Local and global persistence exponents of two quenched continuous-lattice spin models

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Local and global persistence exponents associated with zero-temperature quenched dynamics of two-dimensional *XY* model and three-dimensional Heisenberg model have been estimated using numerical simulations. The method of *block persistence* has been used to find the global and local exponents simultaneously (in a single simulation). Temperature universality of both the exponents for three-dimensional Heisenberg model has been confirmed by simulating the stochastic (with noise) version of the equation of motion. The noise amplitudes added were small enough to retain the dynamics below criticality. In the second part of our work we have studied scaling associated with correlated persistence sites in the three-dimensional Heisenberg model in the later stages of the dynamics. The relevant length scale associated with correlated persistent sites was found to behave in a manner similar to the dynamic length scale associated with the phase ordering dynamics.

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I. INTRODUCTION

Persistence or Persistence probability $[P(t)]$ is an important physical quantity in the field of nonequilibrium statistical mechanics [1–3]. Persistence has the ability to probe into the details of the history of the dynamics. For a general stochastic process, it is defined as the probability that any zero mean stochastic variable $X(t)$ has never changed its sign since the starting time of the dynamics ($t=0$). In an extended dynamical system we study the time evolution of the order-parameter field $\phi(\mathbf{x},t)$, which fluctuates both in space and time. Probability that the local order-parameter field (at a fixed space $\mathbf{x}$) never changed its phase since $t=0$, is known as the local (or site) persistence probability $[P(t)]$. Similarly we can define *global persistence probability* [4,5] for the total value of order parameter (summed over entire space). Spin systems (like Ising, Potts, *XY*, Heisenberg, spin nematic models, etc.) defined on a lattice are nice examples of extended systems. Study of local and global persistence exponents of such systems are of considerable interest in the field of nonequilibrium statistical mechanics [2,6–9]. Persistence probability (local and global) decays with time following the rule $L^{-\theta}(t)$, where $L(t)$ is the dynamical growing length scale associated with the phase ordering process of a quenched system [10]. θ is known as the persistence exponent, and hence there are two exponents: one is the local persistence exponent (θ) and the other is the global persistence exponent (θ_0) [11]. In lattice spin systems, local persistence probability is simply the fraction of spins whose one specified component has never changed sign since $t=0$. Similarly, global persistence is simply the probability that one specified component of the total value of order parameter has never changed sign since $t=0$. Owing to symmetry one must take average of the *persistence probabilities* corresponding to all components and also over several initial configurations. Equally well, one can define persistence probability (local and global) as the probability that none of the components of the vector order parameter has ever changed their signs up to time t since the starting time of the dynamics. However the second definition is clearly different from the first one and leads to a faster decay of the persistence probability. Indeed the second definition should give us persistence exponent n times the exponent obtained using the first definition for O(n) model provided all the spin components are independent stochastic variables. Only in the limit $n\to\infty$, the n components of the vector spins are independent and hence for any large finite value of n, the persistence exponent estimated using the second definition should be approximately n times the exponent obtained using the first definition. In the present work, the persistence properties of O(2) (*XY*) and O(3) (Heisenberg) models have been studied using the first definition. In addition, we have also compared the local persistence exponents of three-dimensional Heisenberg model using both the definitions. Estimation of the global exponent using numerical simulation is a very tedious job as it requires a very large number of initial configurations (Ref. [22] of [11]). Block persistence, introduced by Cueille and Sire [11], provides a natural way to estimate simultaneously both the global and the local persistence exponents in a single simulation. Blocking (phenomenologically same as the blocking operation in Renormalization Group methods [12]) is nothing but summing the order parameters (spins) over blocks of linear size l, where l is much smaller than the lattice size. Block persistence $P_l(t)$ [see Eq. (5)] is the persistence probability associated with the blocked spin variables. (Details may be obtained from the original work of Cueille and Sire [11,13].) Block persistence behaves like both global and local persistence in different limits of time of the coarsening (phase ordering) process. Persistence and scaling in two-dimensional *XY* model has been studied in details in a previous work [8]. However this work was mainly restricted to the studies related to the local persistence associated with the $T=0$ quenched two-dimensional *XY* model.

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The present work is largely motivated by the work of Cueille and Sire [11,13], where the method of block persistence has been applied to two continuous spin models, i.e., the two-dimensional XY model and the three-dimensional Heisenberg model. Using block persistence method, both global and local persistence exponents associated with the zero-temperature quenching dynamics of both the models have been estimated. Another interesting phenomenon we have studied is the scaling associated with the correlated persistence sites of three-dimensional Heisenberg model. The correlated persistence sites do not give rise to any new relevant length scale, in fact it has been observed that it is same as the linear domain length scale in the phase ordering kinetics. We explain in the next section how the method of block persistence is useful in the determination of both the persistence exponents in a single simulation and also discuss its utility in the study of persistence associated with the non-zero-temperature quenching dynamics.

II. METHOD OF BLOCK PERSISTENCE

As mentioned earlier, block persistence is the persistence probability associated with the blocked spin variables. During the initial stages of the dynamics, the dynamical length scale $L(t)$ is small compared to the linear block size (l) and block persistence behaves like global persistence. But at the later stages of the dynamics, $L(t)$ is appreciably large as compared to the linear block size and the block persistence behaves like local persistence. Thus, depending on the value of $L(t)/l$, block persistence $P_l(t)$ behaves like global or local persistence. A single scaling form for block persistence given below gives a very efficient way of determining local (θ) and global (θ_0) persistence exponents simultaneously. The scaling form for $P_l(t)$ associated with the $T=0$ quenched case is given by

$$P_l(t) = l^{-\theta_0} g\left(\frac{L(t)}{l}\right). \tag{1}$$

For $x\to\infty$ ($L(t)\gg l$), $g(x)$ behaves like $x^{-\theta}$ and for $x\to 0$ ($L(t)\ll l$) it behaves like $x^{-\theta_0}$ [where $x=L(t)/l$]. By choosing a proper value of θ_0, one should get good scaling. Slope of the ln-ln plot of $g(x)$ vs x gives the global persistence exponent at the small x region while for the large x region it gives an estimate of the local exponent.

When the system is quenched at temperature $T(\neq 0)<T_c$, then spin components change sign both due to change in phase and thermal excitations. Thermal excitations occur with decay rate given by $\tau\sim\exp[-\Delta E/K_BT]$ (this is known as Arrhenius law), where K_B is the Boltzmann constant and ΔE is the change in energy of the system due to thermal flipping associated with a spin. The energy barrier is of the order of the coupling constant appearing in the microscopic interaction Hamiltonian. For the non-zero-temperature quench the persistence probability falls off much faster than the zero-temperature case. However if the thermal excitations could be eliminated properly, then we should get temperature universality of persistence exponents. Derrida proposed a scheme to eliminate thermal excitations in case of Ising and Potts models [14]. He studied finite temperature persistence exponent associated with nonconserved Ising model and Potts model. In his method, two configurations, one completely uniform (system A) and another completely random (system B), were simultaneously updated (with time) using the same sequence of thermal noise (random numbers) and the same Monte Carlo algorithm (such as Heat Bath, Glauber, or Metropolis). Flips in system A were only due to thermal fluctuations, whereas flips in system B were due to both thermal fluctuations and change in phase. By eliminating simultaneous flips from system B, the temperature-independent persistence probability was estimated and this was same as that of the persistence probability associated with the $T=0$ quench of the system. Using this method Stauffer performed rigorous simulations and found temperature universality of persistence exponent in the case of Ising model [15]. Derrida's method is not suitable for the conserved model and is not trivial enough to be applied directly to the continuous spin models. Moreover it suffers from the problems of damage spreading [16]. The method of block persistence was introduced to overcome the difficulties associated with Derrida's method. It eliminates the short scale thermal excitations in a natural way and is readily applicable to the continuous spin models. With increase in the level of blocking, thermal flippings get eliminated gradually. However, the effect of thermal noise never gets eliminated completely for any finite value of l. We point out that while blocking, the quantity ΔE introduced earlier, becomes roughly the energy required to flip a blocked spin and hence depends explicitly on the linear block size l. This leads to modification of the decay rate in Arrhenius law and the decay rate becomes a function of the linear block size l which is denoted by $\tau(l)$. For the nonzero value of the temperature of quench, the scaling relation for block persistence gets slightly modified and is given by [11]

$$P_l(t) = l^{-\theta_0} f\left(\frac{L(t)}{l}\right)\exp\left(-\frac{t}{\tau(l)}\right), \tag{2}$$

where $\tau(l)$ is the effective decay rate of temperature flipping which depends on the linear block size and temperature as well.

III. PRESENT WORK

In the first part of the present work we have used the method of block persistence to find out the local and global persistence exponents for the two-dimensional XY [O(2)] model and the three-dimensional Heisenberg model [O(3)] quenched at $T=0$ from $T=\infty$ (homogeneous phase). The first part of our work also involves the study of the persistence probability associated with the non-zero-temperature quenching dynamics of three-dimensional Heisenberg model. From the renormalization group results it is known that, there are only two fixed points, i.e., $T=0$ and $T=T_c$. Quenching at any finite temperature below T_C is physically similar to that of zero-temperature quench (same values of dynamical exponents). However, for quenching at $T=T_c$ (known as the critical quench), we get the different values of dynamical exponents. Thus, we should get the same values of both persistent exponents if the temperature of the quench is below critical temperature. It may be noted that two-dimensional XY model is an exception because here we have a continuous series of fixed points (called the KT line) extending from the Kosterlitz-Thouless transition temperature T_{KT} to $T=0$ [17] and therefore we have not studied finite temperature persistence in two-dimensional XY model [18]. In the present study we have numerically simulated the stochastic equation of motion (with noise version of the deterministic equation) to incorporate the non-zero-temperature effect in quenching dynamics of the three-dimensional Heisenberg model. However noise added was small enough to retain the dynamics below critically.

In the second part of our work we have studied the scaling associated with correlated persistence sites of the three-dimensional Heisenberg model quenched at $T=0$ from $T=\infty$. In a previous work we have studied the persistence scaling associated with correlated persistence sites of two-dimensional XY model [8]. We discuss in the next section the simulation procedure in details and present the results obtained. General discussion and mathematical formulations related to scaling associated with correlated persistence sites is given in the later part of the next section.

IV. SIMULATION DETAILS AND RESULTS

The interaction Hamiltonian describing both two-dimensional XY model and the three-dimensional Heisenberg model is given by

$$H = -\sum_{\langle i,j\rangle} (\vec{\phi}_i, \vec{\phi}_j), \tag{3}$$

where $\vec{\phi}$ is the usual two- or three-dimensional spin vectors (for XY or Heisenberg model) and $\langle i,j\rangle$ represents the nearest-neighbor sites. The number of nearest neighbors is four for the two-dimensional XY model and six for three-dimensional Heisenberg model. The general equation of motion for both the models is given by [19]

$$\frac{\partial \vec{\phi}_\mathbf{i}}{\partial t} = \sum_j \vec{\phi}_\mathbf{j} - \sum_j (\vec{\phi}_i, \vec{\phi}_j)\vec{\phi}_i + A\vec{\epsilon}_i, \tag{4}$$

where the sum is taken over all nearest-neighbor sites of i. $\vec{\epsilon}_i$ represents a random vector (components of which are independent random variables uniformly distributed between -1 to $+1$) and is associated with the amplitude factor A. For nonzero values of A, we get the effect of finite temperature of the quench. Nonzero values of A (stochastic equation) have been used to investigate the temperature universality of persistence exponents for the three-dimensional Heisenberg model.

In our simulation, we have used a discretized version of the above partial differential equation and have chosen a value for the time step $\delta t=0.02$. We have extensively simulated the discretized noise-free (or deterministic) version ($A=0$) of above equation to find out the persistence exponents of both the models and scaling associated with the correlated persistence sites for the three-dimensional Heisenberg model. The mathematical representation of the block persistence probability is given by

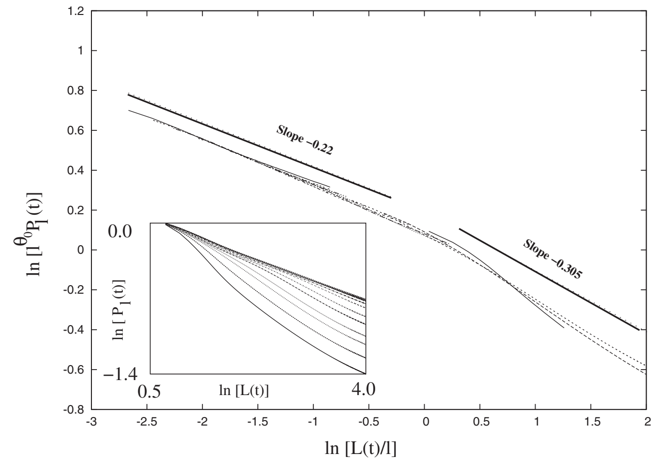

FIG. 1. ln-ln plot of scaling function [$g(x)$ of Eq. (1)] for block persistence probability $p_l(t)$ versus $x=L(t)/l$ for 2400×2400 XY model at $T=0$ for linear block sizes $l=2$, 3, 4, 6, 8, 12, 16, 20, 24, and 30. Best (to eye) scaling or data collapse was obtained by using the value of global exponent $\theta_0=0.22(\pm 0.01)$. The slope of scaled function for smaller values of $x=[L(t)/l]$ is equal to the global exponent $\theta_0=0.22$ and is equal to the local exponent $\theta=0.305(\pm 0.05)$ for large values of x. The inset shows ln-ln plot for block persistence probability $P_l(t)$ versus $L(t)$ (for the values of $l=1$, 2, 3, 4, 6, 8, 12, 16, 20, 24, and 30 from bottom to top).

$$P_l(t) = \text{Prob}[Sb_i(t') \times Sb_i(0) > 0, \quad \forall \quad t' \text{ in } [0,t]], \tag{5}$$

where Sb_i is the i^{th} component of the *blocked spin* for a particular site in the blocked lattice. $\vec{Sb}(t)$ is obtained by summing over all spins $\vec{S}(t)$ situated at the sites of a block of linear size l. The system sizes we have used are 2400×2400 for the XY-model and $144\times 144\times 144$ for the Heisenberg model. Owing to symmetry, we have taken average over all the components of the spin (two for the XY model and three for the three-dimensional Heisenberg model). We have also taken average over all the sites (of the blocked lattice) and several initial configurations. For $l=1$, $P_l(t)$ simply becomes the site persistence probability $P(t)$.

The local persistence exponent for two-dimensional XY model using the block persistence method was found to be $0.305(\pm 0.05)$, which agrees with the value we obtained in our earlier work [8]. In Figs. 1 and 2 we have shown the scaling associated with the block persistence probability (calculated at $T=0$ by setting $A=0$) for the two-dimensional XY model and the three-dimensional Heisenberg model. The global persistence exponent for the two-dimensional XY model was found to be $0.22(\pm 0.01)$ and that for the three-dimensional Heisenberg model was found to be $0.13(\pm 0.01)$, while the local exponents were found to be $0.305(\pm 0.05)$ and $0.50(\pm 0.01)$ for the two models, respectively. In the insets of Figs. 1 and 2, we have plotted $\ln[P_l(t)]$ versus $\ln[L(t)]$ for various values of linear block size l mentioned in the figure caption. One should note that, the growth laws of $L(t)$, the dynamical length scale, are different for two models. For two-dimensional XY model, it grows like $[t/\ln(t)]^{1/2}$ [20,21], whereas for three-dimensional Heisenberg model, it follows the growth law $\sim t^{1/2}$.

In Fig. 3 we have shown the scaling of the block persistence probability. Scaling was good for blocks of linear size

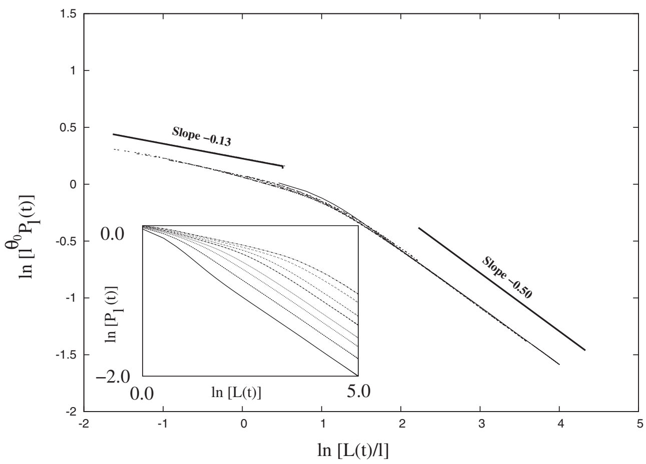

FIG. 2. ln-ln plot of scaling function $[g(x)$ of Eq. (1)] for block persistence probability $p_l(t)$ versus $x=L(t)/l$ for $144\times144\times144$ Heisenberg model at $T=0$ for linear block sizes $l=2$, 3, 4, 6, 8, 12, and 16. Best (to eye) scaling or data collapse was obtained by using the value of global exponent $\theta_0=0.13(\pm0.01)$. The slope of scaled function $[g(x)$ in Eq. (1)] for smaller values of $x=[L(t)/l]$ is equal to the global exponent $\theta_0=0.13$ and is equal to the local exponent $\theta=0.50(\pm0.01)$ for large values of x. The inset shows ln-ln plot for block persistence probability $P_l(t)$ versus $L(t)$ (for the values of $l=1$, 2, 3, 4, 6, 8, 12, and 16 from bottom to top).

greater than 3. Using scaling, we obtained the values of the global and local persistence exponents which were found to be the same as that of the $T=0$ case. This confirms that the persistence exponents do not violate the temperature universality below criticality. We have used the noise amplitude up to 0.07 and got very similar results. In the inset of Fig. 3 we have shown the plot of $\ln[P_l(t)]\sim\ln[L(t)]$ for the three-dimensional Heisenberg model for various linear block sizes

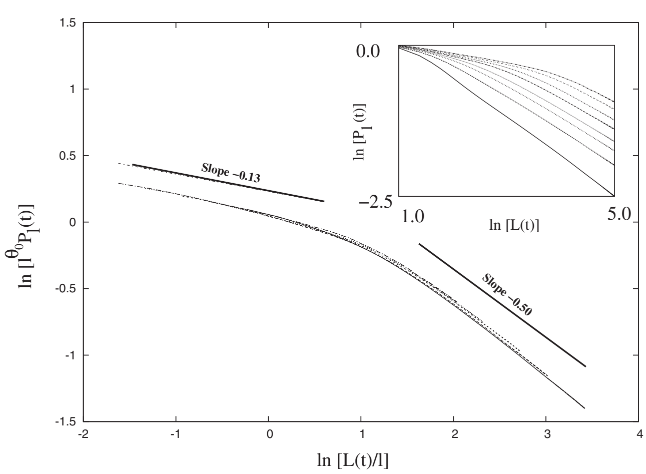

FIG. 3. ln-ln plot for scaling function of block persistence probability $P_l(t)$ for $144\times144\times144$ three-dimensional Heisenberg model for noise amplitude $A=0.05(l>3)$. Good collapse is obtained with global exponent $\theta_0=0.13(\pm0.03)$, which is equal to its zero-temperature value. The slope of scaled function for smaller values of $x=[L(t)/l]$ gives the global exponent $\theta_0=0.13$ and the local exponent $\theta=0.50$ for large values of x. The inset shows ln-ln plot for block persistence probability $P_l(t)$ versus $L(t)$ (for the values of $l=1$, 2, 3, 4, 6, 8, 12, and 16 from bottom to top).

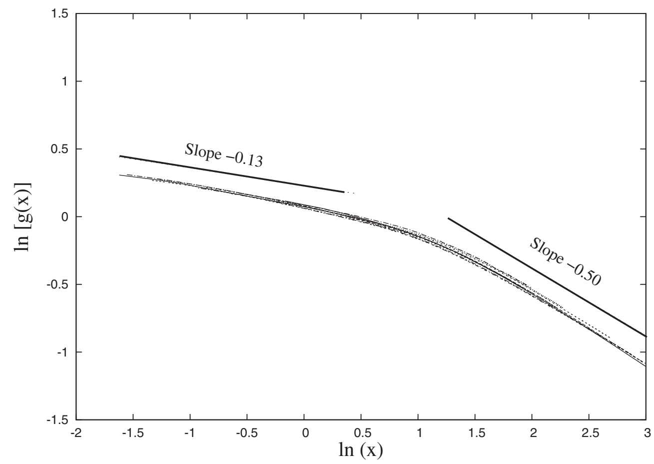

FIG. 4. Simultaneous ln-ln plot of scaling function $g(x)=l^{\theta_0}P_l(t)$ versus $x=L(t)/l$ for the $T=0$ data and scaling function for nonzero temperature multiplied by some constant factor $a_1=1.07$, i.e., $g(x)=a_1 l^{\theta_0}P_l(t)$ versus $x=a_2L(t)/l$, where $a_2=1.02$ is another constant. Best collapse is obtained using $\theta_0=0.13$. The linear block sizes used were greater than 3.

(mentioned in the caption of the figure). The value of the noise amplitude used was $A=0.05$. Clearly on account of non-zero-temperature effect, the decay of the blocked persistence probabilities behaves differently from the zero-temperature case for lower values of linear block sizes (l), and however the behavior of decay is found quite similar as that of $T=0$ case for large values of l. This is because, as mentioned earlier, the effective characteristic time (τ) in the Arrhenius law grows rapidly with l and hence the effect of temperature [see Eq. (2)] has its role much later—well beyond the time up to which we have performed our simulation. Thus for larger block size, scaling is similar to the zero-temperature case and we can safely use the scaling relation for $T=0$, i.e., Eq. (1). Temperature universality of scaling function is shown in Fig. 4. We have shown simultaneously the ln-ln plot of scaling function $g(x)=l^{\theta_0}P_l(t)$ versus $x=L(t)/l$ for the $T=0$ case and scaling function for nonzero temperature multiplied by some constant factor a_1, i.e., $g(x)=a_1 l^{\theta_0}P_l(t)$ versus $x=a_2L(t)/l$, where a_2 another constant. a_1 and a_2 are same for all values of l. The second constant arises due to temperature dependence of the prefactor in the growth law for $L(t)$ [10]. All data shown in various figures were obtained by averaging over 15 initial configurations (except those mentioned in the caption). Errors in the exponents mentioned in the text and the captions of figures for local and global exponent were obtained by roughly estimating the region over which the collapse appears optimal.

In Fig. 5 we have shown how the correlated regions of persistent sites are formed at various times t of the dynamics after the quench (at $T=0$). Scaling and fractal formation by the correlated persistent sites has attracted much interest and have been studied in details by various researchers [8,22–24]. In a previous work we have studied the same for the $T=0$ quenched two-dimensional XY model [8]. Hence in the present work we confine ourselves to the study of the scaling associated with the correlated persistent sites of the three-dimensional Heisenberg model. The persistence correlation function is defined as

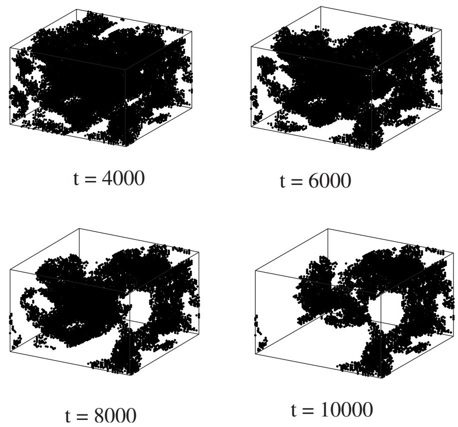

FIG. 5. Correlated persistent sites for $50 \times 50 \times 50$ $T=0$ quenched Heisenberg model for time steps 4000, 6000, 8000, and 10 000.

$$C(r,t) = \frac{\langle \eta_i(t)\,\eta_{i+r}(t)\rangle}{\langle \eta_i(t)\rangle}, \tag{6}$$

where $\langle\ \rangle$ represents average over several initial random configurations, sites, and components of spins. $\eta_i(t)=1$ if site i is persistent at time t of the dynamics, otherwise it is 0. $C(r,t)$ is a measure of the probability that the $(i+r)$th site is persistent when ith site is persistent. The length beyond which the persistent sites are uncorrelated is known as persistent correlation length $[\xi(t)]$. For $r<\xi(t)$ the persistent sites are strongly correlated and the correlation function is found to be independent of t [and hence of $L(t)$]. In the correlated region, $C(r,t)$ shows a power law decay $r^{-\alpha}$ with r. For $r \gg \xi(t)$, $C(r,t)$ becomes equal to $\langle \eta_i(t)\rangle$ or simply $P(t)$, the site persistence probability. At $r=\xi(t)$, continuity demands, $L^{-\theta}(t) = r^{-\alpha}$ [since $P(t) \sim L^{-\theta}(t)$]. So the persistence correlation length $\xi(t)$ should behave like $L^{\zeta}(t)$, where $\zeta=\theta/\alpha$. Mathematically we write $C(r,t)$ as

$$C(r,t) \sim r^{-\alpha} \quad \text{for } r \ll \xi(t)$$

$$= P(t) \quad \text{for } r \gg \xi(t). \tag{7}$$

The scaling form for $C(r,t)$ can be written as follows:

$$C(r,t) = P(t) f[r/\xi(t)], \tag{8}$$

where $f(x)$ is given by

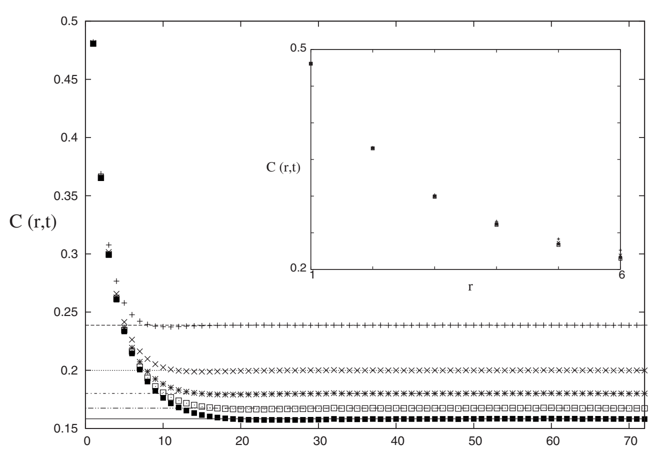

FIG. 6. Variation in $C(r,t)$ with r (up to 72) for $144 \times 144 \times 144$ Heisenberg model. The data shown are for time steps $t=2000$, 4000, 6000, 8000, and 10 000 (from top to bottom). For large r, $C(r,t)$ is same as $P(t)$. The values of $P(t)$ are 0.2387, 0.1999, 0.1801, 0.1675, and 0.1583 for respective time steps. The data shown are averaged over ten initial configurations. Inset shows the variation in $C(r,t)$ (at $t=4000$, 6000, 8000, and 10 000) for small values of r. The overlapping values of $C(r,t)$ confirms that at the later stage of dynamics, $C(r,t)$ is independent of t.

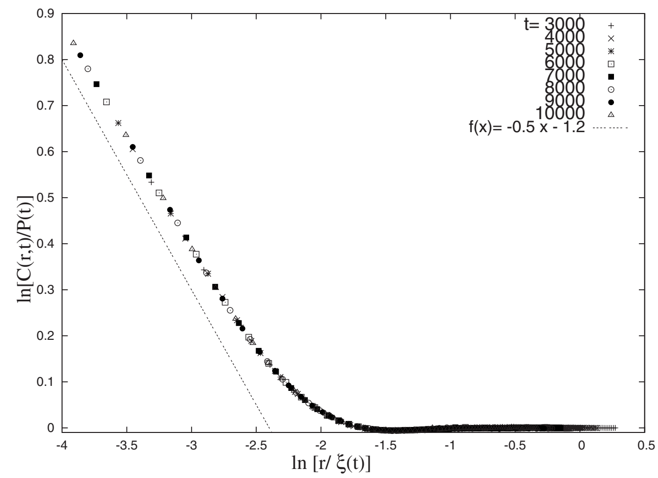

FIG. 7. Plot of $\ln[C(r,t)/P(t)]$ against $\ln[r/\xi(t)]$ for time steps $t=3000$, 4000, 5000, 6000, 7000, 8000, 9000, and 10000. Best collapse was obtained for $\zeta=1(\pm 0.001)$, i.e., $\xi(t) \sim L(t)(t^{1/2})$. The straight line for small values of $r/\xi(t)$ has slope equal to the local or site persistence exponent, 0.50 as expected.

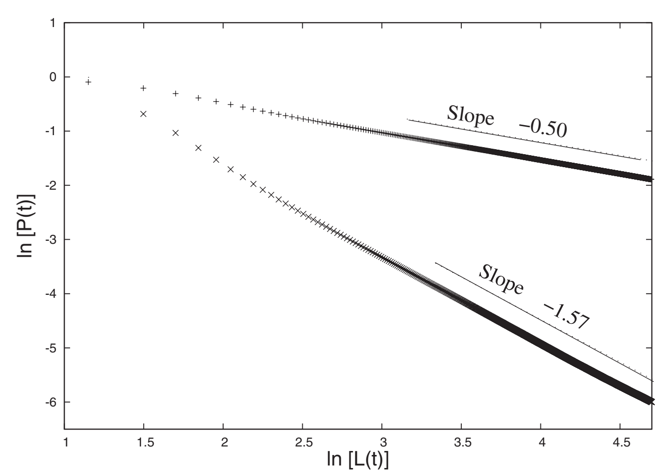

FIG. 8. Simultaneous plot of $\ln[P(t)]$ versus $\ln[L(t)]$ for the $144 \times 144 \times 144$ Heisenberg model obtained using the first and second definitions for local persistence probability. The upper linear decay shows the local persistence exponent (θ) to be equal to 0.50, using the first definition, while the lower linear decay is obtained using the second definition and the local persistence exponent (θ) is found to be 1.57. The exponent using the second definition is 3.14 times the exponent obtained using the first one. Results were obtained by taking average over ten initial configurations.

$$f(x) \sim x^{-\alpha} \quad \text{for } x \ll 1$$

$$= 1 \quad \text{for } x \gg 1 \tag{9}$$

In Fig. 6, we have shown the variation in $C(r,t)$ with r for various values of t (time after quench) for the three-dimensional Heisenberg model. From the figure the time independence of $C(r,t)$ for small values of r is clearly visible (see inset). For large values of r, $C(r,t)$ essentially becomes equal to the persistence probability $P(t)$. In Fig. 7, we have shown, the ln-ln plot of scaling function of $C(r,t)$. We obtained good collapse, for $\zeta = 1(\pm 0.001)$ (and hence $\alpha = \theta$) which implies the persistence correlation length $\xi(t)$ behaves similar to $L(t) = t^{1/2}$. The error in ζ mentioned here is obtained by estimating the region over which the scaling is good to eye. The behavior of $C(r,t)$ and its scaling functions are similar to that for the two-dimensional XY model [8].

We end up this section with a numerical testing of the two definitions of persistence probability mentioned earlier in the introduction. We have evaluated the local persistence exponent for three-dimensional Heisenberg model using both definitions and have shown them in figure 8. Using the second definition, where the local persistence is the probability that none of the components of local order-parameter field has ever changed sign, we have obtained the local persistence exponent to be equal to 1.57. This is slightly more than three times the exponent (0.50) obtained using the first definition [see Eq. (5)]. Probably, the n-times approximation, mentioned earlier is only approximately true, because $n(=3)$ is finite.

V. CONCLUSIONS

We now sum up the findings of the present work. Using the notion of block persistence, we have estimated the local and global persistence exponents for $T=0$ quenched two-dimensional XY model and three-dimensional Heisenberg model. The local persistence exponents were found to be equal to $0.305(\pm 0.05)$ and $0.50(\pm 0.01)$ for the XY and Heisenberg models, respectively, while the global persistence exponents were found to $0.22(\pm 0.01)$ and $0.13(\pm 0.01)$. We have found that in the case of three-dimensional Heisenberg model, the persistence exponents obey the temperature universality. Scaling associated with the correlated persistence sites has been investigated, and we observe that the relevant length scale ξ associated with the correlated persistent sites behaves similar to $L(t)(\sim t^{1/2})$.

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