Control of noise-induced coherent behaviors in an array of excitable elements by time-delayed feedback

Taiji Okano and Kenji Miyakawa
Department of Applied Physics, Fukuoka University, Fukuoka 814-0180, Japan
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We investigate feedback-controlled coherent dynamics in a two-dimensional array of excitable elements. We demonstrate that one can freely enhance or reduce the spatiotemporal coherence of noise-induced oscillation, such as coherence resonance and phase synchronization, by controlling both the delay time and the feedback gain. Furthermore, we find that noise-induced oscillations are entrained by the feedback force in a certain range of the delay time. Experimental observations are approximately reproduced in a numerical simulation with a forced Oregonator model.

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Stochastic resonance is a well-known phenomenon, which is characterized as enhancement of a response to a weak input signal due to a moderate noise level [1–8]. Even in the system without input signal, noise can enhance regular dynamics, called coherence resonance (CR) [9–14]. When an array of excitable elements is subjected to independent local noise, these resonance phenomena are further optimized, known as array-enhanced stochastic resonance [15–18] or array-enhanced coherence resonance (AECR) [19–21]. These results indicate that the interplay between noise and interactions among active elements is crucial for noise-enhanced resonance and synchronization.

Coherent patterns in nonlinear dynamical systems can be efficiently controlled by feedback utilizing the inherent sensitivity to external stimulations. The control of parameters, such as a feedback gain and a delay time, leads to the formation of new spatiotemporal patterns [22–26]. Feedback technique makes it possible to design and control interactions among elements in the array.

In this Brief Report, we experimentally investigate the effect of time-delayed feedback on noise-induced coherent patterns in a two-dimensional array of excitable elements in which the Belousov-Zhabotinsky (BZ) reaction is localized. We find that the spatiotemporal coherence of noise-induced oscillation can be enhanced or reduced by controlling the feedback parameters.

The discrete BZ reaction system was constructed using photolithography-assisted techniques [21,27]. This methodology made it possible to freely control characteristics of the discrete reaction system. In the experiment, reactor units of about 430 μm in diameter and 65 μm in depth were arranged in the lattice [21]. The light sensitive catalyst, tris-(2,2′-bipyridine) ruthenium (II) complex $[Ru(bpy)_3^{2+}]$, was immobilized in silica-gel matrices. We look on a catalyst-doped microgel element in a reactor unit as an oscillator. The measurements were carried out in the array of 10×10 oscillators with the spacing of about 100 μm. The reactor was placed into a chamber that was continuously fed with fresh catalyst-free BZ solution at a pumping rate of 6 ml/h to maintain constant nonequilibrium conditions. The initial concentration of the catalyst-free BZ reaction solution was $[NaBrO_3]=0.37M$, $[NaBr]=0.075M$, $[CH_2(COOH)_2]=0.37M$, and $[H_2SO_4]=0.45M$. At this concentration, the system was initially in an oscillatory regime. When the intensity of light illumination I was increased beyond the threshold $I_{\rm th}=6$ mW, the excitable steady state appeared. Then the period under the dark was estimated by extrapolation to be approximately $T_p=35$ s, which can be regarded as the intrinsic period of oscillation in this system. In the experiment, we fixed I at $I_0=6.5$ mW to sustain the system in an excitable regime close to the bifurcation point. The temperature of the BZ solution was kept at 24 ± 0.5 °C. A computer-controlled video projector was used to illuminate the sample from below through a 460 nm bandpass filter.

The external forcing is introduced via the applied illumination. Then the intensity illuminated on the element (i,j) is expressed as

$$I_{i,j}(t) = I_0 + D\xi_{i,j}(t) + F(t), \tag{1}$$

where I_0 is the background intensity. The second term of Eq. (1) represents the noise with the amplitude D and the random numbers $\xi_{ij}(t)$ distributed uniformly between −1 and 1, i.e., $\langle\xi_{ij}(t)\xi_{lm}(t')\rangle=\delta_{il}\delta_{jm}\delta(t-t')$ and $\langle\xi_{ij}(t)\rangle=0$. The third term of Eq. (1) represents the feedback control $F(t)=kH[B(t-\tau)-B_0]$, where $H(s)$ is the Heaviside step function (zero for $s<0$ and 1 for $s\geq0$), k is the feedback gain, and τ is the delay time. $B(t)$ is the normalized intensity given by $B(t)=(1/N)\Sigma G_{ij}(t)$, where $G_{ij}(t)$ and N are the light intensity of the element (i,j) on an 8 bit gray scale and the total number of elements, respectively. The constant B_0 is the value of $B(t)$ at the time when all reactor units are in the reduction state, which was determined at the beginning of each experiment. Since the third term of Eq. (1) gives always positive value because of positive k, the feedback tends to inhibit the excitability in the system. This means that the system is subjected to the negative feedback. Note that the delayed feedback employed here is nonlinear. The noise pattern and the feedback control were updated at intervals of 4 and 1 s, respectively, and interrupted during 0.1 s every 1 s in order to capture an image of the state of the system.

To evaluate the degree of temporal coherence of noise-induced firings, we use the coherence measure R for N oscillators, defined by

$$R = \frac{1}{N}\sum_{i,j} \frac{\langle T_{ij}\rangle}{\sqrt{\langle T_{ij}^2\rangle - \langle T_{ij}\rangle^2}}, \tag{2}$$

where $\langle T_{ij}^m\rangle = (1/n)\Sigma_{k=1}^{n}(T_k^{ij})^m$, n is the number of firings, and T_k^{ij} is the time interval between the kth and $(k+1)$ th firing events in the oscillator (i,j).

In addition, to characterize the synchronization behavior between oscillators in the array, we introduce a phase of oscillator,

$$\phi_{ij}(t) = 2\pi\frac{t-\tau_k^{ij}}{\tau_{k+1}^{ij}-\tau_k^{ij}} + 2\pi k, \quad \tau_k^{ij} \le t \le \tau_{k+1}^{ij}, \tag{3}$$

where τ_k^{ij} is the time of the kth firing of the oscillator (i,j). The phase difference between oscillators (i,j) and (l,m) is defined as $\Phi_{ij,lm}(t) = \phi_{ij}(t) - \phi_{lm}(t)$. The degree of synchronization between oscillators (i,j) and (l,m) can be measured by a synchronization index as $\gamma_{ij,lm}^2 = \langle \cos\Phi_{ij,lm}\rangle^2 + \langle \sin\Phi_{ij,lm}\rangle^2$, where the angular brackets denote the average over time [28]. The degree of overall synchronization in a $M\times M$ lattice can be characterized by calculating the spatial average,

$$\gamma = \frac{1}{n_p}\sum_{l}^{M}\sum_{m}^{M}\sum_{\langle ij,lm\rangle}\gamma_{ij,lm}, \tag{4}$$

where n_p is the number of coupling pairs. Here, the sum is over nearest-neighbor sites on the lattice.

The τ dependences of R and γ are quite similar to those observed in the array of self-sustained oscillators controlled by time-delayed feedback (see Fig. 3 in Ref. [26]). Both R and γ periodically change with the delay time, and their peak positions entirely coincide with each other. The noise-induced oscillation of the maximal coherence approximately has the period of T_p, which is equal to the period under the dark. The first maximum appears near $\tau=15$ s, which is close to the mean refractory period t_r of oscillation with the period T_p. With further increasing τ, the successive maxima appear at the time interval of about T_p.

We study how the delayed feedback affects the time scales of noise-induced oscillation. Figures 1(a) and 1(b) show typical power spectra of the noise-induced oscillation and the feedback control $F(t)$ in Eq. (1) for $\tau \approx t_r$, respectively. One can see that every spectrum has a pronounced fundamental mode with the frequency of approximately $1/T_p$ and its harmonics. As τ is increased, new modes that change both frequencies and heights with τ appear in the power spectrum of $F(t)$. Since we are interested in a change in time scales of noise-induced oscillation, we describe the behavior of the system in terms of the period rather than the frequency. We introduce the period T_{f} as the inverse of the frequency of the fundamental mode in the spectrum of $F(t)$, and further introduce the period T_0 as the inverse of the highest peak frequency in the spectrum of noise-induced oscillation. Figure 1(c) shows the τ dependence of T_0 and T_{f}. As τ increases from zero, T_0 almost linearly increases along T_{f}, then drops abruptly to a lower branch of T_{f}, and again linearly increases along T_{f} with a smaller slope. These abrupt transitions occur roughly every T_p. Note that the variation of T_0 is limited around $T_p=35$ s. Such a behavior is reminiscent of synchronization phenomena. Accordingly, we can say that noise-induced oscillation is entrained by the delayed feedback force in such a way that the noise-induced oscillation tunes its own basic period to the period of the feedback control. Similar behaviors have been reported in the earlier theoretical study [25].

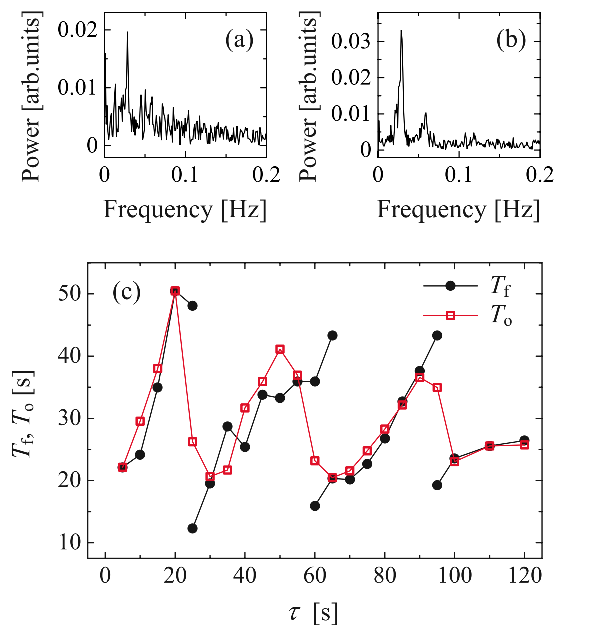

FIG. 1. (Color online) Power spectra of (a) noise-induced oscillations and (b) the feedback control $F(t)$ in Eq. (1) for $\tau=15$ s at $D=0.733$ mW and $k=160$. (c) Basic periods T_0 and T_{f} as functions of τ.

Figure 2 shows the dependence of spatiotemporal coherence on the noise amplitude at $k=160$ for typical values of the delay time. In the absence of the feedback, both R and γ become maximal at $D_{\mathrm{opt}}=0.733$ mW, indicating that the phenomenon of CR occurs, accompanied by phase synchronization. The period of noise-induced oscillation at D_{opt} is approximately T_p. In the presence of the feedback, the spatiotemporal coherence strongly depends on τ. When τ is close to the mean refractory time t_r, i.e., $\tau=15$ s, both R and γ remarkably increase for all values of D. The noise maximizing the coherence is reduced to 0.437 mW, but the period of noise-induced oscillation at this noise level remains almost unchanged. In contrast, when the value of τ is far from

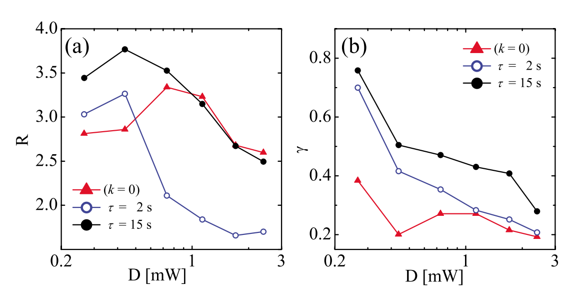

FIG. 2. (Color online) (a) Coherence measure R and (b) synchronization index γ as functions of the noise amplitude D for the cases without delayed feedback $(k=0)$ and with delayed feedback with $k=160$ at $\tau=2$ s and $\tau=15$ s.

t_r, e.g., $\tau=2$ s, the coherence is substantially deteriorated. Thus, the delayed feedback brings about positive or negative effects on the phenomenon of AECR, depending on the value of τ.

It is also possible to enhance the phenomenon of AECR by strengthening the local coupling in the excitable oscillator system without feedback [21]. This means that, under the proper choice of feedback parameters, such as τ and k, the application of delayed feedback is equivalent to strengthening the local coupling.

We attempted to reproduce the experimental results using the three-variable Oregonator model modified to take into account both noise and feedback effects. In our experimental setup, the self-diffusion of the catalyst $Ru(bpy)_3^{2+}$ is negligible since it is immobilized in the silica-gel matrix. The feedback illumination promotes the product of inhibitor Br^- through the photochemical reaction of $Ru(bpy)_3^{2+}$, which influences the excitability of the system. Then the model equations are expressed by

$$\frac{du_{i,j}}{dt} = \frac{1}{\varepsilon}[u_{i,j} - u_{i,j}^2 - w_{i,j}(u_{i,j} - q)] + K_u(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}), \quad (5)$$

$$\frac{dv_{i,j}}{dt} = u_{i,j} - v_{i,j}, \quad (6)$$

$$\frac{dw_{i,j}}{dt} = \frac{1}{\varepsilon'}[fv_{i,j} - w_{i,j}(u_{i,j} + q) + \phi_{i,j}] + K_w(w_{i+1,j} + w_{i-1,j} + w_{i,j+1} + w_{i,j-1} - 4w_{i,j}), \quad (7)$$

where the variables $u_{i,j}$, $v_{i,j}$, and $w_{i,j}$ describe the concentrations of $HBrO_2$, the $Ru(bpy)_3^{3+}$ catalyst, and Br^- in the oscillator (i,j), respectively. ε, ε', and q are scaling parameters; K_u and K_w ($=1.12K_u$) are the coupling strengths; and f is the stoichiometry parameter. These parameters were chosen such that the system was initially in the oscillatory regime: $q=0.002\,75$, $f=1.4$, $\varepsilon=0.01$, $\varepsilon'=0.0001$, and $K_u=0.013$. Then all the periods of oscillators T_p were 7.0, with the refractory period $t_r=3.0$, which can be regarded as the intrinsic time scales in this system. The parameter $\phi_{i,j}$ represents the light-induced production of Br^- on the oscillator (i,j), expressed as

$$\phi_{i,j}(t) = \phi_0 + D\xi_{i,j}(t) + F(t), \quad (8)$$

where ϕ_0 is the background, $D\xi_{i,j}(t)$ is the noise, and $F(t)$ is the feedback force. These terms have the same forms as those of $I_{i,j}(t)$ in Eq. (1). Since in the experiments the oxidized form of the catalyst is monitored, in the calculation of $F(t)$ the normalized intensity $B(t)$ is given by $B(t)=(1/N)\Sigma v_{i,j}(t)$. Increasing ϕ_0 beyond 0.0065 drove the system consisting of 100 oscillators from oscillatory to excitable states. The system was maintained in an excitable regime by fixing $\phi_0=0.007$. The computation was performed by the improved Euler method with time steps $\Delta t=0.0001$. Each oscillator was independently subjected to the noise with the duration time of $8400\Delta t$. The feedback control was updated at the interval of $2100\Delta t$. The spatial separation of the oscillators was taken as $\Delta x=1$. The boundary conditions for both edge elements were taken to be zero flux.

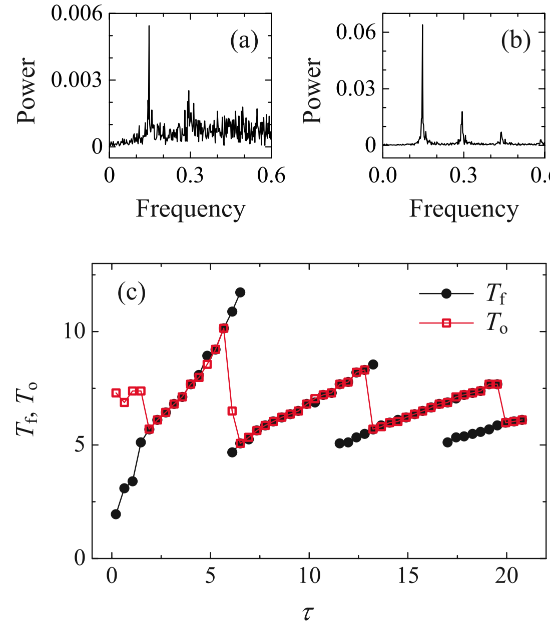

FIG. 3. (Color online) Power spectra of (a) noise-induced oscillation and (b) feedback force $F(t)$ in Eq. (8) for $\tau=t_r$ at $D=0.0015$ and $k=0.015$. (c) Basic periods T_o and T_f as functions of τ.

Both R and γ periodically vary with τ, and their peak positions entirely coincide with each other. Such behaviors are quite similar to those observed in the array of self-sustained oscillators controlled by time-delayed feedback (see Fig. 6 in Ref. [26]). The first maximum appears near $\tau=3.0$, a value close to the intrinsic refractory period t_r, and successive maxima appear at the time interval equal to approximately T_p.

In order to assess the effect of delayed feedback on the time scales of noise-induced oscillation, we calculate power spectra of both noise-induced oscillations and the feedback $F(t)$. Typical power spectra for $\tau \approx t_r$ are shown in Figs. 3(a) and 3(b). One can see that every spectrum is composed of a pronounced fundamental mode with the frequency of approximately $1/T_p$ and its harmonics. We define the period in a similar manner as above, i.e., T_0 for noise-induced oscillations and T_f for $F(t)$. Figure 3(c) shows the τ dependence of both periods. As τ increases from zero, T_0 almost linearly increases along T_f, then drops abruptly to a lower branch of T_f, and again linearly increases along T_f with a smaller slope. These abrupt transitions occur roughly every T_p. These behaviors remind us of entrainment of oscillators by external forces. These results are consistent with the experimental results.

Figure 4 shows the D dependence of the spatiotemporal coherence. In the absence of the feedback ($k=0$), R takes a maximum value R_{max} at $D_{opt}=0.0015$ when $K_u=0.013$, indicating the occurrence of coherence resonance. The period of noise-induced oscillation at D_{opt} is approximately T_p. For a stronger local coupling such as $K_u=0.024$, R_{max} and D_{opt} shifts to larger and smaller values, respectively. When the feedback with $k_c=0.015$ and τ close to t_r is applied, both R and γ are enhanced, even if K_u is fixed at a small value, and

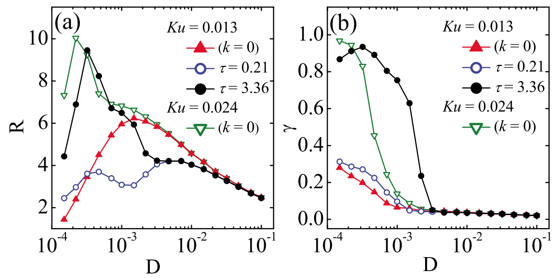

FIG. 4. (Color online) (a) Coherence measure R and (b) synchronization index γ as functions of the noise amplitude D for the cases without delayed feedback ($k=0$) and with delayed feedback with $k=0.015$ at $\tau=0.21$ and $\tau=3.36$.

D_{opt} shifts to a smaller value. Then the period of noise-induced oscillation at D_{opt} remains almost unchanged. When τ is not close to t_r, in contrast, both R and γ are deteriorated. This indicates that the feedback force with the optimal delay time can strengthen the coupling among oscillators, even if the local coupling is weak.

In conclusion, we have demonstrated that time-delayed feedback is effectively applicable to the manipulation of noise-induced coherent dynamics, such as coherence resonance and phase synchronization, in the two-dimensional array of excitable elements. The delayed feedback with a gain more than a threshold can increase or decrease the spatiotemporal coherence, according to whether the delay time τ is close to the mean refractory period t_r or not. These phenomena are evidently associated with a tuning of the period of noise-induced oscillation to the time scale of the feedback force. Such adaptive tuning probably plays a crucial role in the control of noise-induced resonance such as AECR.

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