Control of resonant weak-light solitons via a periodic modulated control field

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We investigate propagation and control of weak-light spatial solitons in a resonant three-level atomic system with a periodic modulated control field. It is shown that the periodic modulation acts like periodic potential which resists the propagation of the soliton in transverse direction. The soliton could be trapped by the periodic potential in the input channel. When the modulation is canceled, the soliton propagates in its initial incident direction. The periodic modulation of control field could be used to control the propagation of the weak-light probe soliton. Due to the good localization efficiency of the periodic potential, an excellent switching is realized for the probe soliton. These properties may have potential applications in all-optical switching, optical information processing and other fields.

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Control of light propagation in periodic configuration medium has attracted intense research interest, due to its fascinating physical phenomena and potential applications in all-optical control and so on [1–5]. Nonlinearity in such configuration may give rise to lattice solitons or discrete solitons [6,7], and they have also been observed in experiments recently [6,8–10]. This periodic can be transverse [2–4,6], longitudinal [11], or simultaneous in transverse and longitudinal directions [12,13]. They all have precision control functions on the propagation of spatial solitons in a homogeneous periodic configuration [2,4] or at the interface of two different periodic materials [7]. However, previous researches mainly utilize far off-resonant excitation and intense laser fields to generate enough nonlinearity and avoid large absorption. By virtue of quantum interference effect, electromagnetically induced transparency (EIT) windows can be generated in resonant medium [14], which provide the possibility to form solitons at weak-light intensity [15–18]. Due to the requirement of low light intensity, weak-light solitons imply superiority in potential applications. Very recently, Hang *et al.* have studied weak-light bright and dark solitons in atomic system with a resonant standing wave control field [19], in which the standing wave field serving as a stabilizing factor for the soliton.

In this paper, we study propagation and control of weak-light spatial soliton in a resonant lambda-type atomic system. Through introducing the periodic modulation on the control field, the refractive index of the medium is modified periodically for the probe field, which is equivalent to create periodic potential in transverse direction. Different from the standing wave field, the periodic modulated control field ensures the medium always in EIT conditions. Then the medium plays the role as a periodic medium and provides some propagation channels for the probe soliton. Whether the soliton propagates in its incident direction or the soliton is well trapped in the input channel could be switched by the periodic modulation. The localization efficiency of the trapped soliton is very close to 1, which shows good switching performance of the periodic potential. These properties may have potential applications in optical information processing, optical engineering and so on.

We carry out our study in a closed resonant three-level atomic system, as illustrated in Fig. 1. A lifetime broadened lambda-type atomic system interacts with a weak, pulsed probe field ($|1\rangle \rightarrow |2\rangle$ transition) and a strong, continuous wave control field ($|3\rangle \rightarrow |2\rangle$ transition) with transverse periodic modulation. Due to spontaneous emission, there exists a decay of each atomic state. In this scheme, we consider that the state $|1\rangle$ is the ground state and the decay rate of the state $|3\rangle$ is very small, which can be realized by adopting a hyperfine ground state or a metastable state. Under rotating-wave approximation, the Hamiltonian of the system in the interaction picture is expressed by

$$H_{\mathrm{int}} = -\hbar(\Delta_1|2\rangle\langle 2| + (\Delta_1 - \Delta_2)|3\rangle\langle 3|) - \hbar(\Omega_p|2\rangle\langle 1| + \Omega_c|3\rangle\langle 2| + \mathrm{H.c.}). \tag{1}$$

From the Hamiltonian and the Liouville equation, we obtain the following motion equations for the density matrix elements describing the atomic response,

$$\dot{\rho}_{11} = i(\Omega_p^*\rho_{21} - \Omega_p\rho_{12}) + \gamma_{21}\rho_{22} + \gamma_{31}\rho_{33}, \tag{2}$$

$$\dot{\rho}_{33} = i(\Omega_c\rho_{23} - \Omega_c^*\rho_{32}) + \gamma_{23}\rho_{22} - \gamma_{31}\rho_{33}, \tag{3}$$

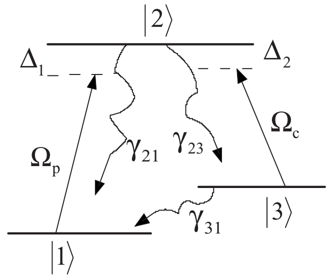

FIG. 1. Energy level diagram and laser excitation scheme.

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$$\dot{\rho}_{21} = i(\Omega_p\rho_{11} - \Omega_p\rho_{22} + \Omega_c^*\rho_{31}) + (i\Delta_1 - \Gamma_{21})\rho_{21}, \quad (4)$$

$$\dot{\rho}_{23} = i(\Omega_p\rho_{13} - \Omega_c^*\rho_{22} + \Omega_c^*\rho_{33}) + (i\Delta_2 - \Gamma_{23})\rho_{23}, \quad (5)$$

$$\dot{\rho}_{31} = i(\Omega_c\rho_{21} - \Omega_p\rho_{32}) + [i(\Delta_1 - \Delta_2) - \Gamma_{31}]\rho_{31}, \quad (6)$$

together with $\rho_{ij}=(\rho_{ji})^*$ $(i\neq j)$ and the conservation condition $\rho_{11}+\rho_{22}+\rho_{33}=1$. Here, $2\Omega_{p(c)}$ is the Rabi frequency of the probe (control) field for the relevant transition, expressed by $\Omega_{p(c)}=|\mu_{21(3)}|E_{p(c)}/(2\hbar)$ with μ_{ij} being the dipole matrix element of the transition $|i\rangle\rightarrow|j\rangle$. $\Delta_1=\omega_p-\omega_{21}$ and $\Delta_2=\omega_c-\omega_{23}$ are the detunings of probe field and control field from corresponding resonant transition (ω is angular frequency). γ_{ij} is the decay rates from state $|i\rangle$ to state $|j\rangle$. The coherence decay rates are defined as $\Gamma_{21}=(\gamma_{21}+\gamma_{23})/2+\gamma_{21}^{col}$, $\Gamma_{23}=(\gamma_{21}+\gamma_{23}+\gamma_{31})/2+\gamma_{23}^{col}$ and $\Gamma_{31}=\gamma_{31}/2+\gamma_{31}^{col}$ with γ_{ij}^{col} arising from the collision broadening.

Considering the probe-field propagation along z axis and diffraction along x axis, it can be described by $\vec{E}_p(x,z,t)=\frac{1}{2}E_p\exp(ik_pz-i\omega_pt)+\text{c.c.}$ with E_p being a slowly varying function of time t and distance z and c.c. standing for complex conjugate operation. The interaction of atoms with fields induces a polarization that oscillates at the frequency of the weak-probe field. By performing a quantum average of the dipole moment over the ensemble of the atoms, we find the polarization

$$\vec{P}(x,z,t) = N\mu_{12}\rho_{21}\exp(ik_pz - i\omega_pt) + \text{c.c.}, \quad (7)$$

where N being the atomic density. The evolution of the weak-probe field is described by the Maxwell equation

$$\nabla^2\vec{E}_p - \frac{1}{c^2}\frac{\partial^2\vec{E}_p}{\partial t^2} = \frac{1}{\varepsilon_0c^2}\frac{\partial^2\vec{P}_p}{\partial t^2}. \quad (8)$$

Under slowly varying envelope approximation, i.e., $\partial E_p/\partial z \ll k_pE_p$ and $\partial E_p/\partial t \ll \omega_pE_p$, then $(\frac{\partial}{\partial z}-\frac{1}{c}\frac{\partial}{\partial t})E_p\cong 2ik_pE_p$ and Eq. (8) is reduced to

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p + \frac{c}{2\omega_p}\frac{\partial^2\Omega_p}{\partial x^2} + \kappa\rho_{21} = 0, \quad (9)$$

here the coefficient $\kappa=N\omega_p|\mu_{12}|^2/(2\varepsilon_0\hbar c)$ with c being the light velocity in vacuum. We consider a steady state propagation regime in which the probe-field envelopes have large enough temporal widths and hence their time evolution could be neglected [16,20,21]. Thus, the probe beam is governed by the following wave equation

$$i\frac{\partial\Omega_p}{\partial z} + \frac{c}{2\omega_p}\frac{\partial^2\Omega_p}{\partial x^2} + \kappa\rho_{21} = 0. \quad (10)$$

The intensity of the probe field Ω_p is taken to be much weaker than that of the control field Ω_c, which dresses the levels $|2\rangle$ and $|3\rangle$ and leads to the destructive interference for the transitions from the ground state to the two dressed levels, then the depletion of the ground state $|1\rangle$ will be not significant. We use the perturbation method to solve the motion equations of density matrix elements in steady state and the density matrix elements are expressed as $\rho_{ij}=\sum_{n=0}^{+\infty}\rho_{ij}^{(n)}$. Assuming the system is prepared initially on the ground state, the zeroth-order solution will be $\rho_{11}^{(0)}=1$ and other elements are equal to zero. Under the weak-probe approximation, we get the matrix element ρ_{21} up to the third order:

$$\rho_{21}^{(1)} = \frac{d_3\Omega_p}{D}, \quad (11)$$

$$\rho_{21}^{(2)} = 0, \quad (12)$$

$$\rho_{21}^{(3)} = \frac{-d_3[2|\Omega_c|^2 + d_2d_3 + d_3^2(4|\Omega_c|^2 + \gamma^2 + \Delta_2^2)/|\Omega_c|^2]}{D|D|^2} \times |\Omega_p|^2\Omega_p, \quad (13)$$

where the new parameters $d_2=\Delta_1+i\gamma$, $d_3=\Delta_1-\Delta_2$ and $D=|\Omega_c|^2-d_2d_3$ are introduced for convenience. In the derivation of Eqs. (11)–(13), we select a metastable state as state $|3\rangle$ which makes γ_{31} be negligible ($\gamma_{31}=0$), and take $\gamma_{21}=\gamma_{23}=\gamma$ for simplicity. In addition, the collision effect in cold dilute atomic gas is very weak so it has been ignored in this paper. Thus we obtain the following nonlinear Shrödinger (NLS) equation governing the propagation of the weak-probe field:

$$i\frac{\partial\Omega_p}{\partial z} + \frac{c}{2\omega_p}\frac{\partial^2\Omega_p}{\partial x^2} + \kappa\frac{d_3}{D}[1 - (m_r + im_i)|\Omega_p|^2]\Omega_p = 0, \quad (14)$$

here $m_i=d_3\gamma/|D|^2$, $m_r=[2|\Omega_c|^2+\Delta_1d_3+d_3^2(4|\Omega_c|^2+\gamma^2+\Delta_2^2)/|\Omega_c|^2]/|D|^2$.

In order to make analysis convenient, we introduce some new variables $\zeta=z/L_d$, $\eta=x/R$, $L_d=\omega_pR^2/c$, $u=\Omega_p/\Omega_{p0}$, $\Omega_{p0}=1/\sqrt{m_0}$ and $m_0=m_r(\Omega_c=\Omega_{c0})$ to further simplify the above NLS equation, where L_d and R are characterization diffraction length and beam radius, respectively. Then Eq. (14) is transformed to the following dimensionless equation:

$$i\frac{\partial u}{\partial\zeta} + \frac{1}{2}\frac{\partial^2u}{\partial\eta^2} + (a_r + ia_i)\left(1 - \frac{m_r + im_i}{m_0}|u|^2\right)u = 0, \quad (15)$$

with

$$a_r = \kappa L_d\frac{[|\Omega_c|^2 - \Delta_1(\Delta_1 - \Delta_2)](\Delta_1 - \Delta_2)}{[|\Omega_c|^2 - \Delta_1(\Delta_1 - \Delta_2)]^2 + (\Delta_1 - \Delta_2)^2\gamma_2^2}, \quad (16)$$

and

$$a_i = \kappa L_d\frac{\gamma_2(\Delta_1 - \Delta_2)^2}{[|\Omega_c|^2 - \Delta_1(\Delta_1 - \Delta_2)]^2 + (\Delta_1 - \Delta_2)^2\gamma_2^2} \quad (17)$$

relating to the control field and detunings. Noticing that $a_i \propto (\Delta_1-\Delta_2)^2$ when $|\Omega_c|^2-\Delta_1(\Delta_1-\Delta_2)\neq 0$, so a_i can be greatly suppressed if let $(\Delta_1-\Delta_2)^2$ sufficiently small. Under this consideration, the absorption of the medium will play no significant role and the system could be approximately thought as a conservative one. In fact, this case can be easily realized by using a strong control field and choosing proper parameters just near the EIT window. With this consideration, $a_r \gg a_i$ and $m_r \gg m_i$ will be simultaneously achieved. If the control field is homogeneous ($\Omega_c=\Omega_{c0}$) and $a_r<0$, Eq. (15) should have the bright soliton solution of the form as $u=\frac{1}{\sqrt{|a_r|}}\frac{1}{l}\sec h(\frac{\eta-\alpha\zeta}{l})e^{i[\alpha\eta+a_r\zeta-(\alpha^2-1/l^2)\zeta/2]}$ with the incident angle

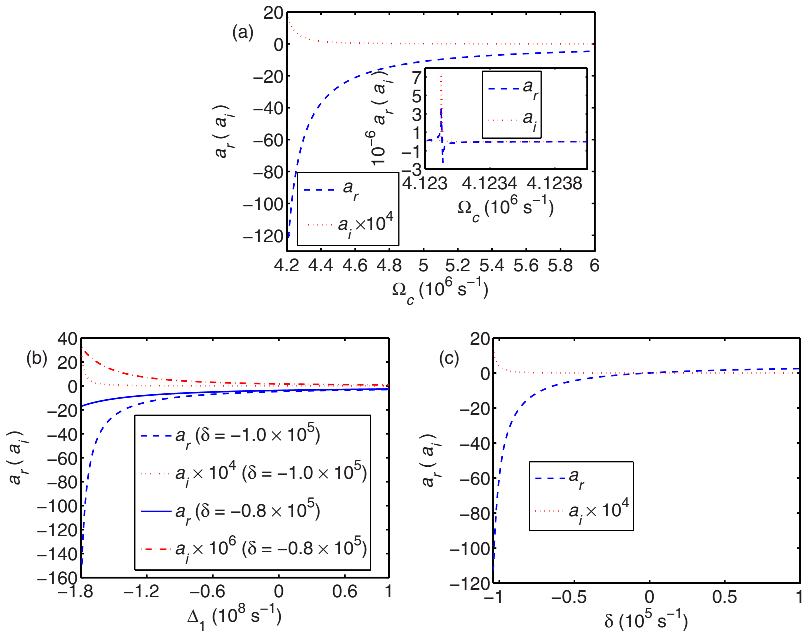

FIG. 2. (Color online) Dependencies of a_r and a_i on (a) the Rabi frequency of the control field Ω_c, (b) probe detuning Δ_1, and (c) two-photon detuning δ. Other parameters are given in the text.

α and the width l of the beam [19]. However, for the inhomogeneous control field, the coefficients a_r and m_r are complicated, and thus the analytical solution of Eq. (15) cannot be obtained.

From the above analysis, we notice that the condition of $a_r<0$ and $a_r \gg a_i$ is necessary for generating bright solitons in this system. Fortunately, this condition is easy to be satisfied by tuning the Rabi frequency of the control field Ω_c, probe detuning Δ_1 and two-photon detuning $\delta=\Delta_1-\Delta_2$, which determine a_r and a_i via Eqs. (16) and (17). We investigate the dependencies of a_r and a_i on those three parameters Ω_c, Δ_1, and δ, and show the results in Fig. 2, where we have considered our model in realistic atomic system via selecting energy levels of $^3S_0$, $^3P_1$ and $^3P_0$ as atomic states $|1\rangle$, $|2\rangle$, and $|3\rangle$ in laser-cooled gas of magnesium atoms, in which the decay time of $|2\rangle$ is about 5.1 ms [22]. In Fig. 2, the parameters $\kappa=4.0\times 10^8$ cm^{-1} s^{-1}, wavelength of probe beam $\lambda_p=457$ nm, $R=40$ μm, and then $L_d=2.2$ cm are employed. By taking $\Delta_1=-1.7\times 10^8$ s^{-1} and $\delta=-1.0\times 10^5$ s^{-1}, the dependency of a_r and a_i on the Rabi frequency of the control field Ω_c is shown in Fig. 2(a), from which one can see that with the increase of Ω_c the magnitude of a_r decreases and gradually approaches to zero, though the absorption coefficient, i.e., a_i, decreases simultaneously. The inset shows the dependency for smaller Ω_c, in which a_r is transformed from $a_r<0$ to $a_r>0$, accompanying greatly enhancement of the absorption coefficient a_i. We plot the dependency of $a_r(a_i)$ on the probe detuning Δ_1 in Fig. 2(b) when $\Omega_c=4.3\times 10^6$ s^{-1}, $\delta=-1.0\times 10^5$ s^{-1}, and $\delta=-0.8\times 10^5$ s^{-1}. It is shown that $a_r(a_i)$ will almost vanish when increasing the probe detuning along positive direction. Figure 2(c) shows the dependency of $a_r(a_i)$ on the two-photon detuning δ with $\Omega_c=4.3\times 10^6$ s^{-1} and $\Delta_1=-1.7\times 10^8$ s^{-1}, from which it can be seen that $a_r(a_i)$ decreases with δ and will keep a small magnitude for positive δ. It should be mentioned that at two-photon resonance ($\delta=0$), i.e., in the center of the EIT window, $a_r=a_i=0$ will be established [as shown in Fig. 2(c)], which is not expected for generating solitons and that is why we should choose parameters near the EIT window. Actually, in all these cases in Fig. 2, a_i is several orders of magnitude smaller than a_r except the case for the inset in Fig. 2(a).

We then consider the control field to be a periodic pattern of $\Omega_c=\Omega_{c0}[1+A\cos(B\eta)]$ with A characterizing modulation depth and B denoting modulation period. Such periodic modulation of the control field may be achieved by using a standing wave field formed by two beams with adverse transverse components of wave vector and same longitudinal components of wave vector. Comparing Eq. (15) with the NLS equation of continuous model describing solitons in optical lattice [4,5], one can see that the periodic modulation of the control field would also arise a periodic potential like the optical lattices or waveguide array for the probe field. So the atomic system may possess the similar properties with optical lattices or waveguide arrays. Each period takes effect as a propagation channel for the soliton. If the soliton is input in one channel with a tilt angle, it will meet the periodic potential barrier in transverse direction during propagation. The potential barrier will resist the transverse propagation of the soliton, and trap the soliton in the incident channel.

Based on the above analysis, the proper parameters to support the formation of the solitons are easy to be realized. By taking $\Omega_{c0}=4.3\times 10^6$ s^{-1}, $\Omega_{p0}=1.79\times 10^5$ s^{-1}, $\Delta_1=-1.7\times 10^8$ s^{-1}, $\delta=-1.0\times 10^5$ s^{-1}, $\kappa=4.0\times 10^8$ cm^{-1} s^{-1}, $\lambda_p=457$ nm, $R=40$ μm, and $L_d=2.2$ cm, $a_r \gg a_i$ and $m_r \gg m_i$ are achieved simultaneously. The weak-light spatial soliton will propagate stably and with almost no loss in this atomic system. According to the bright soliton solution of Eq. (15) with homogeneous control field, we use a hyperbolic secant beam of $u(\eta,0)=u_0 \sec h(\eta)e^{i\alpha\eta}$ with the amplitude $u_0=|a_r(\Omega_c=\Omega_{c0})|^{-1/2}$ as initial probe field.

As the periodic modulation acts like periodic potentials, the probe soliton may be trapped by the periodic potential, which provides a switching effect for the weak-probe soliton. We try to use the modulated control field to switch the probe

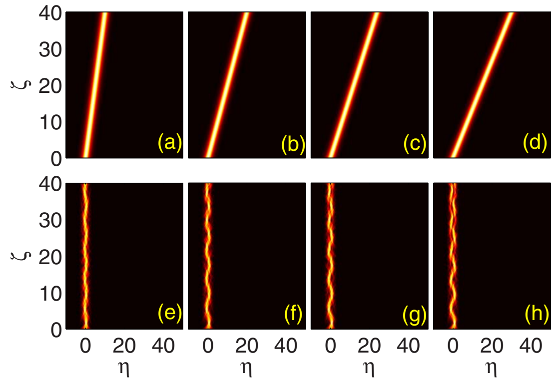

FIG. 3. (Color online) Soliton propagation in the medium with homogeneous control field (top panels) and being trapped by periodic modulated control field (bottom panels). In the subfigures, (a) and (e) $\alpha=0.5$; (b) and (f) $\alpha=1.0$; (c) and (g) $\alpha=1.2$; (d) and (h) $\alpha=1.5$.

soliton in a tilt direction. Top panels of Fig. 3 show the solitons propagate along their own incident direction when it is input at a certain angle and without the periodic modulation of control field. In this case, the medium acts as a homogeneous EIT medium for the probe solitons. However, the periodic potential could prevents the soliton from propagating in its incident direction. It makes the soliton be localized and propagate only in the input channel, as shown in the bottom panels of Fig. 3. Here we take the modulation depth $A=0.2$ and period $B=0.5$ in the derivation of the below panels of Fig. 3. The probe solitons is well trapped by the deep lattice potential and oscillate in a lattice channel. When the lattice width is too small, i.e., for larger periods, parts of the soliton energy would leak into other lattices. Therefore, we consider the lattice width for period $B=0.5$ to be commensurate to the soliton width. If considering smaller modulation period, the probe soliton could also be trapped, but oscillate in a wider lattice channel. For weak modulation of the control field, the probe soliton will traverse the lattice potential and cannot be trapped in the lattice channel. So we only consider the case of deep modulation of the control field here. Therefore, the periodic modulation of the control field may be used as a switching for trapping or releasing the soliton in the propagation direction.

To show clearly the trap of the soliton by the periodic potential, we plot the input and output profiles as well as the periodic modulation in Fig. 4 corresponding the cases of the below panels of Fig. 3. The soliton is completely localized in the region of modulation enhancement of a period, which implies that only the region of modulation enhancement plays the role of the propagation channels for the soliton. Though the input beam initially covers some region of weakened modulation, it will be eventually trapped into the adjacent propagation channel. The output intensity of the probe beam is almost stronger than its input intensity, due to its localization in the channel.

The localization efficiency of the soliton is evaluated by comparing the soliton power trapped in the channel with the total power of the probe beam. The soliton power localized in the input channel and the total power of the beam are calculated by $P_e=\int_{-\pi/2}^{+\pi/2}|u(\eta,\zeta)|^2 d\eta$ and $P_t=\int_{-\infty}^{+\infty}|u(\eta,\zeta)|^2 d\eta$, respectively. We compute the localization efficiency $R_p=P_e/P_t$ for the case of Fig. 3(f), and show the result in Fig. 5. It can be seen that a small rising of the localization efficiency appears at first due to radiation of the beam from the weakened modulation region into the propagation channel. With the propagation, the soliton is well trapped in the input channel and the localization efficiency is approximate to 1, which implies excellent localization performance of the periodic potential. The solitons input in other tilt angles have almost the same localization efficiency with Fig. 5, so they are not plotted in this paper any more.

In conclusion, we have investigated the propagation and control of the weak-light soliton via periodic modulation of the control field in an EIT medium. The periodic modulation modifies the refractive index to be periodic, which acts as periodic potential in transverse direction for the weak-probe

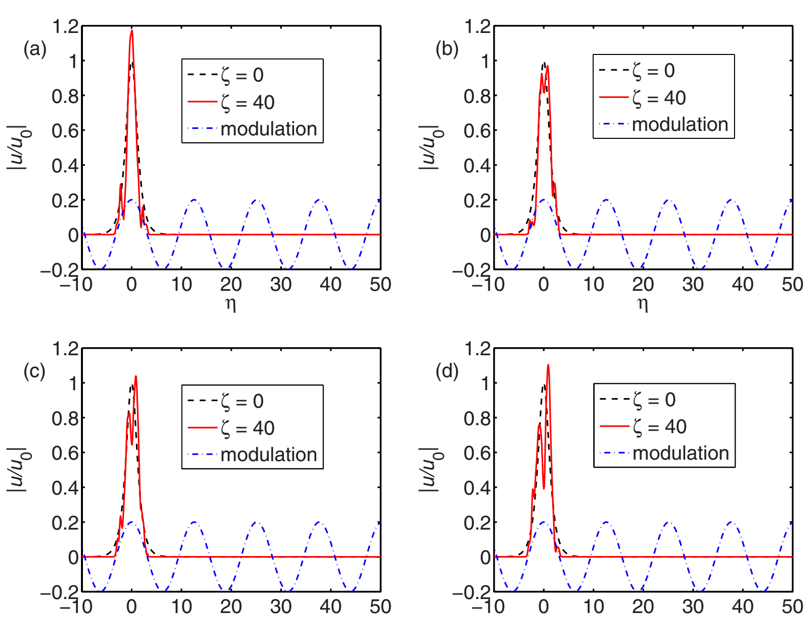

FIG. 4. (Color online) Input and output profiles normalized to the initial amplitude of the soliton and the periodic modulation pattern of the control field plotted corresponding respectively to the cases of Figs. 3(e)–3(h).

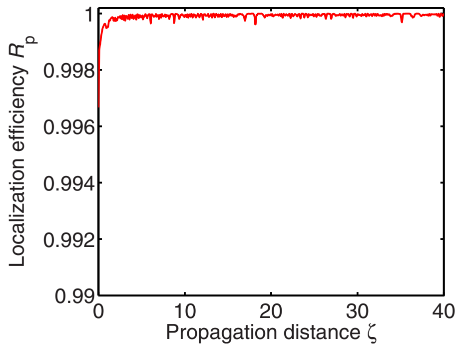

FIG. 5. (Color online) Variation of localization efficiency of the trapped soliton with the propagation distance.

soliton. In fact, the region of the modulation enhancement in a period is equivalent to a propagation channel for the soliton. The probe soliton could be well trapped in the input channel by the periodic potential. If there is no the periodic modulation, the soliton will propagate stably in its input direction. The good localization efficiency of the periodic potential implies its excellent switching performance. Therefore, the periodic modulation could be used for controlling the weak-light soliton, and function as a soliton switching. These properties may provide some advantages in soliton communication and optical information processing.

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