Nonlinear structure of ion-acoustic waves in completely degenerate electron-positron and ion plasma

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A rigorous theoretical investigation has been made of fully nonlinear ion-acoustic waves in nonrelativistic and ultrarelativistic, collisionless, unmagnetized plasma containing of degenerate electrons and positrons, and classical cold ions. In both (nonrelativistic and ultrarelativistic) regimes the electrons and positrons are assumed to follow the corresponding Fermi distribution while the ions are described by the hydrodynamic equations. An energy balancelike equation involving a Sagdeev-type pseudopotential is derived separately for both the regimes. In addition, stationary periodic and solitary waves are also investigated for the two cases. The present work would be helpful to understand the excitation of nonlinear ion-acoustic waves in a degenerate plasma such as in superdense white dwarfs.

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I. INTRODUCTION

Recently, there has been a great deal of interest in the study of electron-positron plasma. Such plasmas are found in solar atmospheres [1,2], in active galactic nuclei (AGN), near the polar cusp regions of the pulsars and neutron star atmospheres, in the inner region of accretion disks surrounding the central black hole, in quasar atmospheres and in the Van Allen belts [3–7]. It is also known that the early prestellar period of the evolution of the Universe was presumably dominated by relativistic electrons and positrons [8]. In the lepton epoch, which occurred 10^{-6} sec $< t < 10$ sec after the Big Bang, temperatures reached the values of 10^9 K $< T < 10^{13}$ K causing annihilation of nucleon-antinucleon pairs resulting in matter which is constituted of electrons, positrons, and photons in thermodynamic equilibrium. Using ultraintense laser pulses the possibility of the production of electron-positron pairs with density 10^{21} cm^{-3} has been shown [9]. Thus, there is a great deal of interest in studying linear as well as nonlinear wave motions in electron-positron plasmas. Tajima and Taniuti [10] and Shukla *et al.* [11] investigated nonlinear interaction of electromagnetic waves and acoustic modes in electron-positron plasma. Medvedev [12] studied thermodynamics and spectral properties of photons in pair plasma. Tsintsadze [13] investigated sound waves in an electron-positron plasma. Moreover, in a relativistically hot electron-positron isothermal plasma, one-dimensional electromagnetic solitons were obtained [14–19]. Dubinov and Sazonkin [20] developed an analytical nonlinear gas dynamic theory of ion-acoustic waves in an electron-positron and ion plasma in which all the plasma components in the wave undergo polytropic compression and rarefaction. The ion-acoustic solitons in electron-positron and ion plasmas were also studied by Popel *et al.* [21] where they presented an investigation of the nonlinear ion-acoustic waves in the presence of cold ions and hot electrons positrons. Besides, some other authors also studied nonlinear structures in plasmas [22–28]. The astrophysical bodies, such as white dwarfs [29] contain very high-density electrons and positrons, i.e., the electron/positron components can be considered as a degenerate ideal gas. In such an environment, one should apply the Thomas-Fermi model for describing the degenerate gas of free electrons and positrons [30–32], while the ion components can be treated as a classical gas. Recently, new quantum kinetic equations [33]. have been derived that include both the degeneracy of the particles and quantum effects [34].

In this paper, we present the properties of nonlinear ion-acoustic solitary waves (IASWs) in a degenerate Thomas-Fermi electron-positron and cold ion plasma. The paper is organized as follows. First in Sec. II we have derived a dispersion relation for ion acoustic wave in nonrelativistic degenerate electron-positron and cold classical ion plasma and then using the fluid model, derived the so called Sagdeev equation. In the same section the stationary nonlinear ion-sound waves are also discussed. In Sec. III, the dispersion relation for ultrarelativistic case is derived. By using the fluid model again the so called Sagdeev equation and the stationary nonlinear ion-acoustic waves in ultrarelativistic regime are obtained. Finally, a brief summary and discussion of our results is given in the last Sec. IV

II. ION ACOUSTIC SOLITARY WAVES IN NONRELATIVISTIC THOMAS-FERMI PLASMA

We consider a collisionless, unmagnetized three-component plasma composed of cold ions, nonrelativistic degenerate electrons and positrons. To derive ion-acoustic wave (IAW) in such a plasma, we use linearized Poisson's equation

$$k^2\varphi = -4\pi e(\delta n_e - \delta n_p - \delta n_i), \tag{1}$$

where δn_e, δn_p, and δn_i are the electrons, positrons and ions perturbed number densities, respectively, k is the wave

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number, φ is the electrostatic potential and e is the magnitude of an electron charge. The equation of continuity and the equation of motion for degenerate Thomas-Fermi plasma are given by

$$\frac{\partial n_j}{\partial t} + \vec{\nabla} \cdot (n_j \vec{u_j}) = 0 \tag{2}$$

and

$$\frac{\partial u_j}{\partial t} + (\vec{u_j} \cdot \vec{\nabla})\vec{u_j} = -\frac{1}{m_j n_j}\vec{\nabla} P_{Fj} - \frac{e_j}{m_j}\vec{\nabla}\varphi, \tag{3}$$

where n_j, m_j, and u_j are the density, the mass and the fluid velocity of the j species ($j = electron, positron$). For degenerate electrons and positrons, the Fermi-pressure is defined as $P_{Fj} = \frac{1}{5}(3\pi^2)^{2/3}\frac{\hbar^2}{m_j}n_j^{5/3}$.

Linearizing Eqs. (2) and (3), i.e., $n_j = n_{0j} + \delta n_j$ and $u_j = \delta u_j$, we obtain

$$\frac{\partial \delta n_j}{\partial t} + n_{0j}\vec{\nabla} \cdot \delta\vec{u_j} = 0 \tag{4}$$

$$\frac{\partial \delta\vec{u_j}}{\partial t} = -\frac{1}{m_j n_j}\vec{\nabla}\delta P_{Fj} - \frac{e_j}{m_j}\vec{\nabla}\varphi. \tag{5}$$

Assuming sinusoidal solution $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, Eqs. (4) and (5) give

$$\left(\omega^2 - \frac{2}{3}\frac{E_{Fe}}{m_e}k^2\right)\frac{\delta n_e}{n_{0e}} = -\frac{k^2 e\varphi}{m_e} \tag{6}$$

$$\left(\omega^2 - \frac{2}{3}\frac{E_{Fp}}{m_p}k^2\right)\frac{\delta n_p}{n_{0p}} = \frac{k^2 e\varphi}{m_p}. \tag{7}$$

The classical cold ions are governed by the equation

$$\omega^2\frac{\delta n_i}{n_{0i}} = \frac{k^2 e\varphi}{m_i}, \tag{8}$$

where n_{0e}, n_{0p}, and n_{0i} are the densities of the electrons, positrons and ions in equilibrium, m_i is the mass of ion and ω is the frequency of ion-acoustic wave, $E_{Fe} = (3\pi^2)^{2/3}\frac{\hbar^2}{2m_e}n_{0e}^{2/3}$ and $E_{Fp} = (3\pi^2)^{2/3}\frac{\hbar^2}{2m_p}n_{0p}^{2/3}$ are the Fermi energies of electrons and positrons respectively.

By employing the Eqs. (6)–(8) in Eq. (1), we can derive the phase velocity of the IAW which is

$$C_{si}^2 = \frac{2\beta}{3(1+\alpha^{1/3})}\frac{E_{Fe}}{m_i}, \tag{9}$$

where $\alpha = n_{0p}/n_{0e}$ and $\beta = n_{0i}/n_{0e}$.

In deriving the expression Eq. (9), we have assumed that the IAW is of long wavelength and the Fermi velocities of electrons u_{Fe} and positrons u_{Fp} are much larger than the ion-acoustic speed C_{si} ($\frac{u_{Fe}}{\sqrt{3}} \gg \frac{\omega}{k}$ and $\frac{u_{Fp}}{\sqrt{3}} \gg \frac{\omega}{k}$). The nonlinear electrostatic ion-acoustic solitary waves (IASWs) in the nonrelativistic Thomas-Fermi gas are governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \tag{10}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} + \frac{e}{m_i}\frac{\partial \varphi}{\partial x} = 0 \tag{11}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e(n_e - n_p - n_i), \tag{12}$$

where the densities of the nonrelativistic degenerate electrons and positrons are given, respectively by the Thomas-Fermi law [30–32].

$$n_e = n_{0e}\left(1 + \frac{e\varphi}{E_{Fe}}\right)^{3/2} \tag{13}$$

and

$$n_p = n_{0p}\left(1 - \frac{e\varphi}{E_{Fp}}\right)^{3/2}. \tag{14}$$

In Eqs. (10)–(14), n_i, n_e, and n_p are the total number densities of ions, electrons and positrons, respectively, u_i is the ion fluid velocity.

In order to investigate the properties of finite amplitude IASWs, we make all the dependent variables in Eqs. (10)–(14) to depend on space coordinate and time as $x - u_0 t$, where u_0 is constant. From Eqs. (10) and (11) the following expression for ion density is obtained

$$n_i = \frac{n_{0i}}{\sqrt{1 - \frac{2e\varphi}{m_i u_0^2}}}, \tag{15}$$

where we have imposed the boundary conditions for the localized disturbances, i.e., $\varphi \to 0$, $u_i \to 0$ and $n_i \to n_{0i}$ at $x \to \pm\infty$

Substituting the expressions for electron, positron and ion densities into the Poisson Eq. (12), we get

$$\frac{\partial^2 \Phi}{\partial X^2} = (1+\Phi)^{3/2} - \alpha(1-\sigma\Phi)^{3/2} - \beta\left(1 - \frac{2\gamma\Phi}{M^2}\right)^{-1/2}, \tag{16}$$

where the following dimensionless quantities have been introduced

$$\sigma = T_{Fe}/T_{Fp}, \quad \Phi = e\varphi/E_{Fe} \quad \text{and} \quad X = x/\lambda_{DFe}.$$

Here $M = u_0/C_{si}$ is the Mach number, $T_{Fe} = E_{Fe}$ and $T_{Fp} = E_{Fp}$ are the Fermi temperatures (in energy units) of electrons and positrons, respectively, $\lambda_{DFe} = (\frac{T_{Fe}}{4\pi e^2 n_{0e}})^{1/2}$ is Thomas-Fermi length of electrons and $\gamma = \frac{3(1+\alpha^{1/3})}{2\beta}$. In equilibrium, we have $\alpha + \beta = 1$ (the neutrality condition of plasma).

By multiplying both sides of Eq. (16) by $d\Phi/dX$, integrating once and imposing the appropriate boundary conditions for localized solutions, namely $\Phi \to 0$ and $d\Phi/dX \to 0$ at $X \to \pm\infty$, we obtain

$$\frac{1}{2}\left(\frac{d\Phi}{dX}\right)^2 + V(\Phi) = 0, \tag{17}$$

where the Sagdeev potential reads as

$$V(\Phi) = \frac{2}{5}[1-(1+\Phi)^{5/2}] + \frac{2\alpha}{5\sigma}[1-(1-\sigma\Phi)^{5/2}] + \frac{\beta M^2}{\gamma}\left[1-\left(1-\frac{2\gamma\Phi}{M^2}\right)^{1/2}\right]. \quad (18)$$

Equation (17) can be regarded as an "energy integral" of an oscillating particle of unit mass, with velocity $\frac{d\Phi}{dX}$ at position Φ in a potential field $V(\Phi)$.

It is clear from Eq. (17) that $V(\Phi)=0$ and $d\Phi/dX=0$ at $\Phi=0$. IASW solutions of Eq. (17) exist if (i) $d^2V/d\Phi^2|_{\Phi=0}<0$, so that the fixed point at the origin is unstable (ii) there exists a nonzero Φ_m, the maximum (or minimum) value of Φ, at which $V(\Phi_m)=0$ and (iii) $V(\Phi)<0$ when Φ lies between 0 and Φ_m. It is of interest to determine the lower and upper limits of the Mach number M for which solitons exist. Applying the condition (i), the minimum Mach number M is 1 which is independent of α, the positron to electron number density ratio. It shows that the ion-acoustic solitary waves in a nonrelativistic dense pair-ion plasma containing degenerate electrons and positrons cannot be subsonic for any value of α.

It may be noted that the Sagdeev potential Eq. (18) is quite different from the usual Sagdeev potential found in electron-ion plasmas in which ions play an important role, but here in Eq. (18), the role of positrons also become relevant. The term 2γ is greater than σ for any concentration of positrons in the plasma. This fact implies that there are two possible values for the upper limit of Mach number M.

Case 1. Suppose, $\Phi_m=\frac{1}{\sigma}$ and by applying the condition (ii), we have

$$\frac{2}{5}\left[1-\left(1+\frac{1}{\sigma}\right)^{5/2}+\frac{\alpha}{\sigma}\right]+\frac{\beta M^2}{\gamma}\left[1-\left(1-\frac{2\gamma}{\sigma M^2}\right)^{1/2}\right]=0. \quad (19)$$

The solution of this equation for real positive Mach number M is

$$M = \frac{2}{5}\left[\left(1+\frac{1}{\sigma}\right)^{5/2}-\left(1+\frac{\alpha}{\sigma}\right)\right] \times \left\{\frac{\gamma\sigma}{\frac{4}{5}\left[\left(1+\frac{1}{\sigma}\right)^{5/2}-\left(1+\frac{\alpha}{\sigma}\right)\right]\beta\sigma-2\beta^2}\right\}^{1/2} \quad (20)$$

for $\alpha\leq 0.3$

Hence, the range of Mach number M can be calculated from the following inequality

$$1 < M < \frac{2}{5}\left[\left(1+\frac{1}{\sigma}\right)^{5/2}-\left(1+\frac{\alpha}{\sigma}\right)\right] \times \left\{\frac{\gamma\sigma}{\frac{4}{5}\left[\left(1+\frac{1}{\sigma}\right)^{5/2}-\left(1+\frac{\alpha}{\sigma}\right)\right]\beta\sigma-2\beta^2}\right\}^{1/2} \quad (21)$$

which holds for $\alpha\leq 0.3$. For any value of Mach number M satisfying the inequality Eq. (21), the term $2\gamma/\sigma M^2$ remains less than one. It is estimated that the last term $(1-2\gamma\Phi/M^2)^{1/2}$ in Eq. (18) remains real for all values of M and Φ even when the term $(1-\sigma\Phi)^{5/2}$ vanishes at $\Phi=1/\sigma$.

Adopting the value of $\alpha=0.1$ and the corresponding values of $\sigma=4.64$ and $\gamma=2.44$, we obtain the range of Mach number as $1<M<1.28$. Increasing α, for example choosing $\alpha=0.3$ (for which the corresponding values of σ and γ are 2.23 and 3.58, respectively), the range of Mach number would extend to $1<M<1.80$. It is clear that an increase of the positron to electron number density ratio does not lead to the propagation of subsonic solitons.

Case 2. Suppose, $\Phi_m=M^2/2\gamma$ and by applying again the condition (ii), we have

$$\frac{2}{5}\left[1-\left(1+\frac{M^2}{2\gamma}\right)^{5/2}\right]+\frac{2\alpha}{5\sigma}\left[1-\left(1-\sigma\frac{M^2}{2\gamma}\right)^{5/2}\right]+\frac{\beta M^2}{\gamma}=0, \quad (22)$$

which contains Mach number M real only for $\alpha>0.3$. The real and positive values of upper critical Mach number obtained from Eq. (22) for different values of α are given below in Table I.

TABLE I. Real and positive values of the upper critical Mach number obtained from Eq. (22) for different values of alpha.

α	0.4	0.5	0.6	0.7	0.8	0.9
M	2.029	2.015	2.007	2.004	2.001	2.000

For any value of M in the above range, it can be observed that the term $\sigma M^2/2\gamma$ should be smaller than unity and that the term $(1-\sigma\Phi)^{5/2}$ of Eq. (18) should be real for all values of M and Φ even when the term $(1-2\gamma\Phi/M^2)^{1/2}$ vanishes.

Abdesalam *et al.* [2] suggested that an increase in the positron-electron number density ratio α may lead to the propagation of subsonic solitons. But as we have seen above, the existence of subsonic solitons in the nonrelativistic Thomas-Fermi electron-positron and classical-ion plasma is not possible at all. This difference seems to arise due to two reasons. First, the ion-acoustic speed in electron ion plasma mostly depends upon the electron temperature and ion mass. But in the case of pair-ion plasma, since positrons are as light particles as electrons, the ion acoustic speed would not only depend on the temperature of electrons, it must also take into account the temperature of positrons. However, Abdesalam *et al.* did not do that. Second, these authors chose $\sigma<1$, which is not correct since $n_{0e}>n_{0p}$ as is evident from the neutrality condition and thus $E_{Fe}>E_{Fp}$ which in turn, implies that σ must exceed unity.

In Figs. 1 and 2, we have numerically analyzed the Sagdeev potential Eq. (18) and investigated how the positron to electron number density ratio α and the Mach number M change the profile of the potential well. An increase in α leads to an increase of both the potential depth and the amplitude.

To study the dynamics of the small-but finite-amplitude IASWs, we consider the case when $\Phi\ll 1$, i.e., the stationary waves have weak nonlinearity. In this case all the terms in Eq. (16) can be expanded in a power series to obtain

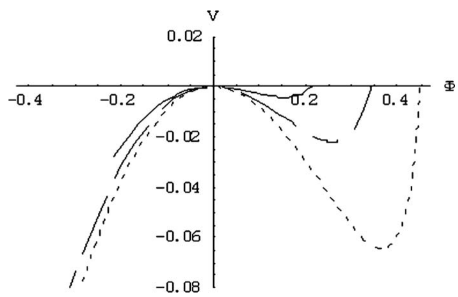

FIG. 1. The Sagdeev potential $V(\Phi)$ [represented by Eq. (18)] against the potential Φ. Solitary pulse for $\alpha=0.1$ (solid curve), $\alpha=0.2$ (dashed curve), and $\alpha=0.3$ (dotted curve).

$$\frac{\partial^2\Phi}{\partial X^2}=\beta\gamma\left(1-\frac{1}{M^2}\right)\Phi-\frac{3}{8}\left(\alpha\sigma^2+\frac{4\gamma^2\beta}{M^4}-1\right)\Phi^2. \quad (23)$$

We may note here in passing that if the last term in Eq. (23) is neglected, there are two possibilities in the linear approximation, one for $M=\gamma m_i u_0^2/E_{Fe}<1$ and other for $M>1$. The former represents the simple harmonic motion with frequency $f=\frac{1}{2\pi}\sqrt{\beta\gamma(\frac{1}{M^2}-1)}$, while the latter condition describes the Debye potential with the characteristic scale length is given by

$$r_D=\frac{u_{Fe}}{\sqrt{2\beta\gamma}\omega_{pe}(1-C_{si}^2/u_0^2)^{1/2}}. \quad (24)$$

It shows that the effect of the Coulomb field extends up to a distance of the order of r_D which plays a role of the Debye screening distance.

Now let us consider the structure of a solitary wave for $M>1$. The solution of Eq. (23) in this case is

$$\Phi=\frac{4\beta\gamma(1-1/M^2)}{(\alpha\sigma^2+4\beta\gamma^2/M^4-1)}\sec h^2\left[\frac{\sqrt{\beta\gamma(1-1/M^2)}}{2}X\right]. \quad (25)$$

Resultantly, using equation (13), we obtain the density

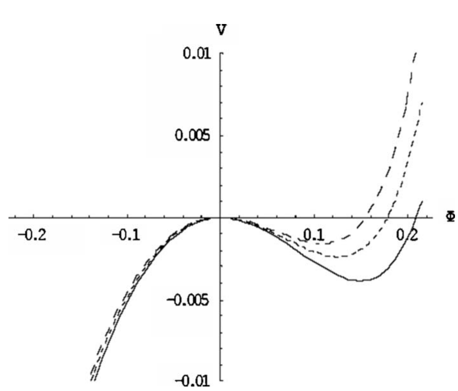

FIG. 2. The Sagdeev potential $V(\Phi)$ [represented by Eq. (18)] against the potential Φ. Solitary pulse for $M=1.2$ (dashed curve), $M=1.23$ (dotted curve), and $M=1.27$ (solid curve). Here, $\alpha=0.1$, $\sigma=4.64$, and $\gamma=2.44$.

$$\frac{n_e}{n_{0e}}=\left\{1+\frac{4\beta\gamma(1-1/M^2)}{(\alpha\sigma^2+4\beta\gamma^2/M^4-1)}\times\sec h^2\left[\frac{\sqrt{\beta\gamma(1-1/M^2)}}{2}X\right]\right\}^{3/2}. \quad (26)$$

We see that $n_e>n_{0e}$ and $n_i>n_{0i}$, since $\Phi>0$. Thus a solitary wave in a quasiequilibrium nonrelativistic Thomas-Fermi electron-positron and cold ion plasma is always a compressional wave.

III. ION ACOUSTIC SOLITARY WAVES IN ULTRARELATIVISTIC THOMAS-FERMI PLASMA

Now we consider a collisionless, unmagnetized three-component plasma composed of cold ions and degenerate ultrarelativistic electrons and positrons. In deriving the velocity of IAW in degenerate dense ultrarelativistic electron-positron gas, the Eqs. (6) and (7) become

$$\left(\omega^2-\frac{1}{3}c^2k^2\right)\frac{\delta n_e}{n_{0e}}=-\frac{c^2k^2e\varphi}{E_{Fe}} \quad (27)$$

$$\left(\omega^2-\frac{1}{3}c^2k^2\right)\frac{\delta n_p}{n_{0p}}=\frac{c^2k^2e\varphi}{E_{Fp}}, \quad (28)$$

where $E_{Fe}=(3\pi^2)^{1/3}c\hbar n_{0e}^{1/3}$ and $E_{Fp}=(3\pi^2)^{1/3}c\hbar n_{0p}^{1/3}$ are Fermi energies of ultrarelativistic electrons and positrons respectively and $\hbar$ is Planck constant.

Proceeding as before in Sec. II, we can derive the dispersion relation of ion acoustic wave in ultrarelativistic Thomas-Fermi plasma as,

$$C_{si}^2=\frac{\beta E_{Fe}}{3m_i(1+\alpha^{2/3})}. \quad (29)$$

The number densities of electrons and positrons in ultrarelativistic regime can be calculated as

$$n_e=n_{0e}\left(1+\frac{e\varphi}{E_{Fe}}\right)^3 \quad (30)$$

and

$$n_p=n_{0p}\left(1-\frac{e\varphi}{E_{Fp}}\right)^3. \quad (31)$$

Using n_e, n_p, and n_i in the Poisson Eq. (12) and proceeding as in the last section, we get

$$\frac{\partial^2\Phi}{\partial X^2}=(1+\Phi)^3-\alpha(1-\alpha^{-1/3}\Phi)^3-\beta\left(1-\frac{2\gamma\Phi}{M^2}\right)^{-1/2}, \quad (32)$$

where $\gamma=\frac{3}{\beta}(1+\alpha^{2/3})$.

In this case, the Sagdeev potential turns out to be

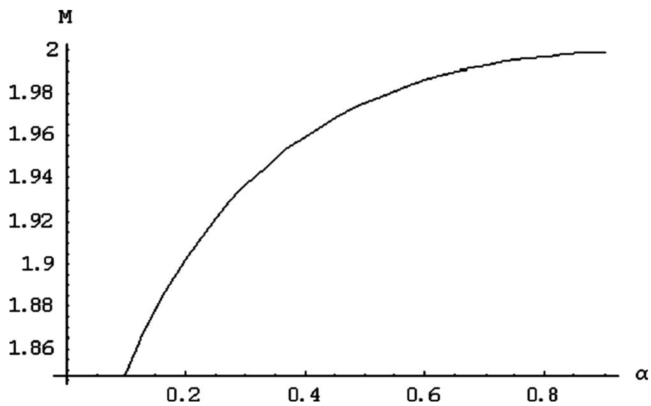

FIG. 3. Variation of M with $\alpha(=n_{0p}/n_{0e})$.

$$V(\Phi)=\frac{1}{4}[1-(1+\Phi)^4]+\frac{\alpha^{4/3}}{4}[1-(1-\alpha^{-1/3}\Phi)^4]+\frac{\beta M^2}{\gamma}\left[1-\left(1-\frac{2\gamma\Phi}{M^2}\right)^{1/2}\right]. \quad (33)$$

From the last term in $V(\Phi)$, it is clear that Φ can have any value but cannot be greater than $M^2/2\gamma$. Therefore, we insert $\Phi_m=M^2/2\gamma$ (the maximum value of Φ at which $V(\Phi)=0$) in Eq. (33) to determine the upper critical Mach number M. Figures 3 and 4 show the variation of M with α and β, i.e., it increases with α but decrease with β. However, the maximum value of M remains less than 2 giving the range $1<M<2$.

In Figs. 5 and 6 we examine how α and the Mach number M change the profile of the potential well. Figure 5 shows that the potential variation with the Mach number M has the same profile as in the nonrelativistic degenerate electron-positron gas, i.e., the potential values associated with the localized excitations expand as the Mach number M acquires higher values, implying that faster pulse excitations will be taller and wider. Figure 6 exhibits that an increase in α leads to a decrease of both the potential depth and the amplitude contrary to what was observed for nonrelativistic plasma.

To study stationary waves with weak nonlinearity., we assume $\Phi\ll 1$ and expand all the terms in Eq. (32) in power series and obtain

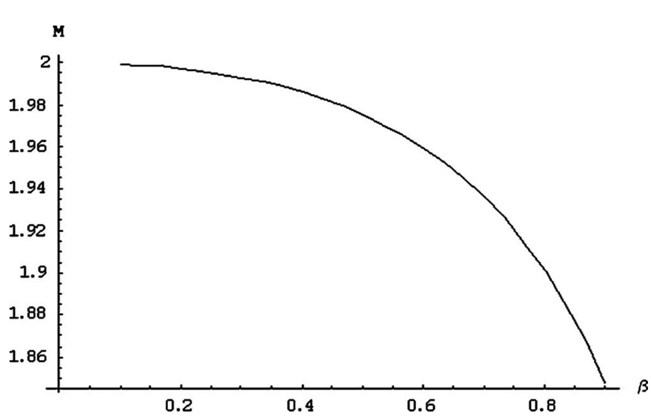

FIG. 4. Variation of M with $\beta(=n_{0i}/n_{0e})$.

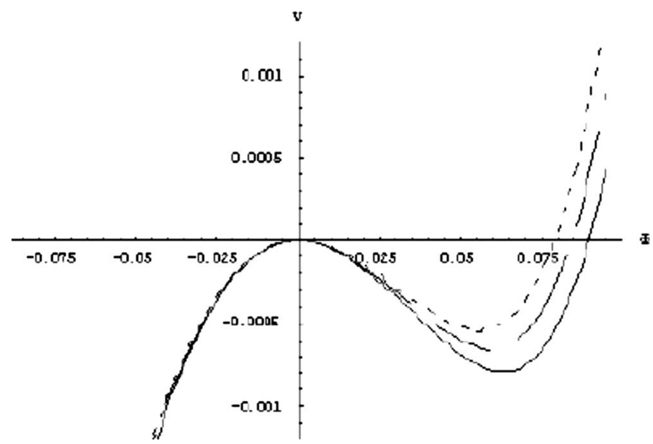

FIG. 5. The Sagdeev potential $V(\Phi)$ [represented by Eq. (33)] against the potential Φ. Solitary pulse for $M=1.15$ (dotted curve), $M=1.16$ (dashed curve), and $M=1.17$ (solid curve). Here, $\alpha=0.1$, $\sigma=2.15$, and $\gamma=4.05$.

$$\frac{\partial^2\Phi}{\partial X^2}=\beta\gamma\left(1-\frac{1}{M^2}\right)\Phi-3\left(\alpha^{1/3}+\frac{\gamma^2\beta}{2M^4}-1\right)\Phi^2. \quad (34)$$

The above equation has the solitary wave solution for $M>1$

$$\Phi=\frac{4\beta\gamma(1-1/M^2)}{(\alpha^{1/3}+\beta\gamma^2/2M^4-1)}\sec h^2\left(\frac{\sqrt{\beta\gamma(1-1/M^2)}}{2}X\right) \quad (35)$$

giving

$$\frac{n_e}{n_{0e}}=\left\{1+\frac{4\beta\gamma(1-1/M^2)}{(\alpha\sigma^2+4\beta\gamma^2/M^4-1)}\times\sec h^2\left[\frac{\sqrt{\beta\gamma(1-1/M^2)}}{2}X\right]\right\}^{3/2}. \quad (36)$$

We again see that $n_e>n_{0e}$ and $n_i>n_{0i}$, since $\Phi>0$. Thus the solitary wave is a compressional wave in ultrarelativistic case also.

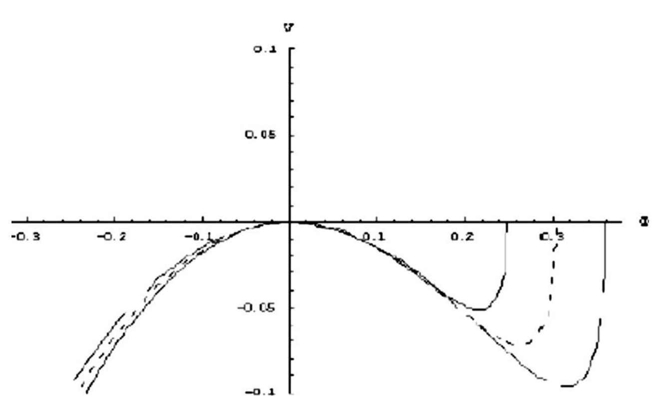

FIG. 6. The Sagdeev potential $V(\Phi)$ [represented by Eq. (33)] against the potential Φ. Solitary pulse for $\alpha=0.2$ (dashed curve), $\alpha=0.3$ (dotted curve), and $\alpha=0.4$ (solid curve).

We also note that the Debye potential in this case has scale length

$$r_D = \frac{\lambda_{DFe}}{\sqrt{\beta\gamma}(1 - C_{si}^2/u_0^2)^{1/2}}, \tag{37}$$

where now all the parameters are defined in the ultrarelativistic regime.

IV. SUMMARY

We have investigated the nonlinear ion-acoustic solitary waves propagating in a collisionless unmagnetized nonrelativistic and ultrarelativistic quantum/dense electron-positron-ion plasma. The electrons and positrons are described by the Thomas-Fermi law, while the ions are described by the ideal hydrodynamic fluid equations. The dispersion relations for ion-acoustic wave in both the regimes, nonrelativistic and ultrarelativistic have been derived. An energy balancelike expression involving a Sagdeev potential has been derived separately for both the cases. Analytical calculations show that in both cases only supersonic ion-acoustic solitary wave can exist for all values of the positron to electron number density ratio. The dependence of the pseudopotential profile and of the potential pulse excitation characteristics on the positron to electron number density ratio and the Mach number has been investigated. It is significant to note that the ion-acoustic speed in the degenerate pair ion plasma does not only depend upon the electron temperature and the ion mass, it also depends upon the concentration of the positrons and the ions in the plasma. An increase in the positron number density or a decrease in the ion number density reduces the ion-acoustic speed and vice versa. It has been found that the role of positrons along with the ions in the Sagdeev potential determined in the degenerate nonrelativistic pair ion plasma becomes relevant. The role of positrons dominates over that of ions when there is the large concentration of ions (i.e., the small concentration of positrons) in the plasma and vice versa. Our results should elucidate the excitation of nonlinear ion-acoustic shock waves in degenerate plasmas, particularly in superdense astrophysical objects, e.g., in the interior of white dwarfs.

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