

# Effects of polarization force and effective dust temperature on dust-acoustic solitary and shock waves in a strongly coupled dusty plasma

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A strongly coupled dusty plasma containing strongly correlated negatively charged dust grains and weakly correlated (Maxwellian) electrons and ions has been considered. The effects of polarization force (which arises due to the interaction between thermal ions and highly negatively charged dust grains) and effective dust temperature (which arises from the electrostatic interactions among highly negatively charged dust and from the dust thermal pressure) on the dust-acoustic (DA) solitary and shock waves propagating in such a strongly coupled dusty plasma are taken into account. The DA solitary and shock waves are found to exist with negative potential only. It has been shown that the strong correlation among the charged dust grains is a source of dissipation and is responsible for the formation of the DA shock waves. It has also been shown that the effects of polarization force and effective dust-temperature significantly modify the basic features (e.g., amplitude, width, and speed) of the DA solitary and shock waves. It has been suggested that a laboratory experiment be performed to test the theory presented in this work.

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## I. INTRODUCTION

About 25 years ago, Ikezi [1] first pointed out that a classical Coulomb plasma with a few micron-sized negatively charged dust grains can readily go into a strongly coupled regime. This is due to high charge and low temperature of these dust grains, which make the coupling parameter  $\Gamma [= (q_d^2/a_d T_d) \exp(-a_d/\lambda_D)]$ , where  $q_d$  is the dust grain charge,  $a_d$  is the intergrain distance,  $T_d$  is the dust temperature in energy unit, and  $\lambda_D$  is the dusty plasma Debye radius] comparable to 1 or even much larger than 1. This theoretical prediction of Ikezi [1] has been conclusively verified by a series of laboratory experiments [2–4] and simulation studies [5]. These laboratory experimental observations [2–4] and simulation studies [5] clearly demonstrate that a dusty plasma [6–10] (plasma with a few micron-sized negatively charged dust grains) can readily go into a strongly coupled regime, and that the charged dust grains organize themselves into crystal structures when  $\Gamma > 171$ . It is also observed by further laboratory experiments [11,12] that due to dust grain heating, the dust crystals first melt and then vaporize, leading to the phase transitions (from solid to liquid and then from liquid to gas). These laboratory dusty plasma experiments [2–4,11,12] and simulation studies [5], therefore, provide an excellent opportunity not only to investigate the phase transitions of interest, but also to study dust-associated waves, viz., dust-acoustic (DA) waves [13] and dust lattice (DL) waves [14] in such a strongly coupled dusty plasma regime.

The linear properties of the DA waves (in which dust particle mass provides the inertia, and electron and ion ther-

mal pressures provide the restoring force) as well as DL waves (in which inertia comes from the dust mass and the restoring force comes from the Debye-Hückel interaction) in such a strongly coupled dusty plasma have been rigorously investigated by many authors during the last decade [14–18] and are now well understood from both theoretical and experimental points of view [14–24].

A limited number of theoretical investigations have also been made on nonlinear propagation of DA and DL waves in strongly coupled dusty plasma [25–29]. Shukla and Mamun [25], Mamun *et al.* [26], Mamun and Shukla [27], Mamun and Cairns [28], and Anowar *et al.* [29] considered weakly coupled Maxwellian electrons [25–29], Maxwellian [25,27,28] or trapped [26,29] ions, and strongly coupled negatively charged dust [25–27,29] or arbitrarily charged dust [28] and studied the DA solitary [26,29] and shock [25–28] waves. Recently, Mamun and Shukla [30] also studied the effects of cylindrical and spherical geometries on DA shock waves in such a strongly coupled dusty plasma. All of these works are valid only when the effects of polarization force [31–33], which arises due to the polarization of plasma particles around the dust grain, and the effective dust temperature [34–36], which arises from the electrostatic interactions among highly negatively charged dust and from the dust thermal pressure, are negligible. However, it has been shown that these effects [31–36] are very important in many space and laboratory dusty plasma situations in order to understand various propagation characteristics of the DA waves.

The concept of polarization force (acting on a charged dust grain) and its importance in dusty plasma physics have been explained by Hamaguchi and Farouki [31,32], and the effects of polarization force on linear propagation of the DA waves have been investigated by Khrapak *et al.* [33]. The

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polarization force ( $\mathbf{F}_p$ ) acting on a dust grain is, mathematically, defined as [31–33]

$$\mathbf{F}_p = -\frac{q_d^2 \nabla \lambda_D}{2\lambda_D^2},$$

where  $\lambda_D = \lambda_{Di}/(1 + \lambda_{Di}^2/\lambda_{De}^2)^{1/2} = \lambda_{Di}/(1 + n_e T_i/n_i T_e)^{1/2}$ ,  $\lambda_{Di(e)}$   $= [T_{i(e)}/4\pi e^2 n_{i(e)}]^{1/2}$ ,  $T_{i(e)}$  is the ion (electron) temperature in energy unit,  $n_{i(e)}$  is the ion (electron) number density, and  $e$  is the magnitude of an electron charge. Now, using  $T_i n_e \ll T_e n_i$  (which is a good approximation [33] for any dusty plasma with highly negatively charged dust, i.e., for  $q_d = -Z_d e$  with  $Z_d$  being the number of electrons residing on the dust grain surface) and  $T_i \nabla n_i = -n_i e \nabla \phi$  (with  $\phi$  being the electrostatic potential), one can simplify the polarization force as  $\mathbf{F}_p = -Z_d e R (n_i/n_{i0})^{1/2} \nabla \phi$ , where  $R = Z_d e^2 / 4 T_i \lambda_{Di0}$  is a parameter determining the effect of polarization force (which arises due to the interaction between thermal ions and highly negatively charged dust grains),  $n_{i0}$  is the ion number density at equilibrium, and  $\lambda_{Di0} = (T_i / 4\pi n_{i0} e^2)^{1/2}$ . It is obvious that (i) the polarization force is independent of the polarity of the dust grain, (ii) it is directed opposite to the electrostatic force ( $\mathbf{F}_e = Z_d e \nabla \phi$ ), and (iii) in a dusty plasma with highly negatively charged dust, the polarization force arises mainly due to the polarization of plasma ions around the dust grain. The polarization force for some other dusty plasma situations has been explained by Hamaguchi and Farouki [31,32] and Khrapak *et al.* [33].

We, in our present work, have taken into account the effects of polarization force and effective dust temperature and have rigorously investigated the DA solitary and shock waves propagating in a strongly coupled dusty plasma containing Maxwellian electrons, Maxwellian ions, and strongly coupled negatively charged dust. The paper is organized as follows. The basic equations governing the strongly coupled dusty plasma system (under consideration) are given in Sec. II. The basic features of the DA solitary waves are investigated in Sec. III, whereas those of the DA shock waves are investigated in Sec. IV. A brief discussion is finally presented in Sec. V.

## II. GOVERNING EQUATIONS

We consider the nonlinear propagation of the DA waves [13] in a strongly coupled dusty plasma whose constituents are negatively charged dust, electrons, and ions. Thus, at equilibrium we have  $Z_d n_{d0} + n_{e0} = n_{i0}$ , where  $n_{d0}$  ( $n_{e0}$ ) is the equilibrium dust (electron) number density. We assume that electrons and ions are weakly coupled due to their higher temperatures and smaller electric charges, and that dust is strongly coupled because of its lower temperature and larger electric charge. Thus, in the presence of the low phase velocity (in comparison with electron and ion thermal velocities) DA waves, the electron and ion number densities obey the Maxwellian distribution, and their densities  $n_e$  and  $n_i$  are, respectively, given by

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right), \quad (1)$$

$$n_i = n_{i0} \exp\left(-\frac{e\phi}{T_i}\right). \quad (2)$$

The dynamics of the nonlinear DA waves in such a strongly coupled dusty plasma is governed by the following well-known generalized hydrodynamic equations [15,37]:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \quad (3)$$

$$D_\tau \left[ m_d n_d D_t u_d - Z_d e n_d \frac{\partial \phi}{\partial x} + Z_d e n_d R \left( \frac{n_i}{n_{i0}} \right)^{1/2} \frac{\partial \phi}{\partial x} + T_{ef} \frac{\partial n_d}{\partial x} \right] = \eta_l \frac{\partial^2 u_d}{\partial x^2}, \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e [n_e - n_i + Z_d n_d], \quad (5)$$

where  $n_d$  is the dust number density,  $u_d$  is the dust fluid speed,  $t(x)$  is the time (space) variable,  $m_d$  is the dust grain mass,  $T_{ef} = (\mu_d T_d + T_\star)$  is the effective dust temperature consisting of two parts: one ( $T_\star$ ) arising from electrostatic interactions among highly negatively charged dust grains and another ( $\mu_d T_d$ ) arising from the dust thermal pressure,  $D_\tau = 1 + \tau_m \partial / \partial t$ ,  $D_t = \partial / \partial t + u_d \partial / \partial x$ ,  $\tau_m$  is the viscoelastic relaxation time,  $\mu_d$  is the compressibility, and  $\eta_l$  is the longitudinal viscosity coefficient. We note that the third term on the left-hand side of Eq. (4) is due to the polarization force (which arises due to the interaction between thermal ions and highly negatively charged dust grains). There are various approaches for calculating these transport coefficients. These have been widely discussed in the literature [15,17,37–39]. The parameter  $T_\star$  (which arises from the electrostatic interactions among highly negatively charged dust), viscoelastic relaxation time  $\tau_m$ , and the compressibility  $\mu_d$  are given by [34,35,37,39]

$$T_\star = \frac{N_{mn} z_d^2 e^2}{3 a_d} (1 + \kappa) e^{-\kappa}, \quad (6)$$

$$\tau_m = \frac{\eta_l}{n_{d0} T_d} \left[ 1 - \mu_d + \frac{4}{15} u(\Gamma) \right]^{-1}, \quad (7)$$

$$\mu_d = \frac{1}{T_d} \frac{\partial P_d}{\partial n_d} = 1 + \frac{1}{3} u(\Gamma) + \frac{\Gamma}{9} \frac{\partial u(\Gamma)}{\partial \Gamma}, \quad (8)$$

where  $N_{mn}$  is determined by the dust structure and corresponds to the number of nearest neighbors (viz., in crystalline state  $N_{mn} = 8$  for bcc lattice,  $N_{mn} = 12$  for fcc lattice, etc.),  $\kappa = a_d / \lambda_D$ ,  $a_d$  is the intergrain distance ( $a_d \approx n_{d0}^{-1/3}$ ), and  $\lambda_D$  is the screening length of the dusty plasma.  $u(\Gamma)$  is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ( $\Gamma < 1$ ) as [15,17]  $u(\Gamma) \approx -(\sqrt{3}/2)\Gamma^{3/2}$ . To express  $u(\Gamma)$  in terms of  $\Gamma$  for a range of  $1 < \Gamma < 100$ , Slattery *et al.* [38] analytically derived a relation

$$u(\Gamma) \approx -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81, \quad (9)$$

where a small correction term due to a finite number of particles is neglected. The dependence of the other transport coefficient  $\eta$  on  $\Gamma$  is somewhat more complex and cannot be expressed in such a closed analytical form. However, tabulated and graphical results of their functional behavior derived from molecular-dynamics simulations and a variety of statistical schemes are available in the literature [37].

### III. SOLITARY WAVES

To derive a dynamical equation for the electrostatic DA solitary waves from our basic equations (1)–(5), we employ the reductive perturbation technique [40]. We first introduce the stretched coordinates [40]

$$\xi = \epsilon^{1/2}(x - V_p t), \quad \tau = \epsilon^{3/2} t, \quad (10)$$

where  $\epsilon$  is a smallness parameter measuring the weakness of the dispersion and  $V_p$  is the phase speed of the DA waves. We can expand the perturbed quantities  $n_d$ ,  $u_d$ , and  $\phi$  about the equilibrium values in power series of  $\epsilon$  as

$$n_d = n_{d0} + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \dots, \quad (11)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \dots, \quad (12)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots. \quad (13)$$

We now use Eqs. (10)–(13) in Eqs. (3)–(5) and develop equations in various powers of  $\epsilon$ . To the lowest order in  $\epsilon$ , i.e., taking the coefficients of  $\epsilon^{3/2}$  from both sides of Eqs. (3) and (4), and  $\epsilon$  from both sides of Eq. (5), one can obtain the first-order continuity equation, momentum equation, and Poisson's equation which, in turn, give

$$n_d^{(1)} = -\frac{Z_d e n_{d0} (1-R) \phi^{(1)}}{m_d V_p^2 - \mu_d T_d - T_\star}, \quad (14)$$

$$u_d^{(1)} = -\frac{Z_d e V_p (1-R) \phi^{(1)}}{m_d V_p^2 - \mu_d T_d - T_\star}, \quad (15)$$

$$\frac{V_p}{C_d} = \sqrt{\gamma_\star + (1-\alpha) \frac{(1-R)}{(1+\alpha\sigma)}}, \quad (16)$$

where  $C_d = (Z_d T_i / m_d)^{1/2}$ ,  $\gamma_\star = T_{ef} / Z_d T_i$ ,  $\alpha = n_{e0} / n_{i0}$ , and  $\sigma = T_i / T_e$ . Equation (16) represents the linear dispersion relation for the DA waves propagating in such a strongly coupled dusty plasma in which dust mass provides the inertia and electron and ion thermal pressures provide the restoring force. It is obvious from Eq. (16) that the phase speed ( $V_p$ ) is decreased by  $R$  (i.e. by polarization force), but increased by  $T_{ef}$  (i.e. by effective dust temperature).

To the next higher order in  $\epsilon$ , i.e. taking the coefficients of  $\epsilon^{5/2}$  from both sides of Eqs. (3) and (4), and  $\epsilon^2$  from both sides of Eq. (5), one can obtain another set of coupled equations for  $n_d^{(2)}$ ,  $u_d^{(2)}$ , and  $\phi^{(2)}$ , which—along with the first set of coupled linear equations for  $n_d^{(1)}$ ,  $u_d^{(1)}$ , and  $\phi^{(1)}$ —reduce to a nonlinear dynamical equation of the form

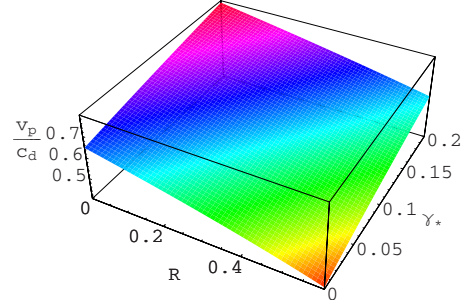


FIG. 1. (Color online) The variation of  $V_p/C_d$  with  $R$  and  $\gamma_\star$  for  $n_{i0} = 7 \times 10^7 \text{ cm}^{-3}$ ,  $n_{e0} = 4 \times 10^7 \text{ cm}^{-3}$ ,  $T_e = 8 \text{ eV}$ , and  $T_i = 0.3 \text{ eV}$ .

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \quad (17)$$

where the nonlinear coefficient  $A$  and the dispersion coefficient  $B$  are given by

$$A = -\frac{Z_d e (1-R) V_p}{\mathcal{E}_d} \left[ 1 - \frac{(1-\alpha)(1-\alpha\sigma^2) \mathcal{E}_d}{2(1+\alpha\sigma)^2 m_d V_p^2} \right] + \frac{\mathcal{E}_d}{2m_d V_p^2} \left( 1 - \frac{(1-\alpha)R}{2(1+\alpha\sigma)} \right), \quad (18)$$

$$B = \frac{\mathcal{E}_d^2}{8\pi e^2 Z_d^2 m_d V_p n_{d0} (1-R)}, \quad (19)$$

where  $\mathcal{E}_d = m_d V_p^2 - \mu_d T_d - T_\star$ . Equation (17) is the well-known KdV (Korteweg-de Vries) equation describing the nonlinear propagation of the DA waves in the dusty plasma system under consideration. It is obvious from Eq. (18) that the nonlinear coefficient  $A$  is always negative since  $0 < \sigma < 1$ ,  $0 \leq \alpha < 1$ ,  $0 \leq R < 1$ ,  $0 \leq \gamma_\star < 1$ , and  $\mathcal{E}_d \leq m_d V_p^2$  must be valid for any dusty plasma system containing electrons, ions, and negatively charged dust.

The stationary solitary wave solution of the KdV equation (17) is obtained by transforming the independent variables to  $\zeta = \xi - U_0 \tau'$  and  $\tau' = \tau$ , where  $U_0$  is the speed of the solitary waves, and imposing the appropriate boundary conditions, viz.,  $\phi^{(1)} \rightarrow 0$ ,  $d\phi^{(1)}/d\zeta \rightarrow 0$ , and  $d^2\phi^{(1)}/d\zeta^2 \rightarrow 0$  at  $\zeta \rightarrow \pm\infty$ . Thus, one can express the stationary solitary wave solution of the KdV equation (17) as

$$\phi^{(1)} = \phi_m^{(1)} \text{sech}^2(\zeta/\delta), \quad (20)$$

where the amplitude  $\phi_m^{(1)}$  and the width  $\delta$  are given by

$$\phi_m^{(1)} = 3U_0/A, \quad (21)$$

$$\delta = \sqrt{4B/U_0}. \quad (22)$$

It is obvious from Eq. (21) that the dusty plasma system under consideration supports the DA solitary waves with negative potential only since  $A$  is always negative for any dusty plasma system containing electrons, ions, and negatively charged dust. Figures 1–6 show how the basic features (phase speed, amplitude, and width) of the DA solitary waves are modified by  $R$ ,  $\gamma_\star$ ,  $\alpha$ , and  $\sigma$ . Figure 1 shows that the phase speed of the DA solitary waves decreases with  $R$ ,

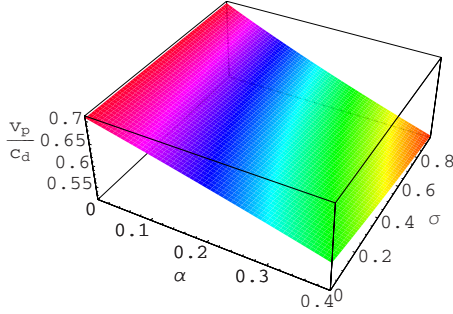


FIG. 2. (Color online) The variation of  $V_p/C_d$  with  $\alpha$  and  $\sigma$  for  $\gamma_*=0.1$  and  $R=0.6$ .

but increases with  $\gamma_*$ . Figure 2 shows that the phase speed of the DA solitary waves decreases with  $\alpha$  and  $\sigma$ . On the other hand, Figs. 3–6 show the variation of the amplitude and width of such DA solitary waves. Figures 3 and 4 imply that the amplitude of the DA solitary waves increases with  $R$  and  $\gamma_*$ , but decreases with  $\alpha$  and  $\sigma$ . On the other hand, Figs. 5 and 6 indicate that the width of the DA solitary waves decreases with  $R$ ,  $\gamma_*$ ,  $\alpha$ , and  $\sigma$ . It may be noted that for  $R=0$  and  $\eta_l=0$ , the results obtained from our present investigation completely agree with those of Mamun and Shukla [36].

IV. SHOCK WAVES

To derive a dynamical equation for the DA shock waves from our basic equations (1)–(5), we again employ the reductive perturbation technique [40] with another stretched coordinates [41],

$$\xi = \epsilon(x - V_p t), \quad \tau = \epsilon^2 t. \tag{23}$$

We now use Eqs. (11)–(13) and (24) in Eqs. (3)–(5) and develop equations in various powers of  $\epsilon$  as before.

To the lowest order in  $\epsilon$ , i.e., taking the coefficients of  $\epsilon^2$  from both sides of Eqs. (3) and (4), and  $\epsilon$  from both sides of Eq. (5), one can obtain the first-order continuity equation, momentum equation, and Poisson’s equation which are exactly identical to Eqs. (14)–(16). Therefore, the variation of the phase speed of the DA shock waves with  $R$ ,  $\gamma_*$ ,  $\alpha$ , and  $\sigma$  will be the same as shown in Figs. 1 and 2, i.e., the phase speed of the DA shock waves decreases with  $R$ ,  $\alpha$ , and  $\sigma$ , but increases with  $\gamma_*$ .

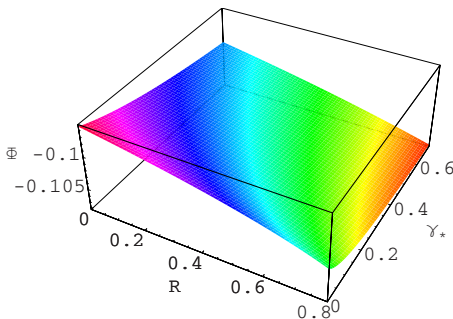


FIG. 3. (Color online) The variation of  $\Phi (=e\phi_m^{(1)}/T_i)$  with  $R$  and  $\gamma_*$  for  $U_0/C_d=0.1$ ,  $n_{i0}=7 \times 10^7 \text{ cm}^{-3}$ ,  $n_{e0}=4 \times 10^7 \text{ cm}^{-3}$ ,  $T_e=8 \text{ eV}$ , and  $T_i=0.3 \text{ eV}$ .

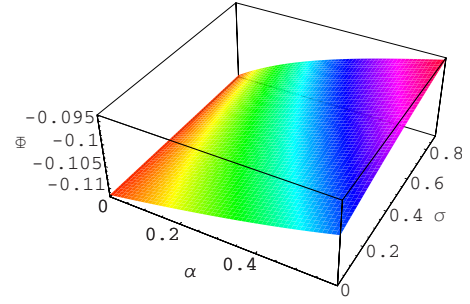


FIG. 4. (Color online) The variation of  $\Phi (=e\phi_m^{(1)}/T_i)$  with  $\alpha$  and  $\sigma$  for  $U_0/C_d=0.1$ ,  $R=0.5$ , and  $\gamma_*=0.1$ .

To the next higher order in  $\epsilon$ , i.e., taking the coefficients of  $\epsilon^3$  from both sides of Eqs. (3) and (4), and  $\epsilon^2$  from both sides of Eq. (5), one can obtain another set of coupled equations for  $n_d^{(2)}$ ,  $u_d^{(2)}$ , and  $\phi^{(2)}$ , which along with the first set of coupled linear equations for  $n_d^{(1)}$ ,  $u_d^{(1)}$ , and  $\phi^{(1)}$  reduce to a nonlinear dynamical equation of the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{24}$$

where the nonlinear coefficient  $A$  is exactly the same as that appeared in Eq. (18), and the dissipation coefficient  $C$  is given by

$$C = \frac{1}{2} \frac{\eta_l}{n_{d0} m_d}. \tag{25}$$

Equation (24) is the well-known Burgers equation describing the nonlinear propagation of the DA waves in the dusty plasma system under consideration. It is obvious from Eqs. (24) and (25) that the dissipative term, i.e., the right-hand side of Eq. (24) is due to the strong correlation among the charged dusts.

We are now interested in looking for the stationary shock wave solution of Eq. (24) by introducing  $\zeta = \xi - U_0 \tau'$  and  $\tau' = \tau$ , where  $U_0$  is the shock wave speed (in the reference frame). This leads us to write Eq. (24), under the steady-state condition ( $\partial/\partial \tau' = 0$ ), as

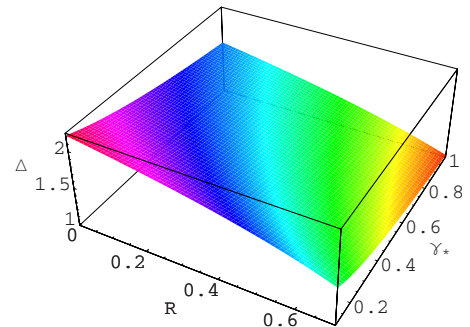


FIG. 5. (Color online) The variation of  $\Delta (= \delta/\lambda_{Di0})$  with  $R$  and  $\gamma_*$  for  $U_0/C_d=0.1$ ,  $n_{i0}=7 \times 10^7 \text{ cm}^{-3}$ ,  $n_{e0}=4 \times 10^7 \text{ cm}^{-3}$ ,  $T_e=8 \text{ eV}$ , and  $T_i=0.3 \text{ eV}$ .

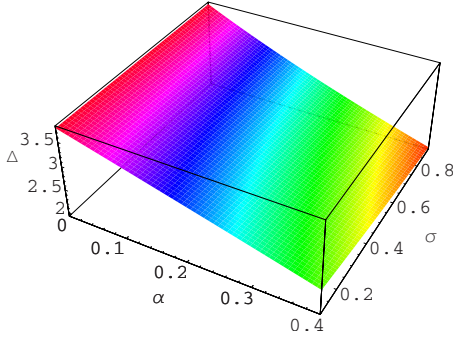


FIG. 6. (Color online) The variation of  $\Delta$  ( $=\delta/\lambda_{D0}$ ) with  $\alpha$  and  $\sigma$  for  $U_0/C_d=0.1$ ,  $R=0.4$ , and  $\gamma_*=0.1$ .

$$-U_0 \frac{\partial \phi^{(1)}}{\partial \zeta} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}. \quad (26)$$

It can be easily shown [42] that Eq. (26) describes the shock waves whose speed  $U_0$  (in the reference frame) is related to the extreme values  $\phi^{(1)}(-\infty)$  and  $\phi^{(1)}(\infty)$  by  $\phi^{(1)}(\infty) - \phi^{(1)}(-\infty) = 2U_0/A$ . Thus, under the condition that  $\phi^{(1)}$  is bounded at  $\zeta = \pm\infty$ , the shock wave solution of Eq. (26) is [42]

$$\phi^{(1)} = \phi_m^{(1)} [1 - \tanh(\zeta/\delta)], \quad (27)$$

where  $\phi_m^{(1)} = U_0/A$  and  $\delta = 2C/U_0$  are the height and the thickness of the shock waves, respectively. It is obvious that the formation of such shock waves is due to the strong correlation among the charged dust grains. It is clear from Eq. (25) that the shock height is independent of the longitudinal viscosity coefficient ( $\eta_l$ ), but the shock thickness is directly proportional to the longitudinal viscosity coefficient  $\eta_l$  (i.e., the shock thickness increases with increasing the longitudinal viscosity coefficient) and is inversely proportional to the equilibrium dust grain mass density ( $\rho_{d0} = n_{d0} m_d$ ). It may be noted that for  $T_* = 0$  and  $R = 0$ , the results obtained from our present investigation completely agree with those of Shukla and Mamun [25]. As we have  $\phi_m^{(1)} = U_0/A$  for shock waves and  $\phi_m^{(1)} = 3U_0/A$  for solitary waves, the height of the shock waves will differ by an amount of 1/3 from that of the solitary waves. The variation of shock height is exactly the same as that of the solitary wave amplitude, i.e., the variation of the shock height can be represented by Figs. 3 and 4.

We note that, in our numerical analysis, we have used the parameter ranges corresponding to the experimental conditions of Bandyopadhyay *et al.* [43]:  $n_{i0} = 7 \times 10^7 \text{ cm}^{-3}$ ,  $n_{e0} = 4 \times 10^7 \text{ cm}^{-3}$ ,  $T_e = 8 \text{ eV}$ ,  $T_i = 0.3 \text{ eV}$ ,  $R = 0-1$ , and  $\gamma_* = 0-1$ . Our numerical analysis shows that for the experimental conditions of Bandyopadhyay *et al.* [43], the magnitude of the amplitude of the DA solitary waves varies from  $\sim 0.095$  to  $\sim 0.105$  and the width of the DA solitary waves varies from  $\sim 1.5$  to  $\sim 3.5$ .

It is important to point out that the strongly coupled dusty plasma system represented by Eqs. (1)–(9) supports both solitary and shock waves, and that solitary (shock) waves are formed due to the balance between nonlinearity and dispersion (dissipation). The dispersion comes from the left-hand-side term of Eq. (5), which is due to the nonzero net charge

density, i.e., due to  $n_i \neq n_e + Z_d n_d$ . On the other hand, the dissipation arises due to the effects of strong correlation, particularly due to the longitudinal viscosity  $\eta_l$  appeared in the right-hand-side term ( $\eta_l \partial^2 u_d / \partial x^2$ ) of Eq. (4). It is usual that when dispersion (dissipation) effect is much more dominant than dissipation (dispersion) effect, and the dissipation (dispersion) effect is neglected, the dust plasma system represented by Eqs. (1)–(9) supports solitary (shock) waves. To neglect the effect of dispersion in comparison with that of dissipation or vice versa, one has to choose a suitable scaling (stretching of coordinates) that we used in our investigation of solitary and shock waves. However, if it would be possible to keep both effects, i.e., possible to derive the KdV-Burgers equation [42] by choosing appropriate stretched coordinates (which have not been found so far), one would have oscillatory shock structures in which the first few oscillations at the wave front will be close to solitons [42]. We note that one could derive the KdV-Burgers equation by using  $\eta_l = \epsilon^{1/2} \eta_0$ , which is not correct from both physical and mathematical points of view since this additional stretching ( $\eta_l = \epsilon^{1/2} \eta_0$ ) leads to the dissipation coefficient to contain the expansion parameter  $\epsilon$ .

It should be mentioned that in our present investigation we have neglected the effects of nonuniform plasma density (density gradient at equilibrium) since we are interested in examining the nonlinear propagation of the long-wavelength dust-acoustic waves. This approximation is valid [33] as long as the wavelength of the dust-acoustic waves is much larger than the dust density inhomogeneity scale length  $L_n = n_{d0} / (\partial n_{d0} / \partial x)$ . However, one can include the effect of dust density inhomogeneity by using a different stretching of the space coordinate ( $x$ ) given by Singh and Rao [44]. It may also be added here that our investigation is valid for small amplitude one-dimensional DA solitary and shock waves and for unmagnetized and uniform dusty plasma limits. However, arbitrary amplitude DA solitary or shock waves in uniform or nonuniform dusty plasma with or without the effects of obliqueness and external magnetic field are also problems of recent interest for many space and laboratory dusty plasma situations, but are beyond the scope of our present investigation.

## V. DISCUSSION

We have considered a consistent and realistic dusty plasma system containing Maxwellian electrons and ions and negatively charged dust fluid, and studied the effects of polarization force and effective dust thermal energy on DA solitary and shock waves. The findings of our present investigation can be pointed out as follows:

(1) The phase speed of the DA waves,  $V_p$  (which is the critical phase speed of the DA waves for which the DA solitary or shock waves are formed), is decreased by the effects of polarization force ( $R$ ), free-electron number density ( $\alpha$ ), and ion temperature ( $\sigma$ ), but is increased by the effective dust temperature ( $\gamma_*$ ).

(2) The DA solitary waves, which are due to the balance between nonlinearity and dispersion, exist with negative potential only. The amplitude (width) of the DA solitary waves

increases (decreases) with  $R$  and  $\gamma^*$ , keeping the product of the amplitude and square of the width constant. However, both the amplitude and the width of the DA solitary waves decrease with  $\alpha$  and  $\sigma$ .

(3) The DA shock waves, which are due to the balance between nonlinearity and dissipation, exist with negative potential only. The strong correlation among the charged dust grains is the source of dissipation and is, therefore, responsible for the formation of the DA shock structures.

(4) The shock height is increased by the effects of polarization force ( $R$ ) and effective dust temperature ( $\gamma_*$ ), but is decreased by the effects of free-electron number density ( $\alpha$ ) and ion temperature ( $\sigma$ ).

(5) The shock thickness is directly proportional to the longitudinal viscosity coefficient ( $\eta_l$ ) and is inversely proportional to the equilibrium dust mass density ( $\rho_{d0}$ ).

To conclude, the results of the present investigation should be useful for understanding the basic features of the localized DA solitary and shock waves in space and laboratory dusty plasmas. We finally suggest that a laboratory experiment be performed to test the theory presented in this work.

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