Optimal transport on supply-demand networks

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In the literature, transport networks are usually treated as homogeneous networks, that is, every node has the same function, simultaneously providing and requiring resources. However, some real networks, such as power grids and supply chain networks, show a far different scenario in which nodes are classified into two categories: supply nodes provide some kinds of services, while demand nodes require them. In this paper, we propose a general transport model for these supply-demand networks, associated with a criterion to quantify their transport capacities. In a supply-demand network with heterogeneous degree distribution, its transport capacity strongly depends on the locations of supply nodes. We therefore design a simulated annealing algorithm to find the near optimal configuration of supply nodes, which remarkably enhances the transport capacity compared with a random configuration and outperforms the degree target algorithm, the betweenness target algorithm, and the greedy method. This work provides a start point for systematically analyzing and optimizing transport dynamics on supply-demand networks.

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I. INTRODUCTION

Network transport has attracted increasing attention in recent years (see the review papers [1,2] and the references therein). Indeed, it describes a large number of natural phenomena and technological processes, such as substance flow in a metabolic network, power transmission in an electric network, information propagation in the Internet, and so on. A matter of prime importance is to make the transport processes more effective and efficient, corresponding to maximizing the global capacity and minimizing the average delivery time. Previous works that addressed this issue can be roughly classified into two categories: one concerns the optimization of underlying topology [3–5], while the other focuses on the design of highly efficient routing protocols [6–10].

A latent assumption in most previous works is that every node in a transport network plays the role of a host; that is to say, every node has the ability of creating a certain kind of substance, energy, or information. However, the real world is far from this assumption. For example, in an electric network [11,12], there are two kinds of nodes: power stations and transformer substations. The power is generated in the former nodes, flowing to the latter ones, and then imported to customers through them. Therefore, power stations behave as a kind of suppliers, while the transformer substations are customers holding demands. In some Internet serving systems, such as music libraries (e.g., audioscrobbler.com; see Ref. [13]), movie-sharing systems (e.g., Netflix.com; see Ref. [14]), and online viewing site (e.g., YouTube.com; see Ref. [15]), all the resources are located in a few servers, while other connected nodes, usually personal computers, only regale themselves with those services. Those examples give rise to a general concept of *supply-demand network*, whose nodes are classified into two categories: supply nodes provide some kinds of services, while demand nodes play the role of customers. Analysis of supply-demand networks has found its applications in various real systems, ranging from the power grids [16,17] to supply chain networks [18,19].

In this paper, we propose a general model for the transport on a supply-demand network (see the Appendix for the discussion about searching process on supply-demand networks, which is similar yet different from the transport dynamics), whose capacity is very sensitive to the locations of supply nodes. By applying a simulated annealing (SA) algorithm, we obtained the near optimal locations of supply nodes subject to the maximal network transport capacity. The proposed algorithm performs obviously better than the random selection, degree targeted, betweenness targeted, and greedy methods.

II. MODEL

Consider a network consisted of N nodes, which are classified into two categories: one is called the *supplier* that provides a certain kind of service, and the other is called the *customer* who requires this service. Here, the service is an abstract concept and can stand for substance, energy, information, etc. For simplicity, we use the language of the Internet; that is to say, every customer needs some information packets (resource), and only the suppliers can generate these packets. We assume that the demands are uniformly distributed, namely, each customer needs a unit number of packets (one can simply say one packet). For a given customer, we suppose that this packet is always sent by one of the nearest

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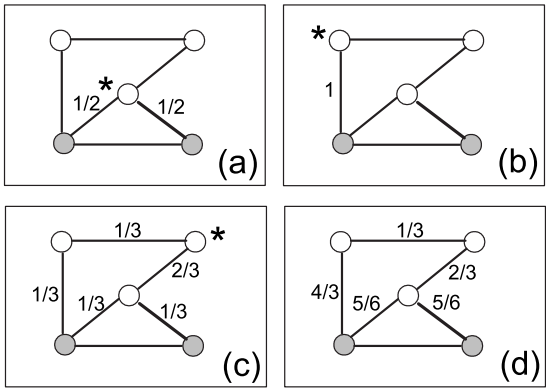

FIG. 1. Illustration of the distribution of edge loads in a supply-demand network. The gray solid and hollow circles denote supply and demand nodes, respectively. In each of (a)–(c), the circle marked by a star is the target demand node, and the resulting loads are labeled besides corresponding edges. Integrating (a)–(c), the distribution of edge loads can be obtained, as shown in (d). Here, $L_{\max}=4/3$.

suppliers. However, in general case, there are several nearest suppliers and for each there are several shortest paths. In the real implementation, one of those shortest paths should be randomly picked, and the packet will follow this path from the supplier to the customer. In the numerical calculation, to reduce the fluctuation, if there are in parallel k shortest paths from a customer to the suppliers (generally, those paths aim to several nearest suppliers), we assume that the packet is divided into k pieces, each goes through one shortest path and contributes $1/k$ to the traffic load (see an illustration in Fig. 1).

If the bandwidth (i.e., traffic capacity) of each edge is identical, the maximal edge load $L_{\max}$ is the key factor determining the traffic condition. Actually, the traffic congestion will occur when $L_{\max}$ exceeds the bandwidth. Therefore, given a limited bandwidth, the smaller $L_{\max}$ corresponds to higher transportation capacity. Analogously, in the previous studies [3,7], the maximal node load is used to quantify the system's performance: the smaller the maximal node load, the higher the transport capacity. In this paper, we use edge load instead of node load because in the real systems, such as the Internet and the highway, the congestion usually happens along the edges, not at the nodes [20].

Given network structure and the number of suppliers, we aim at finding out the optimal configuration of suppliers (i.e., the locations of suppliers) making $L_{\max}$ as small as possible. This is an optimization problem with $L_{\max}$ being the objective function, and the algorithm presented in this paper (see below) can be directly extended to the case with maximal node load being the objective function. In addition, since many real transport networks have heterogeneous degree distribution (see the examples shown in Refs. [21,22]), we use scale-free networks to mimic their topologies.

III. ALGORITHM

In a supply-demand network of N nodes and M suppliers, there are in total $\binom{N}{M}$ different configurations for suppliers' locations. Finding the optimal solution by evaluating all the possible configurations is infeasible when $N \gg M \gg 1$. The optimization of a system with many degrees of freedom with respect to a certain objective function is a frequently encountered task in physics and beyond (a similar optimization problem, named *p-median problem*, was proved to be nondeterministic polynomial-time (NP) hard [23], where the optimal configuration of M suppliers is required to minimize the total distance of all demand nodes to their nearest suppliers). One special class of algorithms used for finding the high-quality solutions to those optimization problems is the so-called nature inspired algorithms, including *simulated annealing* [24,25], *genetic algorithms* (GAs) [26,27], *genetic programming* (GP) [28], *extremal optimization* [29,30], and so on. Here, we adopt the SA algorithm, whose procedure is as follows:

(i) Randomly choose an initial configuration, denoted by S^0. Calculate its maximal edge load $L^0_{\max}$ and set the best solution as $S^{\text{best}}=S^0$ and $L^{\text{best}}_{\max}=L^0_{\max}$. Set the system time as $t=1$.

(ii) Randomly pick one supplier from the configuration S^{t-1} and change its location randomly; denote this new configuration as S^t. Calculate its maximal edge load $L^t_{\max}$.

(iii) If $L^t_{\max}<L^{\text{best}}_{\max}$, then set $S^{\text{best}}=S^t$ and $L^{\text{best}}_{\max}=L^t_{\max}$. If $L^t_{\max}\leq L^{t-1}_{\max}$, we accept the current configuration, that is, set $t\leftarrow t+1$ and repeat (ii). Otherwise, if $L^t_{\max}>L^{t-1}_{\max}$, the current configuration is accepted with probability $e^{-\Delta/T}$, where T is a temperature-like parameter and $\Delta=L^t_{\max}-L^{t-1}_{\max}$. When a configuration is rejected, the algorithm directly goes back to (ii) and keeps the system time t unchanged.

To obtain the high-quality solution, one shall repeat step (ii) as long as desired. In this paper, we terminate the algorithm if the variance of $L^t_{\max}$ in the latest 10^4 time steps is smaller than a threshold 10^{-6}. Note that one time step corresponds to one implementation of step (ii), which is different from the system time t. The parameter T is crucial for the algorithmic efficiency. According to the Metropolis' guidance [31], in the initial stage, the accepting probability of a new configuration should be close to 1. Therefore, we first choose a relatively low temperature T_0 and numerically calculate the corresponding accepting probability, resulted from a random change of one supplier's location in a completely random configuration. The temperature is doubled until the accepting probability reaches a threshold quantile of 0.50. During the optimizing process, the temperature should slowly decrease [25]; here we adopt the simplest method, that is, we set $T\leftarrow\alpha T$ after every Q time step, where the parameter α is 0.90 and the period is set as $Q=0.1NM$.

For comparison, we also implement some other algorithms. A brief introduction is as follows: *Random allocation* (RA)—the locations of suppliers are selected completely randomly. *Degree target algorithm* (DTA)—the suppliers are the M nodes with the highest degrees. *Betweenness target algorithm* (BTA)—the suppliers are the M nodes with the highest betweennesses (see Refs. [32,33] for the definition and calculation of node betweenness). *Greedy method* (GM)—first, we consider the case with only one supplier and find out the optimal location of this supplier that minimizes the corresponding $L_{\max}$. Then, we add one supplier and find out its optimal location under the condition that the location

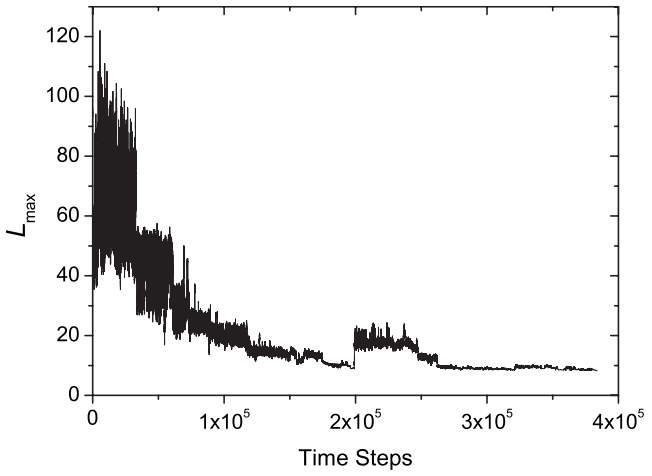

FIG. 2. The objective function L_{max} vs time step in the optimizing process of the SA algorithm. This figure illustrates a typical result on a BA network of size $N=1000$ and average degree $\langle k\rangle=6$. The number of suppliers is set as $M=10$.

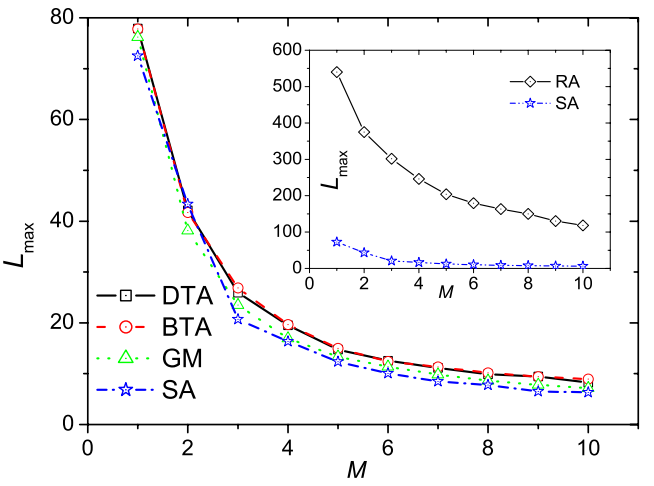

FIG. 3. (Color online) Algorithmic performance for BA networks. The main plot shows a comparison among DTA, BTA, GM, and SA, while the inset reports a comparison between RA and SA. The number of suppliers, M, varies from 1 to 10, while the network size $N=1000$ and the average degree $\langle k\rangle=6$ are fixed. All the data points are obtained by averaging over 100 independent network configurations.

of the first added supplier is fixed. Repeating this operation, that is, at the kth step, we add the kth supplier and find out its optimal location subject to minimal L_{max} under the condition that the locations of former $k-1$ suppliers are fixed. This algorithm is terminated when M suppliers are added already. Notice that the above-mentioned algorithms are presented in this paper to address the optimization problem for transportation capacity of supply-demand networks; however, the ideas of DTA and BTA have already been applied in intentional attack [34,35], epidemic immunization [36,37], pinning control of synchronization [38,39], and so on.

IV. RESULTS

In this paper, all the numerical simulations are implemented based on the Barabási-Albert (BA) model [40], which is one of the minimal models reproducing the heterogeneous degree distribution of real-world networks. Figure 2 reports a typical optimizing process, during which the objective function L_{max} fluctuates strongly in the early stage and approaches a relatively stable value lately. The proposed SA can reduce the objective function L_{max} by more than ten times compared with its initial value corresponding to a random selection of suppliers. We implement SA in BA networks for different M from 1 to 10 and take the average over 100 independent network configurations. As shown in the inset of Fig. 3, SA performs much better than RA. We also compare SA with the above-mentioned algorithms, DTA, BTA, and GM, and the results have demonstrated that SA performs the best. We report two examples, $M=5$ and $M=10$, in Table I. The improvement is in general about 10%. Note that, although SA performs the best, it spends the longest running time. Actually, the time complexity obeys the inequality $O(\mathrm{SA})>O(\mathrm{GM})>O(\mathrm{BTA})>O(\mathrm{DTA})$. Since GM performs not so bad, it is a strong candidate especially for huge-size networks, that is, GM might be a considerable trade-off of time complexity and accuracy of solution.

Note that, although BA model has successfully captured the degree heterogeneity of real networks, it lacks some other important structural properties, such as the community structure [41] and rich-club phenomenon [42]. DTA might perform worse if the network has strong community structure or presents the rich-club phenomenon. The reason is that a good algorithm should prefer to allocate suppliers to different communities rather than putting them together in a community containing many very-large-degree nodes, and if the very-large-degree nodes are closely connected to form a rich club, selecting them as a whole is of low efficiency since the increasing suppliers cannot substantially reduce the average distance from customers to suppliers. As a start point, we only discuss here simulation results on BA networks and leave the investigations of algorithmic performance on more complicated topologies as an open issue.

The DTA and BTA have almost the same performance and give out very similar selections of suppliers since in BA networks betweenness and degree are very strongly correlated [43,44]. To provide insights of the solution by SA, in Fig. 4, we give a scatter plot of betweenness versus degree and mark by red the selected suppliers by SA. Although SA also prefers large-degree (large-betweenness) nodes, the selected suppliers are remarkably different from those by DTA or BTA; actually, moderate-degree (moderate-betweenness)

TABLE I. Comparison of the maximal edge load obtained by DTA, BTA, GM, and SA. The underlying networks are BA networks with $N=1000$ and $\langle k\rangle=6$, and all the data are obtained by averaging over 100 network configurations.

M/Algorithm	DTA	BTA	GM	SA
5	14.73	14.98	13.32	12.37
10	8.25	8.92	7.17	6.31

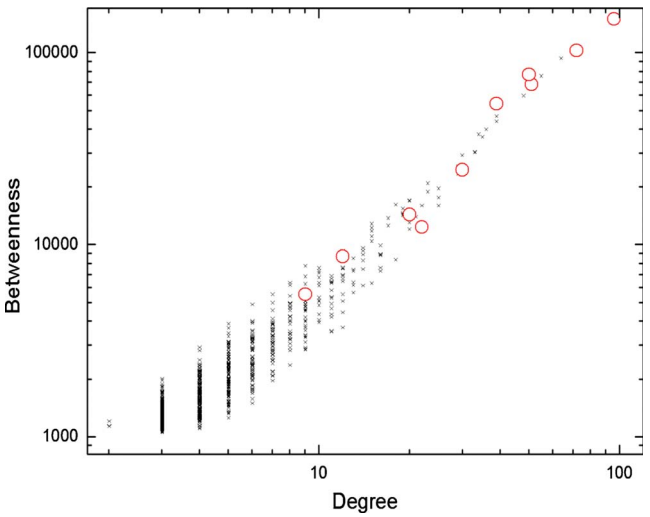

FIG. 4. (Color online) Scatter plot of betweenness vs degree in a BA network with $N=1000$ and $\langle k\rangle=6$. Each small black fork represents a node. These ten red circles denote the selected suppliers by SA. The smallest degree of suppliers is 9, and the second smallest one is 12.

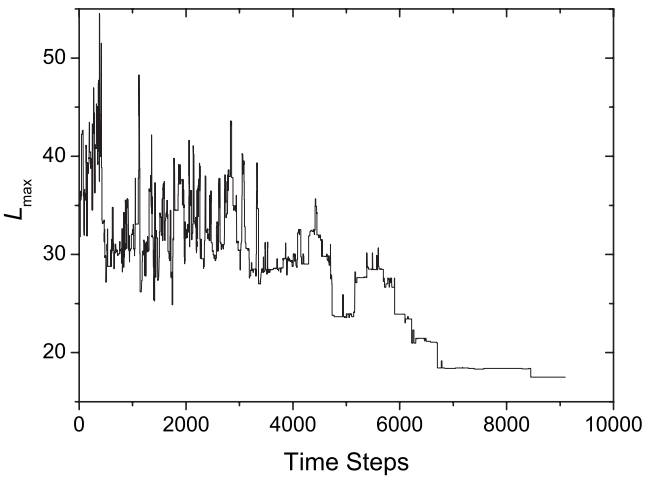

FIG. 5. The objective function $L_{\max}$ vs time step in the optimizing process of the SA algorithm for the collaboration network of network scientists. The number of suppliers is set as $M=10$.

nodes also have a chance to be selected by SA. We have checked the solutions given by SA and found that, in most cases, only the top 40% large-degree nodes have the chance to be included. Therefore, in the algorithmic procedure, we could restrict the candidates of suppliers in those 40% nodes [i.e., in algorithm step (ii), only the nodes of top 40% degree have the chance to be selected as a supplier], and then the number of possible solutions is largely reduced from $\binom{N}{M}$ to $\binom{0.4N}{M}$. When $N \gg M$, the reduced solution space is about 2.5^M times (e.g., $2.5^5 \approx 10^2$ and $2.5^{10} \approx 10^4$) smaller than the original one. We have checked this restriction in BA networks with $N=1000$, $\langle k\rangle=6$, $M=5$, $\alpha=0.90$, and $Q=500$, which gives out equivalently good solution while it requires about ten times shorter CPU time. Notice that the absolute time of SA is less meaningful since it strongly depends on M, α, and Q. The comparison reported here is only to show the benefit due to the restriction.

To further validate our algorithm, we tested it on a network of coauthorships between scientists who are themselves publishing on the topic of networks [45]. This network contains 1589 scientists, and 128 of which are isolated. Its connectivity is not good. It is consisted of 268 connected components. We only consider the giant component, which contains 379 nodes and 914 edges (more detailed structural information about the giant component can be found in Table II of Ref. [46]). As shown in Fig. 5, analogous to the case of BA networks, $L_{\max}$ can be sharply reduced after the SA process. Since the target network is highly clustered, more homogeneous than BA networks in degree distribution, and of clear community structure [45,46], comparing with the DTA, the advantage of SA algorithm is remarkable. For $M=5$ and $M=10$, the values of $L_{\max}$ obtained by SA are 25.33 and 16.38, respectively, while they are 51.97 and 25.14 by DTA.

V. CONCLUSION AND DISCUSSION

In this paper, we proposed a generic model of transport in the supply-demand network, which is consisted of suppliers (supply nodes) and customers (demand nodes). Accordingly, a measure of edge load is given, under the assumption that every customer only requires service from the nearest supplier. In such a network with heterogeneous degree distribution, its transport capacity is very sensitive to the locations of supply nodes. We therefore design a simulated annealing algorithm to find out the near optimal configuration of supply nodes, which remarkably enhances the transport capacity and outperforms the degree target algorithm, the betweenness target algorithm, and the greedy method. This work provides a start point for systematically analyzing and optimizing transport dynamics on supply-demand networks. Even though the model and algorithm are simple, we get some nontrivial result, that is, simply picking up those nodes of highest degrees may be not a good strategy; in fact, some moderate-degree nodes also have chance to be selected as suppliers.

In our model, every customer requires the same amount of resource, which is not in accordance with *the elephants and mice phenomenon* [47] found in the real Internet, where a small fraction of flows contribute to most of the traffic. Corresponding to the current model, a flow stands for the resource transported from a supplier to a customer, and thus each flow has the same size although the one passing longer paths contributes more to the total load. In addition, the proposed algorithm does not fully take into account and make use of the topological features. We have already mentioned in the last section that the mesoscopic structure, such as communities and the rich club, may highly influence the solutions. That structural information should be extracted prior to the optimizing algorithm and be embedded in the algorithmic procedure in some way to improve the efficiency and/or the resulting network capacity. All those blemishes listed above can be treated as open problems worth of future exploration.

In the end, we emphasize that many real systems can be better described by the current supply-demand network model, instead of the much simpler assumption [7] that every node simultaneously plays the roles of supplier and customer. We have already mentioned some examples, such as power grids [16,17] and supply chain networks [18,19]; an-

other typical example is the software supporting systems in the Internet, where a system usually has set up several servers in different locations, and users from everywhere can ask for downloading of some software. The locations of those servers play the crucial role in determining the efficiency and capacity the software supporting system.

This study also provides some complementary information for relevant phenomena in disparate systems. For example, social scientists have studied how to determine who should be integrators in a given social communication network to better solve problems, and they have found that people having extensive relations (i.e., of very large degrees) may not be the suitable information integrators; instead, the highest efficient structure makes the distance of all nodes from the obvious integrator the shortest [48], which is—to some extent—similar to the results of this paper since the shorter distance from the integrator statistically corresponds to higher transportation capacity if the network is not very heterogeneous (see the Appendix). In addition, empirical studies show that the public service facilities are not just located in the place of the densest population, but somehow more uniformly distributed to make the total travel distance between people and facilities shorter [49,50]. As a final remark, we note that a very recent work has considered the network-based transport with multiple sources and sinks [51], which shows different yet relevant motivation to the current work.

ACKNOWLEDGMENTS

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APPENDIX: TRANSPORT VERSUS SEARCHING

Searching and navigation describe the problem of how to find the shortest path connecting a pair of nodes. In the trivial case where the global topology is known, the shortest paths can be obtained by the breadth-first search. However, in the real world, usually only local structural information is available and the problem changes to how to design a navigation strategy to make the average required time, often measured by the average path length, as short as possible [52–55]. The searching efficiency can be largely improved by making use of some additional information, such as the metric space information [56,57] and labels of nodes [58]. For supply-demand networks, even with the knowledge of global topologies, the searching problem is not trivial. In such a case, the optimal configuration of suppliers minimizes the average path length form a demand node to its nearest supplier. This problem is widely known as the p-median problem, which has been proved to be NP hard [23].

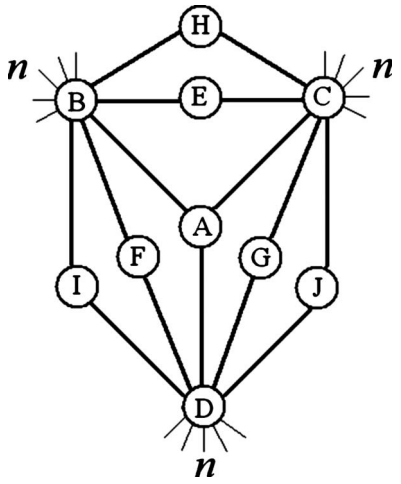

FIG. 6. Illustration of the different concerns for searching and transport. In this network, nodes B–D are, respectively, connected to n other nodes. In the large limit of n, if there is only one supplier, for searching, the optimal location of this supplier is A with $\langle d \rangle \approx 2$. In comparison, for transport, the optimal location is B or C or D, with $L_{\max} \approx \frac{2n}{3}$, while if A is the supplier, $L_{\max} \approx n$.

Searching and transport in supply-demand networks are two similar problems, both of which ask for an optimal configuration of supply nodes. Their essential difference is that the possible congestion is not considered in the searching process, while in the transport dynamics to avoid congestion is the first-priority requirement. As indicated by some previous works [3,7], the maximal load plays the key role in transport congestion. When the real transport exceeds the bandwidth of an edge, congestion happens and will soon spread to neighboring edges. Therefore, for searching, the system performance can be characterized by the average distance, while for transport the maximal edge load is used to quantify the system throughput.

Note that the sum of loads over all edges is equal to $(N-M)\langle d \rangle$, where $N-M$ is the number of demand nodes and $\langle d \rangle$ is the average path length from a demand node to its nearest supplier. Therefore, in a network with homogeneous edge load distribution, the objects to minimize $\langle d \rangle$ and $L_{\max}$ may lead to more or less the same result. In contrast, for general networks, the resulted optimal configurations for searching and transport may be far different. Figure 6 illustrates an example.

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