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Collective behavior of inanimate boats

Nobuhiko J. Suematsu,[1,2] Satoshi Nakata,[1] Akinori Awazu,[1] and Hiraku Nishimori[1,2]
[1]*Department of Mathematical and Life Sciences, Graduate School of Science, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8526, Japan*
[2]*Meiji Institute for Advanced Study of Mathematical Sciences (MIMS), 1-1-1 Higashimita, Tamaku, Kawasaki 214-8571, Japan*
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We propose an inanimate system composed of camphor boats in an annular water channel in order to understand the collective motions. The boats move on the water surface spontaneously and interact with one another through the concentration of the camphor molecules on the water. We observed several modes of collective motion, e.g., behaviors analogous to traffic flow or an ant trail. Our system provides a convenient experimental setup for the investigation of a variety of collective motions.

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I. INTRODUCTION

Spatiotemporal collective motions of several organisms have attracted considerable attention in a wide range of fields, e.g., biology [1], nonlinear physics of multibody systems [2,3], and engineering of autonomous systems [4]. Several types of models have been proposed to understand the universal and individual aspects of such collective motions [3–8]. It is, however, difficult to provide experimental means for the exploration of the intrinsic mechanisms of collective motions due to the complexity of the motions and the difficulty of controlling the behavior of individual organisms. Therefore, reduced systems are extremely important for clarifying the mechanism of such phenomena since they can be useful not only for controlling the experimental parameters, but also for creating advanced artificial systems that can change their features by sensing and adapting to the environment.

Several artificial systems with an annular field have already been proposed for experimental investigation of collective motion, e.g., granular system of bronze sphere driven by reciprocating rotation of the annular container [9], multiple robots system [10], and ideal traffic system [11]. In these systems, the interactions between the particles were demonstrated by collision, virtual pheromone realized using computer, and assessment of driver, respectively. The collective behavior of interacting particles depended on the number density in all of the systems, and as a result, density wave [9,11] and cluster formation [10] were observed.

In addition to collective behavior, spontaneous motion, which we refer to as "self-motion" hereinafter, is also one of the characteristic features of organisms participating in collective motion and a similar behavior can be observed in inanimate systems [12]. Several artificial systems which exhibit self-motion have already been studied from experimental [13–15] and theoretical [16,17] points of view, e.g., droplet systems [13] and solid systems [14,15] at immiscible interfaces. In particular, camphor boats constitute a promising system for the investigation of collective motion by providing a means for changing the number of particles and the physicochemical conditions due to controllability of the direction of motion and simple interaction between the boats [14]. The driving force of the camphor boat arises from the difference in surface tension, which is determined by the concentration of camphor molecules on the surface. Therefore, the direction of motion is controlled by the design of the boat and the interaction is carried out through the field of concentration of the camphor molecular layer.

Here, we propose an inanimate system composed of camphor boats in an annular water channel in order to construct an experimental setup for collective motion. The boats affected behind boats through a chemical field on a surface without any artificial feedback. Several (up to several tens of) camphor boats floated in water ring chamber [Figs. 1(a) and 1(b)]. The boats could not overtake each other due to the narrowness of the chamber. As a result, three types of collective behavior were observed with varying the number of the boat and physicochemical conditions.

II. EXPERIMENTAL SETUP

Camphor and glycerol were purchased from Wako Chemicals (Kyoto, Japan). Water was first distilled and then purified with a Millipore Milli-Q filtering system (pH of the obtained water: 6.3, resistance: $>20\ \mathrm{M\Omega}$). A camphor disk

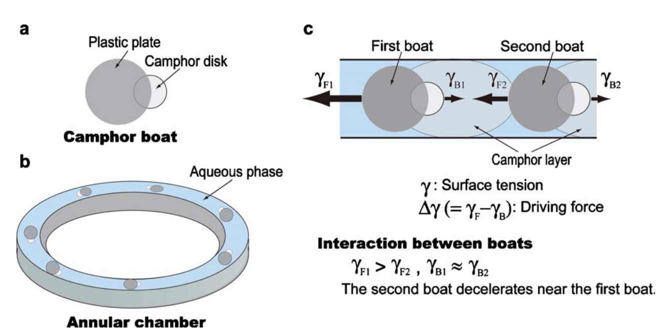

FIG. 1. (Color online) Illustration of experimental setup. (a) Design of the camphor boat. A plastic boat was made by cutting a polyester film into a discoid shape and a camphor disk was stuck on one edge of the plastic disk with an adhesive. (b) Boats were floated on a glass container with a circular route filled with water or glycerol aqueous solution of 5 or 10 mM. (c) Illustration of the interaction between the camphor boats through the camphor layer. The driving force of following boat decreases due to the camphor layer developed from the first boat.

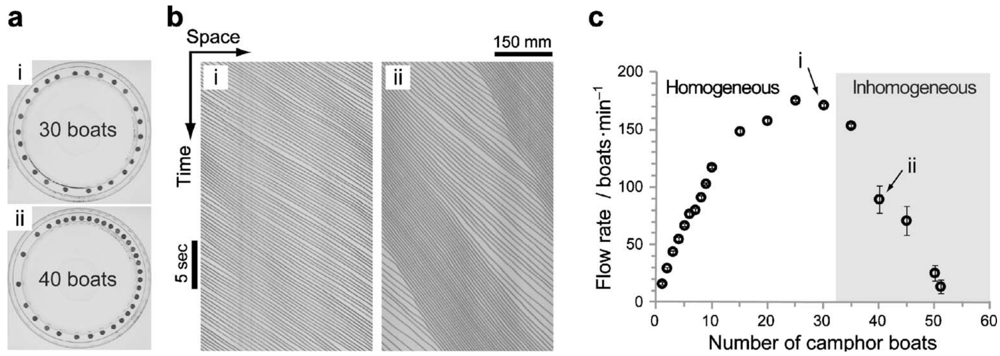

FIG. 2. Results of experimental observation to demonstrate collective motion of camphor boats. (a) Snapshots and (b) space-time diagram of camphor boats floated on water (i: 30, ii: 40 boats). Two types of states were observed, i.e., i: homogeneous and ii: inhomogeneous states. (c) Fundamental diagram, which is flow rate of camphor boat depending on the number of boats N. The critical number N_c was 30, over which the state became inhomogeneous.

(diameter: 3 mm, thickness: 1 mm, mass: 5 mg) was prepared using a pellet die set for Fourier transform infrared spectroscopy (FTIR). A plastic boat was made by cutting a polyester film (thickness: 0.1 mm) into a discoid shape (diameter: 6 mm) and the camphor disk was stuck on one edge of the plastic disk with an adhesive [Fig. 1(a)]. A glass container with a circular route (455 mm in peripheral length) had 5 mm in depth of water phase, 10 mm in the width, and 145 mm in inner diameter. Since the boat had spaces about 2 mm on each side, the neighboring boats cannot pass.

Several camphor boats floated in water ring chamber, where all boats were set to move in clockwise direction [Figs. 1(a) and 1(b)]. The number of camphor boats, N, was changed from 1 to 51. After floating the boats, the movement of the boat was monitored up to 5 min. In addition, 40 boats were floated on glycerol aqueous solutions of 5 or 10 mM in separated experiments to search an effect of viscosity of water phase. The movement of the camphor boat was monitored with a digital video camera (SONY DCR-VX700, minimum time resolution: 1/30 s) in an air-conditioned room at 298 ± 2 K and then analyzed by an image-processing system (ImageJ 1.41, National Institutes of Health, USA).

III. RESULTS AND DISCUSSION

A. Collective behavior of camphor boats similar to traffic flow

1. Experimental observation

First, we focus on the global behavior of the boats depending on the number of boats N, in particular, the spatiotemporal behavior and the transitions between characteristic states. The collective motion of camphor boats exhibited two different states depending on N, which were (i) a homogeneous state and (ii) an inhomogeneous state [18]. When N was less than 30, the boats dispersed homogeneously and moved with a constant velocity [Fig. 2, i], while for $N > 30$ the boats showed characteristic collective motion similar to a traffic congestion [Fig. 2, ii]. In the homogeneous state, the spacing between the boats and the velocity of each boat was almost constant both spatially and temporally, with the exception of small fluctuations. As a result, the boats advanced with a constant average velocity (125 mm s^{-1} for a single boat). In contrast, the velocity oscillated between relatively high and low values in the inhomogeneous state (49 and 13 mm s^{-1} for 40 boats). In addition, a shock wave was observed in the line of boats [Fig. 2(b), ii] similar to traffic congestions on highways [5]. The shock wave was based on the following individual behavior of boats: the first boat in a congestion region accelerated and passed through the free flow region at high speed, after which it reached the tail of the congestion of boats. Both the homogeneous and the inhomogeneous state were maintained for at least 5 min.

The state of the collective motion was estimated on the basis of the flow rate of the camphor boats, i.e., the number of boats passing through a fixed point per unit of time [Fig. 2(c)]. This estimation was analogous to that in traffic flow, where the plot of the flow rate versus the number density of cars is referred to as "fundamental diagram" [5]. In Fig. 2(c), with the increase in the number of camphor boats, the flow rate increased up to the critical number of boats (N_c: 30 boats), while it decreased for $N > N_c$.

The collective behavior of camphor boats is caused by the indirect interaction between individual boats through the camphor concentration field [14]. When two boats are sufficiently close, the driving force of the boat at the back decreases due to the camphor molecules developing from the first boat [Fig. 1(c)] [14]. As a result, the distance between the boats is adjusted spontaneously. Since camphor molecules on water surface undergo sublimation, the surface tension increases with time. Thus, the first boats has little effect on the boat at the back, even if the spacing between individual boats is sufficiently large. In other words, the interaction also depends on the total length of the chamber and the important factor is the mean distance between the boats [14].

2. Mathematical model

We present a mathematical model for the collective motion of camphor boats in a narrow circular chamber. This model is based on the Nagayama model, which reproduces the self-motion of a single camphor disk [17]. In our model, all boats move in a one-dimensional system which has a length R and a periodic boundary. The motion of the ith boat is assumed to be expressed by the following Newtonian equation:

$$m\frac{\partial^2}{\partial t^2}x_i = -\mu\frac{\partial x_i}{\partial t} + l\{\gamma[c(x_i+L,t)] - \gamma[c(x_i,t)]\}, \quad (1)$$

where m (kg) is the mass of the camphor boat, x_i is the position of the stern of in the one-dimensional system, μ (kg s^{-1}) is a constant of viscous resistance, γ (N m^{-1}) is the surface tension of water, c (mol m^{-2}) is the surface concentration of the camphor molecular layer, and l (m) and L (m) are the width and the length of the camphor boat, respectively. The second term on the right-hand side represents the driving force of a boat, which corresponds to the difference in surface tension between the front and rear edges of the boat. The surface tension $\gamma(c)$ is assumed to be expressed as a sigmoidal function of the surface concentration of the camphor molecular layer $c(x,t)$ as obtained from experimental results [19]. In this model, we assume

$$\gamma(c) = \frac{1}{2}(\gamma_{\text{water}} - \gamma_{\text{camphor}})\{\tanh[-(c - c_{\text{threshold}})] + 1\} + \gamma_{\text{camphor}}, \quad (2)$$

where γ_{water} and γ_{camphor} are the surface tensions of water and that of the saturated camphor solution, respectively, with the relation $\gamma_{\text{water}} > \gamma_{\text{camphor}}$ and $c_{\text{threshold}}$ is a phenomenologically estimated threshold of concentration of the camphor molecular layer over which the surface tension of the camphor solution rapidly decreases.

The surface concentration of the camphor molecular layer $c(x,t)$ is constructed by considering four factors: development on water surface, sublimation into the air, slight dissolution into the water, and supply from the camphor disk. Thus, $c(x,t)$ can be assumed to obey the reaction-diffusion equation as follows:

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2}{\partial x^2}c(x,t) - k_0 c(x,t) + \alpha\sum_{i=1}^{N}\delta(x_i - x), \quad (3)$$

where D is the diffusion coefficient of the camphor layer on the water surface, k_0 is the sum of the rates of sublimation (k_1) and dissolution into water (k_2) of the camphor layer ($k_0 = k_1 + k_2$), α is a positive constant, and N is the number of camphor boats. The delta function, δ, denotes that the camphor molecules are supplied only to the position x_i (the stern of the boat) where the camphor disk contacts with the water surface.

The results of the mathematical model were calculated with an initial condition where the boats are distributed randomly. The mathematical model qualitatively reproduces the experimental results, as shown below. First, there are two types of distribution, namely, (i) homogeneous and (ii) inhomogeneous with a shock wave depending on N [Fig. 3(a)]. Second, the fundamental diagram shows that the flow rate increases as N increases up to N_c (22 boats at $m = 15.0$), after which it starts decreasing [Fig. 3(b)]. The critical number N_c depends on the parameter conditions.

Mathematical analysis of this model uncovers the characteristics of the collective motion of camphor boats and its basic mechanism. With the assumption that the right-hand side of Eq. (3) equals 0 and the decay length of $c(x,t)$ is much shorter than the boat length L, the profile of $c(x,t)$ behind each boat remains constant in the moving coordinate system. Therefore, the interaction is determined only by the spacing between the neighboring boats. As a result, our model is reduced to the optimal velocity (OV) model which describes highway traffic flows [20]. The details of the derivation is described in the Appendix.

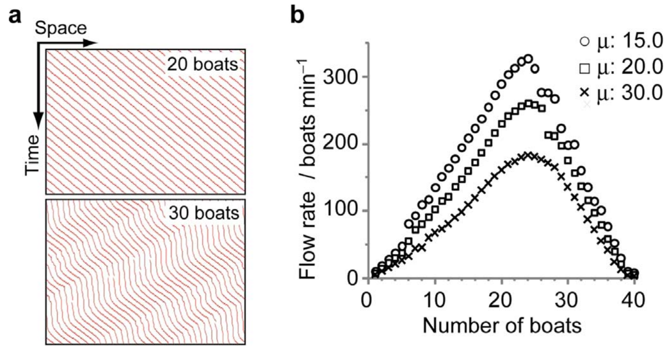

FIG. 3. (Color online) Results of numerical calculation. (a) Space-time diagram and (b) fundamental diagram obtained by numerical simulation.

3. Stability of the homogeneous state

The linear stability analysis has clarified that the stability of the homogeneous state in the OV model depends on both the average distance between the boats and a sensitivity parameter for adjusting their velocities to the optimal velocity [20]. As both the mean distance and the sensitivity increase, the homogeneous state becomes unstable and the system enters the inhomogeneous congestion mode. In our model, the mean distance corresponds to R/N and the constant sensitivity corresponds to the viscous resistance μ of the aqueous phase. In both the experiments and numerical calculations, the dependence of the stability of the homogeneous state on the mean distance was obtained (Figs. 2 and 3).

In order to clarify the influence of μ, the collective motion of 40 boats was observed by changing the viscosity of the aqueous phase. Figure 4(a) shows space-time diagram for 40 boats floating in (i) 0 mM (pure water), (ii) 5 mM, and (iii) 10 mM aqueous glycerol solution. The stable congestion mode in the pure water phase transferred to the homogeneous state concurrent with the increase in the viscosity. The respective viscosities of the aqueous glycerol solutions (5 and 10 mM) were 2×10^{-3} and 10×10^{-3} Pa s and their surface tensions were similar to that of pure water [17]. The model presented here also reproduces the change in the mode of distribution of camphor boats depending on the value of μ [Fig. 4(b)]. Both the experimental results and the numerical calculations indicated that the collective motion of camphor boats under the appropriate conditions can be considered as a system analogous to a traffic flow system.

B. Movement in "clusters" like an ant trail

From the condition that each boat affects two or more of the following boats due to excessive decay length, clusters of boats were observed in both the experimental results as

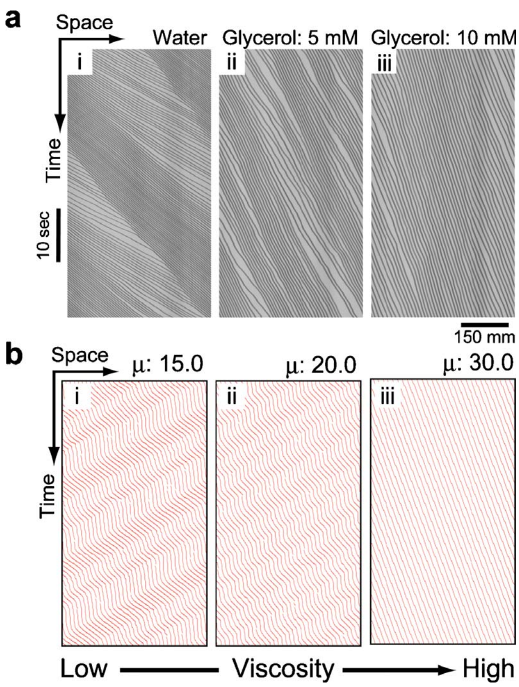

FIG. 4. (Color online) Influence of viscosity on the state of boats. (a) Experimental observation. The aqueous phases are i: pure water (1×10^{-3} Pa s), ii: 5 mM, and iii: 10 mM glycerol aqueous solution (2×10^{-3} and 10×10^{-3}Pa s, respectively). (b) Numerical calculation. The values of μ are i: 15, ii: 20, and iii: 30.

shown in Fig. 5[18] and the numerical calculation. In contrast to the congestion flow system, the first boat of the cluster did not accelerate and thus the boats moved in a group. Therefore, no shock waves were observed, even though the distribution of the boats was inhomogeneous.

The cluster formation depends on the condition that the first boat is slower than the following boats. This particular condition can be explained not by the characteristics of the first boat, but by the sigmoidal relation between $\gamma(c)$ and $c(x,t)$ [Fig. 5(a)]. In the case of excessive decay length of the camphor layer, the camphor concentration increases at the following boat. When the concentration around the following boats reaches at the inflection point of the sigmoidal function, the driving force of the following boat can become larger than that of the first boats [$\Delta\gamma_1<\Delta\gamma_2$ in Fig. 5(a)], even though the difference in camphor concentration is constant. As a result, a slow first boat induces the cluster mode.

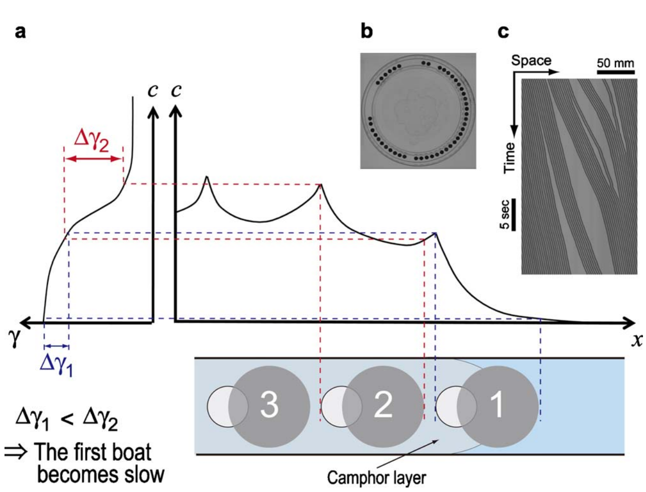

FIG. 5. (Color online) Experimental results and phenomenological explanation of cluster mode. (a) Illustration of phenomenological scenario to explain the cluster formation. (b) Snapshot and (c) space-time diagram of cluster mode.

A behavior similar to that of the cluster mode has been observed in the ant trail model [6,10]. Both camphor boats and ants interact through the sublimation of volatile chemicals (pheromones in the case of ants) in their surrounding environments. Although their mechanisms of action are opposite, namely, repulsion in the case of camphor and attraction in the case of pheromones, interaction through a chemical field can yield a head-slow situation, resulting in a cluster behavior.

IV. CONCLUSION

An inanimate system exhibiting collective motion was constructed by using several tens of camphor boats, which were floated in a periodic field and were interacted each other through the chemical field. In this camphor boats system, at least three types of collective motion were observed, i.e., a homogeneous state, congestion flow, and cluster flow, depending on the number of boats, the viscosity of the aqueous phase, and other physicochemical conditions. Although these collective behaviors have been observed in completely dissimilar systems, i.e., traffic flow and ant trail, we demonstrated in a single system by changing their physicochemical conditions. Namely, it is revealed that the several types of collective behavior could be represented by self-driven particles interacted through any chemical field. In the future, it is desired to uncover the detail conditions to determine the types of collective motion of camphor boats. Furthermore, our system could be extended to two-dimensional system that will demonstrate swarming behavior as is investigated theoretically [3].

ACKNOWLEDGMENTS

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APPENDIX: MATHEMATICAL ANALYSIS

Here we show that the present theoretical model corresponds, in a certain condition, to the OV model to describe the traffic on highway. First, we assume that the relaxation time of $c(x,t)$ in Eq. (1) is sufficiently shorter than the time needed for each boat, in its migration process, to experience definite change of the camphor concentration of surrounding solution. Then, quasisteady-state approximation of Eq. (3),

$$0 = D\frac{\partial^2}{\partial x^2}c(x,t) - k_0 c(x,t) + \alpha \sum_{i=1}^{N} \delta(x_i - x), \tag{A1}$$

holds. The solution of Eq. (A1),

$$c_{qs}(x,t) = \sum_{i=1}^{N} c_0 \exp\left(\frac{-|x - x_i|}{\mu}\right), \tag{A2}$$

is a superposition of those obtained from the contribution of each x_i, where $c_0 = \frac{\alpha}{2}\sqrt{\frac{1}{Dk_0}}$ is camper concentration at the rear edge of boat (for the isolated boat system) and $\mu = \sqrt{\frac{D}{k_0}}$ is the correlation length between boats through camphor field. If μ is sufficiently shorter than the interval between contiguous boats, $c_{\text{steady}}(x)$ at the rear and front edges of boat i is dominantly influenced by the neighboring boats, that is,

$$c_{qs}(x_i + L,t) = c_0 \exp\left[\frac{-|x_{i+1} - (x_i + L)|}{\mu}\right] + c_0 \exp\left[\frac{-|x_{i-1} - (x_i + L)|}{\mu}\right] + c_0 \exp\left(\frac{-L}{\mu}\right), \tag{A3}$$

$$c_{qs}(x_i,t) = c_0 \exp\left(\frac{-|x_{i+1} - x_i|}{\mu}\right) + c_0 \exp\left(\frac{-|x_{i-1} - x_i|}{\mu}\right) + c_0. \tag{A4}$$

Assuming, in addition, that μ is close to or shorter than L, the second terms in the right-hand side of Eq. (A3) are neglected like

$$c_{qs}(x_i + L,t) \cong c_0 \exp\left[\frac{-|x_{i+1} - (x_i + L)|}{\mu}\right] + c_0 \exp\left(\frac{-L}{\mu}\right) = \hat{c}(|x_{i+1} - x_i - L|,t)$$

and that $c_0 \geq c_{\text{threshold}}$ holds which means $\gamma[c_{qs}(x_i,t)] \cong \gamma_{\text{camphor}}$, then Eq. (1) has the form

$$m\frac{\partial^2}{\partial t^2}x_i = -\mu\frac{\partial x_i}{\partial t} + l|\gamma[\hat{c}(|x_{i+1} - x_i - L|,t)] - \gamma_{\text{camphor}}| = -\mu\frac{\partial x_i}{\partial t} + \frac{l}{2}(\gamma_{\text{water}} - \gamma_{\text{camphor}})\{\tanh[-\hat{c}(|x_{i+1} - x_i - L|,t) + c_{\text{threshold}}] + 1\}. \tag{A5}$$

This is just a typical form of OV model which has a qualitative fit to the data widely extracted from highway traffic.

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