Experimental investigation of the enhancement factor for microwave irregular networks with preserved and broken time reversal symmetry in the presence of absorption

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We present the results of the experimental study of the two-port scattering matrix $\hat{S}$ elastic enhancement factor $W_{S,\beta}$ for microwave irregular networks simulating quantum graphs with preserved and broken time reversal symmetry in the presence of moderate and strong absorption. In the experiment, quantum graphs with preserved time reversal symmetry were simulated by microwave networks which were built of coaxial cables and attenuators connected by joints. Absorption in the networks was controlled by the length of microwave cables and the use of microwave attenuators. In order to simulate quantum graphs with broken time reversal symmetry we used the microwave networks with microwave circulators. We show that the experimental results obtained for networks with moderate and strong absorption are in good agreement with the ones obtained within the framework of random matrix theory.

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Quantum graphs of connected one-dimensional wires were introduced more than seventy years ago by Pauling [1]. This idea was used later on by Kuhn [2] to describe organic molecules by free electron models. Quantum graphs are often considered as idealizations of physical networks in the limit where the widths of the wires are much smaller than their lengths, i.e., assuming that propagating waves remain in a single transversal mode. Among systems modeled by quantum graphs one can find, e.g., mesoscopic systems [3,4], quantum wires [5,6], excitation of fractons in fractal structures [7,8], electromagnetic optical waveguides [9,10], and microwave networks [11,12].

It has been shown that quantum graphs are excellent systems for studying quantum chaos [11–22] and localization phenomena [23–25]. Additionally, to the ideal graphs, without absorption, other interesting objects—open quantum graphs with leads—have been analyzed theoretically in details in [14,15]. From the experimental point of view more complicated and more realistic systems—microwave networks (graphs) with moderate absorption strength $\gamma=2\pi\Gamma/\Delta\le 7.1$, where Γ is the absorption width and Δ is the mean level spacing, have been investigated in [26,27]. Quite recently, the distribution of the reflection coefficient $P(R)$ and the distributions of the Wigner reaction matrix [28] (in the literature often called K matrix [29]) for networks with time reversal symmetry (β=1) in the presence of strong absorption ($\gamma>19$) have been studied experimentally and numerically in [12].

In this paper we present the results of the experimental study of the two-port scattering matrix $\hat{S}$ elastic enhancement factor $W_{S,\beta}$ [30,31] for microwave irregular networks simulating quantum graphs with preserved time reversal symmetry (symmetry index β=1) and with broken time reversal symmetry (β=2) in the presence of moderate and strong absorption.

In the case of the two-port scattering matrix

$$\hat{S}=\begin{bmatrix} S^{aa} & S^{ab} \\ S^{ba} & S^{bb} \end{bmatrix} \qquad (1)$$

the elastic enhancement factor $W_{S,\beta}$ is defined by the following relation [30,31]

$$W_{S,\beta}=\frac{\sqrt{\mathrm{var}(S^{aa})\mathrm{var}(S^{bb})}}{\mathrm{var}(S^{ab})}, \qquad (2)$$

where $\mathrm{var}(S^{ab})\equiv\langle|S^{ab}|^2\rangle-|\langle S^{ab}\rangle|^2$ denotes the variance of the scattering matrix element S^{ab}. One of the most important property of the enhancement factor $W_{S,\beta}$ for $\gamma\gg 1$ is connected with the fact that it should not depend on the direct processes present in the system [31,32]. The reciprocal quantity $\Xi_{S,\beta}=1/W_{S,\beta}$ was also considered theoretically and measured in the function of frequency for a chaotic microwave cavity with time reversal symmetry (TRS) [32]. The variances of the fluctuations of multiport impedance parameters which are necessary to estimate the enhancement factor $W_{S,\beta=1}$ were also studied within electromagnetic scattering theory in [33].

There is a general agreement that for $\gamma\gg 1$, $W_{S,\beta}=2/\beta$ [30–32]. However, calculations performed within random matrix theory (RMT) indicate that for the other values of the parameter γ the enhancement factor $W_{S,\beta}$ might depend both on the coupling to the system and the parameter γ itself [32]. In the more particular case of the stochastic environment characterized by a statistically isotropic scattering matrix (occurring in RMT in the case of the perfect coupling when all transmission amplitudes are equal to unity) the enhancement factor should have the universal value [33].

In the experiment we use microwave networks to simulate quantum graphs. The analogy between quantum graphs and microwave networks is based upon the equivalency of the Schrödinger equation describing the quantum system and the telegraph equation describing the microwave circuit [11].

A general microwave network with preserved TRS consists of N vertices connected by bonds, e.g., coaxial cables. A coaxial cable consists of an inner conductor of radius r_1 surrounded by a concentric conductor of inner radius r_2. The space between the inner and the outer conductors is filled with a homogeneous material having a dielectric constant ε. For a frequency ν below the onset of the next TE_{11} mode only the fundamental TEM mode can propagate inside a coaxial cable. The cutoff frequency of the TE_{11} mode is $\nu_c \simeq \frac{c}{\pi(r_1+r_2)\sqrt{\varepsilon}} = 32.9$ GHz [34], where $r_1=0.05$ cm is the inner wire radius of the coaxial cable (SMA-RG402), while $r_2=0.15$ cm is the inner radius of the surrounding conductor, and $\varepsilon \simeq 2.08$ is the Teflon dielectric constant [35,36].

The microwave networks with broken time reversal symmetry can be constructed using microwave circulators. A microwave circulator is a nonreciprocal three port passive device. A wave, which enters port 1 of a circulator exits port 2, a wave into port 2 exits port 3, and finally a wave into port 3 exits port 1 (see Fig. 2 for a schematic representation of a circulator).

From the experimental point of view absorption of the networks can be changed by the change of the bonds (cables) length [11] or more effectively by the application of microwave attenuators [26,27]. In the numerical calculations weak absorption inside the cables can be described with the help of the complex wave vector [11] while strong absorption inside an attenuator can be described by a simple optical potential [25]. The corresponding mathematical theory has been developed in [37].

Figure 1(a) shows the experimental setup for measuring the two-port scattering matrix $\hat{S}$ of fully connected hexagon microwave networks. We used Agilent E8364B microwave vector network analyzer to measure the scattering matrix $\hat{S}$ of the networks in the frequency window: 4–14 GHz. The networks were connected to the vector network analyzer through the leads—HP 85133–616 and HP 85133–617 flexible microwave cables—connected to the six-joint vertices. The five joints were the other four vertices of the networks. Each bond of the network presented in Fig. 1(a) contains a microwave attenuator.

Figure 2 shows the setup for measuring the two-port scattering matrix $\hat{S}$ of microwave networks with broken time reversal symmetry which contain four microwave circulators and microwave attenuators. The microwave circulators are enlarged to show the directions of traveling waves. In the experiment Anritsu PE8403 microwave circulators with the operating frequency range 7–14 GHz and low insertion loss ($i_{12} \simeq i_{23} \simeq i_{31} \simeq 0.4$ dB) were used.

Properties of such networks with microwave circulators but without microwave attenuators were investigated using the integrated nearest-neighbor distribution $I(s)$. The distribution $I(s)$ for microwave networks with broken TRS is shown in Fig. 3 (full circles). In order to minimize absorption of the networks the measurements were performed in the narrower frequency range 7–9 GHz. The distribution $I(s)$ was averaged over 20 microwave network configurations. In this way 1378 eigenfrequencies of the networks were used in the calculations of the distribution $I(s)$. Figure 3 shows that the experimental distribution $I(s)$ is close to the theoretical prediction for Gaussian unitary ensemble (GUE) in RMT (full line). This finding clearly demonstrates that microwave networks with microwave circulators, which introduce only very small absorption, can be used to simulate quantum graphs with broken TRS. This should be contrasted with the results of [11] obtained for microwave networks without circulators containing only microwave cables and joints which

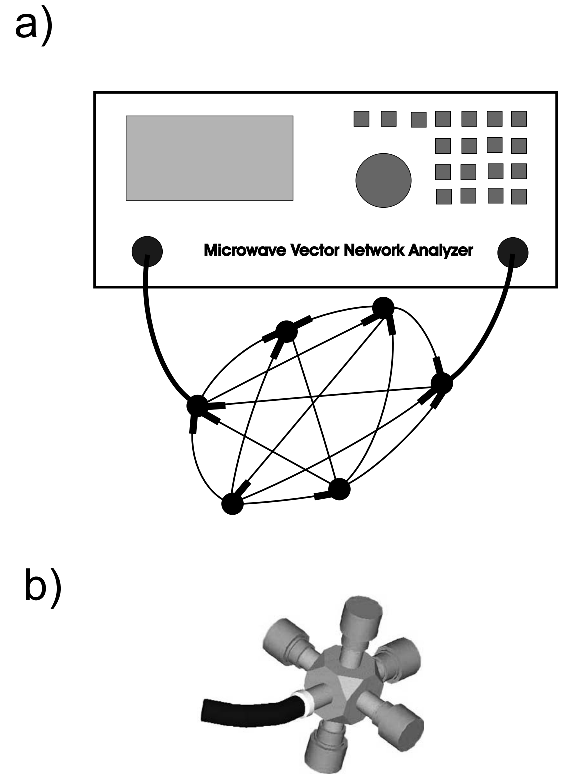

FIG. 1. (a) The scheme of the experimental setup for measuring the two-port scattering matrix $\hat{S}$ of fully connected hexagon microwave networks. The measurements were performed in the frequency window: 4–14 GHz. The microwave networks were connected to the vector network analyzer through the leads connected to the six-joint vertices. The other four vertices of the networks were the five joints. Each bond of the network presented in this figure contains a microwave attenuator. (b) The scheme of the setup used to measure the radiation scattering matrix S_r^{kk} of the six-joint connector. Instead of a microwave network five 50 Ω loads were connected to the six joint.

show agreement of the integrated nearest-neighbor distribution $I(s)$ with the theoretical prediction for Gaussian orthogonal ensemble (GOE) in RMT characteristic for the systems with preserved TRS. In Fig. 4 the enhancement factor $W_{S,\beta}$ of the two-port scattering matrix $\hat{S}$ of the microwave networks simulating quantum graphs with preserved and broken TRS, respectively, is shown in the function of the parameter γ.

The experimental values of the parameter $\gamma = (\gamma^{(a)} + \gamma^{(b)})/2$ were estimated for each realization of a network by adjusting the theoretical mean reflection coefficients

$$\langle R \rangle_{th}^{(k)} = \int_0^1 dRRP(R), \tag{3}$$

to the experimental ones $\langle R \rangle^{(k)} = \langle s^{kk} s^{kk\dagger} \rangle$ obtained after eliminating the direct processes [12]. Here the index $k=a,b$ denotes the port a or b. In the impedance approach [38,39] the scattering matrix s^{kk} of a network for the perfect coupling case (no direct processes present) can be extracted from the formula

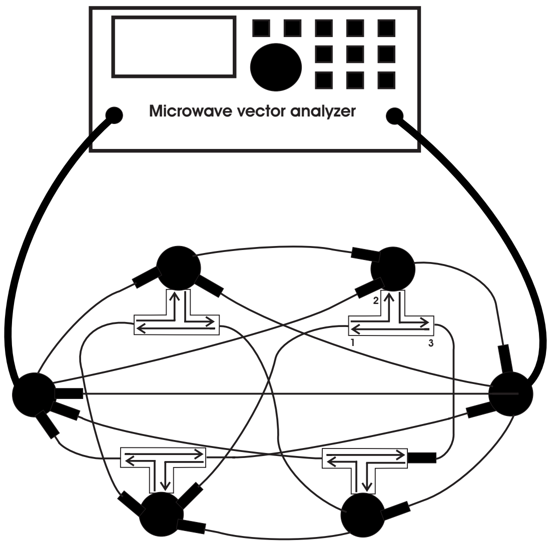

FIG. 2. The scheme of the experimental setup for measuring the two-port scattering matrix $\hat{S}$ of microwave networks with broken time reversal symmetry. The network additionally to the attenuators contains four microwave circulators. The microwave circulators are enlarged to show the input ports and the directions of traveling waves. The measurements were performed in the frequency window: 7–14 GHz.

$$s^{kk} = (1 - z^{kk})/(1 + z^{kk}), \tag{4}$$

where the normalized impedance z^{kk} of a chaotic microwave network is given by

$$z^{kk} = \frac{\mathrm{Re}\, Z^{kk} + i(\mathrm{Im}\, Z^{kk} - \mathrm{Im}\, Z_r^{kk})}{\mathrm{Re}\, Z_r^{kk}}. \tag{5}$$

In the formula (5) $Z^{kk}=Z_0(1+S^{kk})/(1-S^{kk})$ and $Z_r^{kk}=Z_0(1+S_r^{kk})/(1-S_r^{kk})$ are the network and the radiation impedances

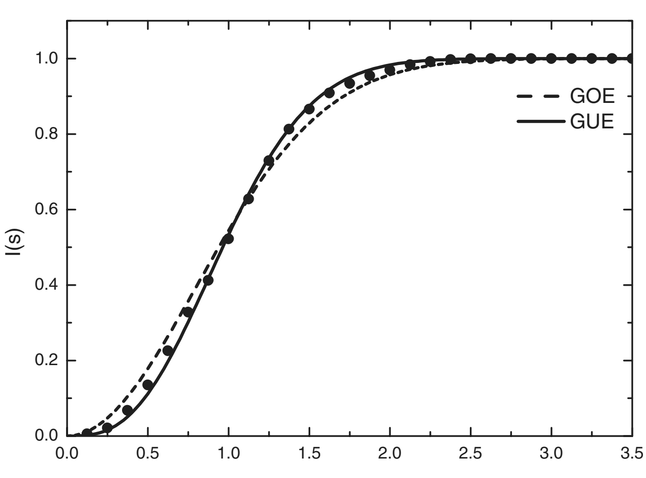

FIG. 3. The integrated nearest-neighbor distribution $I(s)$ obtained for microwave networks with broken TRS (full circles). The measurements were performed in the frequency range 7–9 GHz. The distribution $I(s)$ was averaged over 20 microwave network configurations. The experimental distribution is compared to the theoretical predictions for GOE (broken line) and GUE (full line).

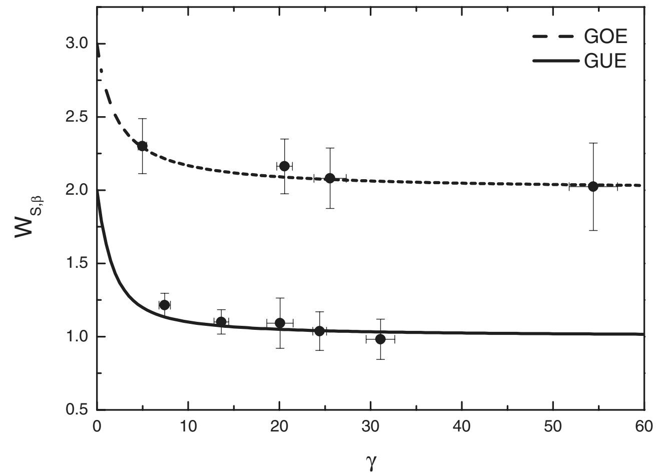

FIG. 4. The enhancement factor $W_{S,\beta}$ of the two-port scattering matrix $\hat{S}$ of the microwave networks simulating quantum graphs with preserved and broken TRS, respectively, in the function of the parameter γ. The experimental results are compared to the theoretical predictions for GOE (broken line) and GUE (full line).

expressed, respectively, by the network S^{kk} and the radiation S_r^{kk} scattering matrices. Z_0 is the characteristic impedance of the transmission lines feeding the six-joint vertices. The radiation scattering matrix S_r^{kk} is the scattering matrix measured at the input of the coupling structure for the same coupling geometry, but with the vertices (walls) of the system removed to infinity. The scheme of the setup used to measure the radiation scattering matrix S_r^{kk} of the six-joint connector is shown in Fig. 1(b). The five 50 Ω loads are connected to the microwave joint to simulate the vertices removed to infinity.

For systems with preserved TRS ($\beta=1$), the explicit analytic expression for the distribution $P(R)$ of the reflection coefficient R is given in [40]. The explicit form of the distribution $P(R)$ for systems with broken TRS ($\beta=2$) is presented in [41,42].

The measurements were done for the networks with preserved TRS containing no attenuators (the smallest value of the parameter $\gamma \simeq 5.0$) as well as for the ones containing fifteen 1 dB attenuators ($\gamma \simeq 20.6$), nine 1 dB and six 2 dB attenuators ($\gamma \simeq 25.6$) and finally fifteen 2 dB attenuators ($\gamma \simeq 54.4$). In all of the cases our results were averaged over 60 microwave network configurations. The total "optical" length of the networks including joints and attenuators was varied for different network configurations from 538 to 681 cm. The experimental results for the networks with preserved TRS are in general good agreement with the theoretical ones predicted by [30,31]. Even for moderate absorption $\gamma \simeq 5$ the experimental result $W_{S,\beta=1}=2.30\pm 0.19$ is very close to the theoretical one. In order to show the spread of the results obtained for different graph configurations the assigned errors were calculated using the definition of the sample standard deviation. Because of absorption of microwave cables (network bonds) we could not test experimentally predicted by the theory [30,31] increase in the enhancement factor $W_{S,\beta=1}\to 3$ for small values of the parameter γ. The measurements of the enhancement factor for this range of the parameter γ has been recently reported in [43]. In the experiment a flat microwave cavity with Ohmic losses was used. However, due to relatively large spread of the experimental results and their big uncertainties the limit $\lim_{\gamma\to 0} W_{S,\beta=1} \simeq 3$ still remains to be tested.

The experimental studies of the enhancement factor $W_{S,\beta}$ of the two-port scattering matrix $\hat{S}$ for the systems with broken TRS have not been reported so far. However, the paper [43] has already reported a weak change of the enhancement factor due to partially broken time invariance in microwave cavity. In Fig. 4 we show the results for the networks with broken TRS for the range of the parameter $7<\gamma<32$. The five experimental points were obtained for the networks containing four microwave circulators and different number of microwave attenuators. Beginning from the lowest absorption, the measurements were done for the microwave networks containing no attenuators ($\gamma \simeq 7.4$), seven 1 dB attenuators ($\gamma \simeq 13.6$), fifteen 1 dB attenuators ($\gamma \simeq 20.1$), nine 1 dB, and six 2 dB attenuators ($\gamma \simeq 24.4$) and finally for the ones containing fifteen 2 dB attenuators ($\gamma \simeq 31.1$). In all of the cases our results were averaged over 80 microwave network configurations. The total "optical" length of the networks including joints, circulators and attenuators, was varied for different network configurations from 528 to 699 cm.

Also in this case the experimental results are in good agreement with the theoretical ones $W_{S,\beta=2} \simeq 1$ predicted by [30–32] for moderate and strong absorption, although for the smallest value of the parameter $\gamma \simeq 7.4$ the experimental value of $W_{S,\beta=2} = 1.21 \pm 0.08$ lies slightly above the theoretical one while for the largest value $\gamma \simeq 31.1$ the experimental value of $W_{S,\beta=2}$ is below the theoretical one.

In summary, we studied experimentally of the two-port scattering matrix $\hat{S}$ elastic enhancement factor $W_{S,\beta}$ for microwave irregular networks simulating quantum graphs with preserved time reversal symmetry (symmetry index $\beta=1$) and with broken time reversal symmetry ($\beta=2$) in the presence of moderate and strong absorption. We show that the experimental results for networks with preserved and broken TRS in the presence of strong absorption are, respectively, in good agreement with the theoretical ones $W_{S,\beta=1} \simeq 2$ and $W_{S,\beta=2} \simeq 1$ predicted by [30–32] within the framework of RMT. Some deviations of the experimental results from the theoretical ones are visible both for moderate and strong absorption, which might suggest that the measurements of the enhancement factor $W_{S,\beta}$ could be influenced by some nonuniversal processes, such as coupling of the measuring system to the microwave graphs.

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