Formation of a magnetic hole above the mirror-instability threshold in a plasma with sloshing ions

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Within the framework of paraxial approximation it is shown that in an anisotropic plasma with sloshing ions confined an open-ended system a magnetic hole is formed near the turning point of the sloshing ions above the threshold of the mirror instability. The magnetic field experiences a jump at the hole boundary from the side of the magnetic mirror. For a small excess over the mirror instability threshold, the surface of the discontinuity has the shape of a truncated paraboloid, and the magnitude of the magnetic field jump at the system axis is proportional to the radius of the hole and gradually decreases to zero away of the axis. It is argued that disappearance of the magnetic hole because of the widening of the sloshing ions angular spread in the course of the neutral beam injection results in abrupt anticorrelated changes of the diamagnetic signals measured near the turning point of the sloshing ions and near the midplane of the gas-dynamic trap.

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The inequality

$$\frac{\partial}{\partial B}\left(p_{\perp}+\frac{B^2}{8\pi}\right) > 0, \tag{1}$$

which guaranties the stability against the mirror modes, is commonly recognized as necessary condition for the boundary value problem of the plasma equilibrium in an open-ended domain to be well posed [1]. It also restricts applicability of a paraxial approximation to the case of sufficiently low transversal plasma pressure $p_{\perp}$ [2].

In spite of these directions, we admit that the paraxial approximation still can be used for computation of the plasma equilibrium in an open-ended system even above the threshold of the mirror instability almost in the entire bulk of a plasma column except for some small regions. A similar situation was addressed by K. Lotov [3]. In his example, a magnetic hole with exactly zero magnetic field is formed around the axis of isotropic plasma column in a superconducting expander of an axisymmetric confinement system.

We consider a plasma configuration, typical for systems with an intense slope injection of high-energy neutral atoms, but address a much mode dense plasma as compared to earlier experimental studies described, e.g., in Ref. [4]. The atoms are trapped into a target, relatively cold plasma through the change exchange process, thus forming a population of fast sloshing ions. The ions bounce off the magnetic mirrors building up narrow pressure peaks at the turning points. As the pressure of the sloshing ions rises in the course of injection, the criterion Eq. (1) breaks starting from the plasma axis, where the pressure is maximal.

Following a standard paraxial approach, we assume that the magnetic field B inside the plasma column can be related to the external magnetic field H by the equation

$$p_{\perp}+\frac{B^2}{8\pi}=\frac{H^2}{8\pi}, \tag{2}$$

where the transversal pressure, $p_{\perp}=p_{\perp}(\Phi,B)$, is interpreted as a given function of the magnetic flux Φ and the actual magnetic field B. The approximate Eq. (2) is derived from the exact equation of the transversal equilibrium,

$$\frac{\partial}{\partial n}(B^2+8\pi p_{\perp})=\kappa(2B^2+8\pi p_{\perp}-8\pi p_{\parallel}), \tag{3}$$

by dropping the right-hand side, proportional to the field line curvature κ, and integrating the left-hand side over the direction of the normal n to the magnetic field line. In the paraxial approximation, the curvature is assumed to be small, and the vacuum field $H=H(z)$ a given function of the coordinate z along the axis of symmetry.

In the same approximation, the magnetic flux Φ as a function of r and z can be implicitly determined from the equation

$$\frac{\partial r^2}{\partial \Phi}=\frac{1}{\pi B(\Phi,H)}, \tag{4}$$

where $B(\Phi,H)$ stands for a root of Eq. (2). Outside the plasma column, $B=H$, so that a non-trivial part of the problem resides in the interval $0\le\Phi\le\Phi_p$, where $p_{\perp}$ is distinguished from zero. For a known function $B(\Phi,H)$, Eq. (4) can be formally solved in quadratures as follows

$$r^2(\Phi,H)-r_p^2(H)=\int_{\Phi}^{\Phi_p}\frac{d\Phi}{\pi B(\Phi,H)}, \tag{5}$$

where the radius of the column, $r_p(H)$, is determined from the condition $r(0,H)=0$.

The described method for solving Eqs. (2) and (4) works fine, if the condition Eq. (1) holds and, hence, the dependency of B on H is one valued. An uncertainty appears if Eq. (2) has more than a single root regarding B. We assume below that one has to select a maximal root in this case. Such a choice is based on the proposition that the magnetic field

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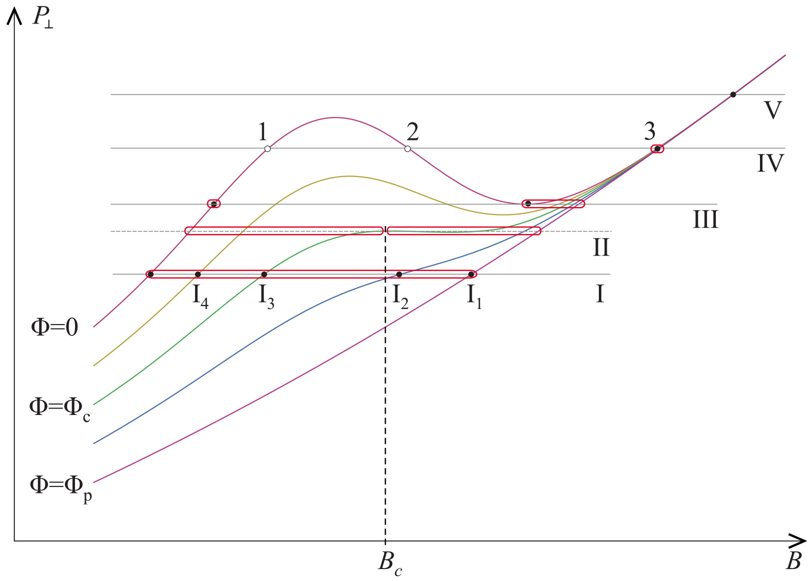

FIG. 1. (Color online) The dependency $P_\perp$ on B is nonmonotonic above the mirror instability threshold. It exhibits a peak, most prominent at the plasma axis ($\Phi=0$), which dissolves toward the column edge ($\Phi=\Phi_p$). The intersections of the curve $P_\perp(\Phi,B)$ with horizontal lines $P_\perp=H^2/8\pi$ correspond to the roots of Eq. (2). Nonrealizable roots are shown with empty circles, and the actual roots in solid circles. Allowed ranges of the magnetic field variation within the plasma column at a given section H=const are encircled by elongated ovals.

$H(z)$ outside the plasma column is not affected by the field restructuring in the plasma interior; this is true, if the plasma cross section is small as compared to the cross section of a conducting chamber surrounding the plasma.

Indeed, let us consider the total pressure $P_\perp(\Phi,B)=B^2/8\pi+p_\perp(\Phi,B)$ as function of B for various values of Φ as shown in Fig. 1 and imagine that we mentally integrate Eq. (4) with a step-by-step method, starting from the plasma edge at $\Phi=\Phi_p$ and moving toward the system axis at $\Phi=0$. Performing the integration, we move along a horizontal line $P_\perp(\Phi,B)=H^2/8\pi=$const from the right to the left. For example, moving along the line I, we sequentially go over the points $I_1,I_2,I_3\ldots$, which correspond to sequentially increasing values of $p_\perp$ as the plasma pressure monotonically increases toward the plasma axis.

The line $P_\perp=H^2/8\pi$ crosses any curve $P_\perp(\Phi,B)$ at a given Φ only once if H is noticeably larger than the magnetic field B_* at the turning point (line V). Passing to a lower vacuum field, from the line V to IV, we get three cross points, which correspond to three roots of Eq. (2). However, there is no visible reason at the moment for jumping to newly appeared smaller roots (indicated by open circles). The jump is forced when the maximal root 3 merges with the intermediate root 2 and disappears (line III). At first, the jump occurs for the uppermost curve that correspond to $\Phi=0$ (line III). However for a smaller H a similar jump occurs also at $\Phi>0$. Its magnitude gradually diminishes and the jump finally dissolves at some critical value of the magnetic flux Φ_c and B_c (line II). In the critical point,

$$\frac{\partial P_\perp}{\partial B}=0, \quad \frac{\partial^2 P_\perp}{\partial B^2}=0. \tag{6}$$

Since $p_\perp+B^2/8\pi$ is constant across the plasma column, the jump in B is accompanied by a jump in $p_\perp$. The magnetic field is smaller and the plasma pressure is larger at the inner side of the surface of discontinuity. Thus, a magnetic hole, filled with dense plasma, is formed near the system axis, if the stability criterion against the mirror modes is broken in the vicinity of the turning point. No such hole has been observed in laboratory experiments so far but both magnetic holes and magnetic humps are discovered in cosmic plasmas (see [5] and references therein).

The parameter Φ_c determines the radial size of the magnetic hole, which is roughly proportional to $\sqrt{\Phi_c}$. In the rest of the Brief Report, we consider the case of shallow magnetic hole where $\Phi_c\ll\Phi_p$, and, hence, the hole radius r_c is also small as compared with the plasma radius, $r_c\ll r_p$.

We assume that the sloshing ions have a narrow angular distribution with the angular width $\Delta\theta\ll 1$ around the angle of injection θ_*. Introducing the mirror ratio $b=B/B_0$ as the ratio of B to the magnetic field B_0 at the location of the injection in the minimum of magnetic field, one can show that the transversal pressure $p_\perp$ is peaked near the turning point located at the mirror ratio $b\approx b_*=1/\sin^2\theta_*$. Near the maximal value, $p_{\perp *}$, the transversal pressure as a function of B varies on the scale of $\Delta B=B_0\Delta b$, where $\Delta b=2\sqrt{b_*-1}b_*\Delta\theta$ (see, e.g., [6]). Evaluating maximal negative value of $\partial p_\perp/\partial B$ as $-p_{\perp *}/\Delta B$, we readily find that the condition Eq. (1) breaks if

$$p_{\perp *}>p_{\perp c}\sim B_*\Delta B/4\pi. \tag{7}$$

Thereafter, the corresponding critical value $\beta_c\sim\Delta b/b_*$ of the parameter $\beta=8\pi p_{\perp *}/B_*^2$ turns out to be small since $\Delta\theta\ll 1$.

In a typical plasma configuration, $p_\perp$ is monotonically decreasing function of Φ, which is maximal at $\Phi=0$. When $p_\perp$ at $\Phi=0$ slightly exceeds the critical value $p_{\perp c}$, the variation of the magnetic field on the size of the hole is small as compared with ΔB. This justifies expansion of the function $P_\perp(\Phi,B)$ to the Taylor series around the critical values Φ_c and B_c. Putting the expansion to the left-hand side of Eq. (2) yields

$$\psi\frac{\partial P_\perp}{\partial\psi}+\psi\zeta\frac{\partial^2 P_\perp}{\partial\zeta\,\partial\psi}+\frac{\zeta^3}{6}\frac{\partial^3 P_\perp}{\partial\zeta^3}=\frac{H^2(z)}{8\pi}-P_\perp, \tag{8}$$

where $\psi=(\Phi_c-\Phi)/\Phi_p$, $\zeta=(B-B_c)/\Delta B$, and the function $P_\perp$ and its derivatives are evaluated at $\psi=\zeta=0$. By order of magnitude, all the derivatives of $P_\perp$ are equal to $p_{\perp *}$ with minor refinement that $\partial^2 P_\perp / \partial\zeta\partial\psi$ is negative. Being a rising function of z, the right-hand side of Eq. (8) passes through zero near the turning point. Let it occurs at $z=0$. Expanding the right-hand side around $z=0$, we can write it in the form $\frac{1}{8\pi}H_c^2 z/L$, where L stands for the gradient length of the vacuum magnetic field, and H_c is the vacuum magnetic field in the plane $z=0$. Dividing Eq. (8) by $\partial^3 P_\perp/\partial\psi^3$ finally yields

$$\tfrac{1}{6}\zeta^3 - \alpha\psi\zeta = z/\ell - \gamma\psi, \tag{9}$$

where

$$\alpha = -\frac{\partial^2 P_\perp}{\partial\zeta\,\partial\psi}\bigg/\frac{\partial^3 P_\perp}{\partial\zeta^3}, \qquad \gamma = \frac{\partial P_\perp}{\partial\psi}\bigg/\frac{\partial^3 P_\perp}{\partial\zeta^3},$$

$$\ell = \left(\frac{\partial^3 P_\perp}{\partial\zeta^3}\bigg/\frac{1}{8\pi}H_c^2\right)L.$$

Note that $\ell \sim \beta_c L$, and one of the coefficients α and γ can be equated to 1 by renormalization of the parameter ψ; using this opportunity, we set $\gamma=1$ below.

The parameter ψ can take negative or positive values not exceeding $\psi_c=\Phi_c/\Phi_p$. For $\psi>0$, Eq. (9) has 3 real roots, if its right-hand side falls in the range from $-(2\alpha\psi)^{3/2}/3$ to $(2\alpha\psi)^{3/2}/3$ near zero, otherwise it has a single real root. We assume that multiple roots are numbered in ascending order so that ζ_1 designates a minimal real root whereas ζ_3 stands for the maximal one. By continuity, we keep these notations for single roots matching corresponding multiple roots. Thus, there remains a single root ζ_1 if ζ_2 merges with ζ_3, and there remains ζ_3 if ζ_2 merges with ζ_1. The roots are arranged so that

$$\zeta_1 \le -\sqrt{2\alpha\psi} \le \zeta_2 \le \sqrt{2\alpha\psi} \le \zeta_3,$$

where the equality occurs if ζ_2 merges either with ζ_1 or with ζ_3.

Following the logic, described above, the intermediate root ζ_2 should never be chosen, and a "regular" solution $\zeta(\psi,z)$ is glued from ζ_3 and a part of ζ_1. The regular solution jumps from $\zeta_1=-2\sqrt{2\alpha\psi}$ on the "internal" side of the surface discontinuity to $\zeta_3=\sqrt{2\alpha\psi}$ at the "outer" side of the surface. Accordingly, the magnetic field jumps by the quantity of $3\sqrt{2\alpha\psi}\Delta B$.

At the surface of discontinuity, the left-hand side of Eq. (9) takes the value $-\frac{1}{3}(2\alpha\psi)^{3/2}$. Consequently, the position of the discontinuity in the coordinates (ψ,z) is computed from the equation

$$z/\ell = \psi - \tfrac{1}{3}(2\alpha\psi)^{3/2}. \tag{10}$$

Regular solution monotonically rises radially from the plasma axis toward its periphery, and axially from the system midplane toward the magnetic mirror. It means that no local mirror trap appears in the plasma as it occurs in the result of mirror instability development in a homogeneous magnetic field. In this particular sense, the inhomogeneity stabilizes the mirror instability.

To find the shape of the magnetic field lines in the coordinates (r,z), we put $B=B_0[b_c+\Delta b\zeta]$ to the right-hand side of Eq. (4) and write down it in the form

$$\frac{\partial r^2}{\partial\psi} = -a^2 + a^2\frac{\Delta b}{b_c}\zeta, \tag{11}$$

where $a=\sqrt{\Phi_p/\pi B_0 b_c}\sim r_p$. A formal integrations with the boundary condition $r^2=0$ at $\psi=\psi_c$ gives

$$r^2 = a^2[\psi_c-\psi] + a^2\frac{\Delta b}{b_c}\int_{\psi_c}^{\psi}\zeta(\psi,z)d\psi. \tag{12}$$

Since $\zeta(\psi,z)$ experiences a jump at the discontinuity surface, the function $r(\psi,z)$ has a kink there; hence, the derivative $\partial r/\partial z$, that characterizes a slope angle of the field line to the system axis, breaks at the surface of discontinuity.

The integral in Eq. (12) can be computed with the aid of Eq. (9). The result of computation,

$$\int_{\psi_c}^{\psi}\zeta(\psi,z)\mathrm{d}\psi = \left(\frac{\alpha z}{2\ell} - \frac{\zeta^2}{8}\right)\zeta^2 + F(z),$$

involves a function $F(z)$ to be determined separately for the regions inside and outside the hole, where $\zeta=\zeta_1$ and $\zeta=\zeta_3$, respectively.

The two expressions for $F(z)$ are determined from the condition that the entire integral is zero for $\psi=\psi_c$ and is also continuous at the surface of discontinuity. Simple calculations give

$$r^2 = a^2[\psi_c-\psi] + a^2\frac{\Delta b}{b_c}\left[\frac{\alpha z}{2\ell}(\zeta_1^2-\zeta_{1c}^2) - \frac{1}{8}(\zeta_1^4-\zeta_{1c}^4)\right], \tag{13}$$

inside the hole, and

$$r^2 = a^2[\psi_c-\psi] + a^2\frac{\Delta b}{b_c}\left[\frac{\alpha z}{2\ell}(\zeta_3^2-\zeta_{1c}^2) - \frac{1}{8}(\zeta_3^4-\zeta_{1c}^4) - \frac{9}{2}\frac{\alpha^2 z^2}{\ell^2}\right], \tag{14}$$

outside it; here $\zeta_{1c}=\zeta_1(\psi_c,z)$. The shape of the hole is found by substituting $\zeta_1=-2(2\alpha z/\ell)^{1/2}$ to Eq. (13) or $\zeta_3=(2\alpha z/\ell)^{1/2}$ to Eq. (14). Excluding ψ and ψ_c with the aid of Eq. (9) gives

$$r_h^2 = a^2\frac{(z_c-z)}{\ell} + \frac{2^{3/2}\alpha^{3/2}a^2}{3}\frac{(z_c^{3/2}-z^{3/2})}{\ell^{3/2}} - \mathcal{O}(\Delta b/b_c), \tag{15}$$

where z_c is the coordinate where the surface of discontinuity intersects the axis z; it is determined from the equation

$$\psi_c = \frac{z_c}{\ell} + \frac{1}{3}\left(\frac{2\alpha z_c}{\ell}\right)^{3/2}, \tag{16}$$

which follows from Eq. (12) at $\psi=\psi_c$, $z=z_c$, and $\zeta=-2(2z_c/\ell)^{1/2}$. The discontinuity surface has approximately the shape of a truncated paraboloid of revolution that terminates at the plane $z=0$; thus, z_c is the length of the hole, and $r_c=a\sqrt{\alpha z_c/\ell}$ is its radius.

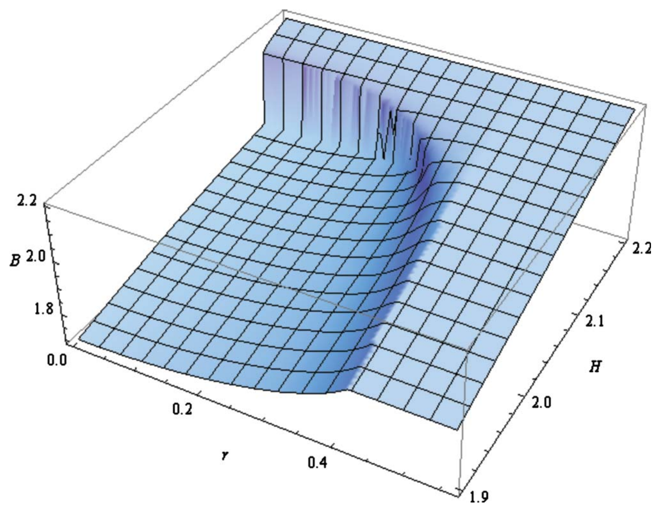

FIG. 2. (Color online) Magnetic hole near the turning point of sloshing ions for the case $\beta=0.1$ at $b=1$, $b_*=2$, $\Delta b=0.1$, $\Phi_p=1$, and $B_0=1$.

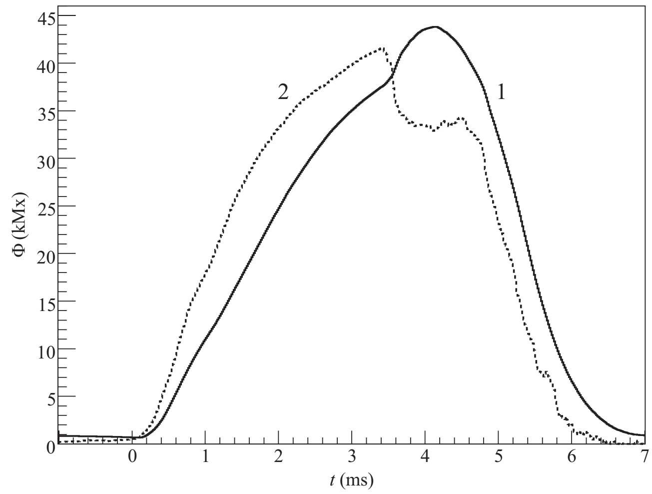

FIG. 3. Diamagnetic signals from two diamagnetic loops located near the midplane (solid curve 1) and near the turning point (dotted curve 2) of sloshing ions.

The validity of the paraxial approximation is limited by two requirements.

First, it is necessary for the slope of the magnetic field lines to be small, $|\partial r/\partial z| \ll 1$. The derivative $\partial r/\partial z$ is formally infinite at $z \to 0$, near the nose of the paraboloid, but it becomes sufficiently small if

$$z_c - z \gg z_c(a/L)^2. \tag{17}$$

Second, it is necessary to justify the use of approximate equality Eq. (2) instead of the exact Eq. (3). Since the derivative $\partial r/\partial z$ breaks on the surface of discontinuity, the curvature κ contains a delta function, $\kappa=\{\partial r/\partial z\}\delta(z-z_h)$, where $\{\partial r/\partial z\}$ denotes the magnitude of the jump, and $z_h=z_c-(r_h/a)^2\ell$ its coordinate. Hence, $P_\perp$ has a jump on the surface of discontinuity. This jump is negligible provided the condition

$$z_c - z \gg a^2/4\ell \tag{18}$$

holds, which is more stringent than Eq. (17).

Figure 2 shows a deep magnetic hole near the turning point of sloshing ions for a special case when all ions have same energy and their angular distribution is described by the Gaussian function with the angular width such that $\Delta b=0.1$ and the turning point is located at $b_*=2$.

In conclusion, we emphasize that the mechanism of the magnetic hole formation, discussed in this Letter, might be responsible for similar phenomena in cosmic plasmas [5], where the origin of the magnetic holes and humps is still disputable. We also note that in a pulsed experiment, such as gas-dynamic trap (GDT) [7,8], the angular spread $\Delta\theta$ and, hence, the parameter Δb increases with time, so that the magnetic hole finally disappears, even if the total number of sloshing ions continues to rise. The bifurcation from the magnetic hole to a smooth field profile occurs in a very narrow range of values of Δb. Thus, pushing a large number of fast ions from the magnetic hole looks like an abrupt event. We assume that this phenomenon is observed in GDT, where β in the turning point exceeds 50%, although only direct measurements of the spatial structure of the magnetic field could prove that the magnetic hole is indeed formed. Figure 3 shows the diamagnetic signals from two loops located at the mirror ratios $b=1$ and $b=2$. They exhibit anticorrelated abrupt changes at the end of neutral beam injection after $t=3.5$ ms. Accompanying measurement of the neutron flux in the experiment with deuterium plasma shows no significant change of the total flux, which means that the total number of sloshing ions is conserved though the bifurcation.

[1] Harold Grad, Proc. Symp. Appl. Math. **XXVIII**, 162 (1967).
[2] I. M. Lansky, *Budker INP 93–96* (Budker INP, Novosibirsk, 1993).
[3] K. V. Lotov, Phys. Plasmas **3**, 1472 (1996).
[4] T. J. Orzechowski, S. L. Allen, J. H. Foote, R. K. Goodman, A. W. Molvik, and T. C. Simonen, Phys. Fluids **26**, 2335 (1983).
[5] T. L. Zhang, W. Baumjohann, C. T. Russell, L. K. Jian, C. Wang, J. B. Cao, M. Balikhin, X. Blanco-Cano, M. Delva, and M. Volwerk, J. Geophys. Res. **114**, A10107 (2009).
[6] I. A. Kotelnikov and D. D. Ryutov, Sov. J. Plasma Phys. **11**, 655 (1985) [Fiz. Plazmy **11**, 1155 (1985)].
[7] P. A. Bagryansky, A. A. Ivanov, E. P. Kruglyakov, A. M. Kudryavtsev, Yu. A. Tsidulko, A. V. Andriyash, A. L. Lukin, and Yu. N. Zouev, Fusion Eng. Des. **70**, 13 (2004).
[8] T. C. Simonen, A. Anikeev, P. Bagrynsky, A. Beklemishev, A. Ivanov, A. Lizunov, V. Maximov, V. Prikhodko, and Yu. Tsidulko, in *Innovative Confinement Concepts Workshop* (Princeton University, Princeton, NJ, 2010).