal synchronization. (b,c) Variation of the rescaled peak frequencies with I_a. f_{RO} and its harmonics decrease with I_a, whereas f_{MB} and mirrors of RO at MB increase slightly. Within I_a=38.20–38.92 (69.24–69.56) mA, shaded gray, f_{RO} and f_{MB} are synchronized at a ratio of 1:3 (1:5).

time τ_c of the slave oscillations is finally obtained by calculating $\tau_c=\int_0^\infty C^2(\tau)d\tau$.

We first characterize the quasiperiodic motion in the free running slave laser. As described previously [15], the coexisting internal pulsations undergo mutual locking at integer winding numbers $\theta=f_{MB}/f_{RO}=2$ up to $\theta=6$ when setting the control parameters appropriately. Here, we focus specifically on the synchronization of f_{RO} and f_{MB} as function of the amplifier current, all other currents and the temperature are fixed. Typical power spectra are shown in Fig. 2(a) for f_{RO} and f_{MB} resonant and mutually locked, and off resonant. In addition to the two fundamental frequencies f_{RO} and f_{MB}, the nonlinear amplitude-phase coupling in the laser gives rise to various extra features such as higher harmonics $(2f_{RO},3f_{RO},\ldots)$ and mirror peaks $(f_{MB\text{-}RO},f_{MB\text{-}2RO}=f_{MB}-2f_{RO},\ldots)$. The latter directly evidence an internal interaction between the two fundamental oscillations. As seen in Figs. 2(b) and 2(c), all peaks when properly rescaled follow either RO or MB when changing the amplifier current. The MB frequency at $\theta\approx3$ is around 30 GHz and at $\theta\approx5$ around 37 GHz. Obviously, the $\theta=3$ resonance is stronger as indicated by the larger locking range in frequency and amplifier current. After elaboration of the internal synchronization scenario, we are able to study the response to the external force selectively, in the presence or absence of internal locking.

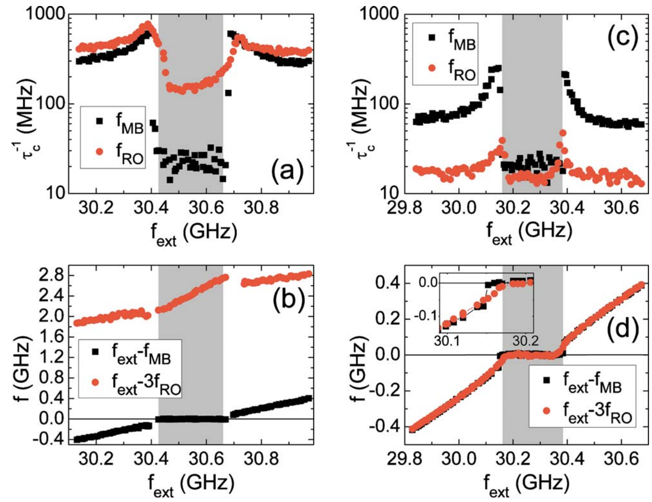

FIG. 3. (Color online) Synchronization of f_{MB} to f_{ext} at θ =3.24 (a,b) and θ=3.00 (c,d). (a,c) Inverse coherence time τ_c^{-1} of f_{RO}, red (gray) circles, and f_{MB}, black squares. See text for details. (b,d) f_{RO} and f_{MB} rescaled with respect to f_{ext}. Shaded gray: locking range of f_{MB}, only one peak within ±0.5 GHz. Inset (d): detail of locking border. f_{MB} is locked earlier than f_{RO}.

We now couple master and slave. The optical emission of the master with frequency f_{ext} is injected into the slave to synchronize f_{MB}. Linear tuning of f_{ext} without hysteresis is achieved by tuning one of the dc pump currents of the master. We observe that a linear increase in the injected optical power leads to a linear increase of the locking range of f_{MB} to f_{ext}, indicating a regime of weak forcing. This locking range of up to a few 100 MHz is small compared to the peak frequency separation of a few GHz. The optical injection leads to several new lines in the high-resolution power spectrum due to the presence of nonlinear mixing within the slave, there are no significant unidentified peaks. Power spectra in the vicinity of each f_{RO} and f_{MB} are recorded during the experiment.

We first investigate synchronization of f_{MB} to f_{ext} at $\theta=f_{MB}/f_{RO}=3.24$. As can be seen in the center of Figs. 3(a) and 3(b) only f_{MB} is locked by the injected f_{ext}, within a range of 252±11 MHz. For f_{RO} a significant increase in coherence by a factor of more than two is visible, although it is nonresonant. Additionally the RO slow down while f_{MB}, locked to f_{ext}, increases. While a direct interaction of f_{ext} with f_{RO} cannot be excluded, nonresonant conditions indicate that the internal coupling between MB and RO is responsible for the transfer of coherence and change in frequency.

The winding number is now adjusted to θ=3, see Figs. 3(c) and 3(d). This corresponds to internal synchronization of RO and MB in the gray shaded center of Fig. 2(b). f_{MB} locks to f_{ext} within 217±7 MHz. Three regimes can be identified in Figs. 3(c) and 3(d) and inset of Fig. 3(d). On the right and left side of Figs. 3(c) and 3(d), f_{RO} and f_{MB} are internally locked. In the gray shaded center, both RO and MB are locked to f_{ext}. On both borders of this center region a small parameter interval is present where no internal reso-

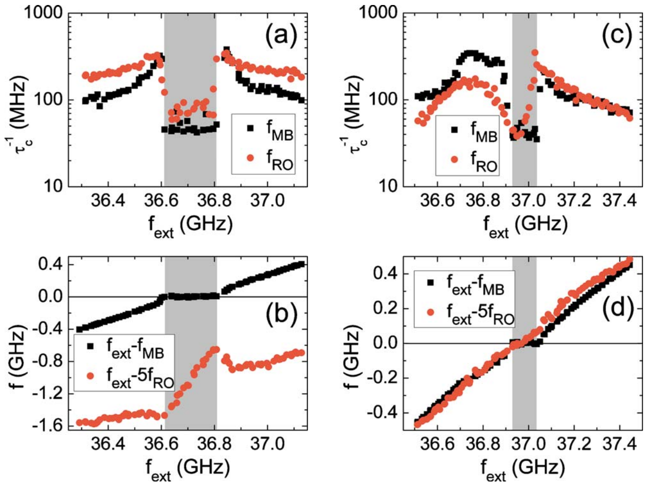

FIG. 4. (Color online) Synchronization of f_{MB} to f_{ext} at $\theta=4.91$ (a,b) and $\theta=5.00$ (c,d). (a,c) Inverse coherence time τ_c^{-1} of f_{RO}, red (gray) circles, and f_{MB}, black squares. (b,d) f_{RO} and f_{MB} rescaled with respect to f_{ext}. Shaded gray: locking range of f_{MB}, only one peak within ± 0.5 GHz. Note the significant reduction for $\theta=5$ in (c,d).

nance between f_{RO} and f_{MB} exists, and f_{MB} is locked to f_{ext}, inset of Fig. 3(d). As in the oscillator models [9,14], the internal locking is first broken before the regime of synchronization of all three frequencies is established. This route to synchronization is clearly different from that of a classical limit cycle, harmonics of which always remain at integer multiples. As can be seen in Fig. 3(c), the coherence of the RO is higher than of the MB. In the locking range shaded gray, the coherence of the RO nearly reaches the coherence separately measured for f_{ext}, while f_{MB} is less coherent than both f_{ext} and f_{RO}.

The point of operation of the slave is now adjusted to $\theta \approx 5$. The internal coupling strength of the torus is decreased compared to $\theta \approx 3$, while the coupling strength of the external frequency f_{ext} remains the same.

The results for $\theta=4.91$, Figs. 4(a) and 4(b), are qualitatively equivalent to the case $\theta=3.24$. The locking range from f_{MB} to f_{ext} is smaller at 196 ± 17 MHz, because the MB frequency now is faster at ≈ 37 GHz instead of ≈ 30 GHz. Again, for all peaks the coherence increases significantly in the center of the synchronization region of f_{MB} to f_{ext}.

Now we adjust to internal resonance at $\theta=5$, Fig. 4. Synchronization differs markedly from the case of $\theta=3$ in Figs. 3(c) and 3(d). f_{MB} locks to f_{ext} within a considerably reduced range of 106 ± 13 MHz and shows an increase in coherence. f_{RO} however remains almost stationary when f_{MB} is synchronized to f_{ext}, see Figs. 4(c) and 4(d). At the given internal coupling strength, synchronization of both RO and MB to f_{ext} as for $\theta=3$ is not achieved. In contrast to synchronization of a classical limit cycle where all frequency peaks would be expected to shift, only f_{MB} locks to the external signal. If f_{RO} is not changed, a small interval has to exist where f_{RO} is again resonant to the quasiperiodic oscillation now formed by the mutually locked f_{MB} and f_{ext}. This explains the increase in coherence of f_{RO} within the locking range of f_{MB} to f_{ext}. Though only a few selected values of θ have been examined above, the results are representative in the following sense. The similarity between $\theta=3.24$ and $\theta=4.91$ in the nonresonant case demonstrates that the scenario does not qualitatively depend on the exact value of θ. Second, $\theta=3$ and 5 just define the two fundamental synchronization regimes of weak and strong internal resonances, respectively [9]. The external synchronization of the intermediate resonance $\theta=4$ was not successful. It is interesting to note that this behavior also seems to reflect the classification of quasiperiodic resonances in the mathematical literature: ($\theta \geq 5$) and ($\theta \leq 4$) are weak and strong resonances, respectively, with $\theta=4$ as a peculiar case regarding its stability [1,2]. The resonances $\theta=2$ and $\theta \geq 6$ are not accessible with our experimental setup.

Two experimental conditions have not yet been addressed. First, the intensity pulsations used in the experiment are modulated waves with an optical carrier frequency which is orders of magnitude faster. Synchronization is nontrivial already for periodic intensity self-pulsations, as both locking of the intensity modulation and locking in the optical domain may appear, if the respective frequencies are close [17]. When coupling the light of the master into the slave, interactions in the optical domain have to be avoided. We have found that intensity beating of optical modes and synchronization in the optical domain only play a role below 1 nm separation between the two emission wavelengths. In the present measurements the emission wavelengths of master and slave are detuned by about 15 nm, far exceeding this range. Second, the mean power in the slave is modified by the injected light even for weak coupling. All intensity pulsation frequencies are decreased by a small amount, locking cones therefore are tilted to lower frequencies with increasing strength of external coupling [18]. The operating parameters of the slave have to be readjusted slightly for the locked torus, as the mutual locking region of both torus frequencies is shifted. The change in the point of operation does not seem to influence the scenario described in Fig. 2 besides the small shift in frequency.

We now discuss the generality of the observed phenomena. The bifurcation scenario for synchronization of a quasiharmonic oscillation in the presence of internal resonance has been explored by Anishchenko *et al.* [9]. Two coupled Van der Pol oscillators are used as model system. They find that the internal resonance always is first broken up when synchronizing to an external harmonic force and a finite parameter region with three independent frequencies is observed. We indeed observed that for $\theta=3$ the internal resonance first breaks up, the locking range to f_{ext} of the RO is slightly smaller than the one of f_{MB}. The magnitude of the internal coupling strength then decides between two possible scenarios. Given a sufficiently strong internal coupling both frequencies are entrained by f_{ext}, first the frequency near f_{ext}, then the other one. For weak internal coupling this second frequency cannot be synchronized and the internal resonance remains broken. We confirmed the proposed dependence on the internal coupling strength in the experiment. While for $\theta=3$ synchronization of all three frequencies can be achieved, the lower internal coupling strength associated with $\theta=5$ generally does not allow this. But even when the internal resonance remains broken at first, if the unlocked frequency is stationary, there is at least one point where again all three frequencies are resonant. This is the reason for the increase of coherence seen in Fig. 3(d). For a sufficiently low internal coupling strength and without noise, this region eventually could not be detected anymore in [9]. Outside of regions of internal synchronization, only one frequency can be synchronized by f_{ext} as would be expected. In summary, the locking behavior of our complex system is qualitatively in agreement with the prediction for more basic models [9]. This is a strong indication for universality of the phenomena we have observed.

In conclusion, quasiperiodic self-pulsations of a semiconductor laser have been synchronized to optically injected periodic pulses emitted by another laser. New regimes due to the interplay between internal and external synchronization processes have been found. Concerning frequency locking, they are in agreement with recent theoretical predictions by Anishchenko *et al.* [9], despite the significant increase of the system complexity. In particular, they confirm that the route to synchrony of a closed orbit on a torus differs from that of a classical limit cycle. As a consequence, the presence of a resonant torus can be verified by synchronization and distinguished from a nonlinear oscillation with higher harmonics. Completely new and surprising effects have been observed when studying the coherence properties of the oscillations. In particular, external coherence is transferred to both internal oscillations even if they have very different frequencies and are not synchronous to each other. This feature opens up a novel way to increase the quality of not directly accessible oscillatory subsystems. All together, our findings might be highly relevant for multiscale systems in chemistry, biology and technology, e.g. [19]. In general, complex oscillating processes with several independent natural frequencies are expected to show further new phenomena, e.g., various resonances and even chaotic regimes on the locking borders [9].

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