

Emergence of heterogeneity and political organization in information exchange networks

Nicholas Guttenberg and Nigel Goldenfeld

Department of Physics, University of Illinois at Urbana–Champaign, 1110 West Green Street, Urbana, Illinois 61801-3080, USA

(Received 24 November 2009; published 22 April 2010)

We present a simple model of the emergence of the division of labor and the development of a system of resource subsidy from an agent-based model of directed resource production with variable degrees of trust between the agents. The model has three distinct phases corresponding to different forms of societal organization: disconnected (independent agents), homogeneous cooperative (collective state), and inhomogeneous cooperative (collective state with a leader). Our results indicate that such levels of organization arise generically as a collective effect from interacting agent dynamics and may have applications in a variety of systems including social insects and microbial communities.

DOI: [10.1103/PhysRevE.81.046111](https://doi.org/10.1103/PhysRevE.81.046111)

PACS number(s): 05.65.+b, 89.65.-s

I. INTRODUCTION

To understand systems of social and political organization, it is tempting to begin by trying to understand the individuals that form them. This approach quickly runs into a problem—the behavior of individuals is very hard to predict. The behavior of any given individual depends upon a large number of factors: their culture, their experiences to date, their genetics, the events they are currently experiencing, their education, their economic status, and so on. It seems as if understanding the behavior of a group is an impossible goal if predicting the behavior a single person is so difficult. However, models of group behavior through agent-based modeling [1] have been reasonably successful despite this, reproducing generic properties of the dynamics of crowds, mobs, and riots [2–4]; collective opinion formation [5–7]; the structure of social groups [8–15]; and financial markets [16]. When large numbers of people interact, there exists the possibility for the emergence of collective effects which—surprisingly—are insensitive to the details of the elements which comprise them. When this occurs, the interactions between agents overwhelm their individual dynamics; and although their may well be many factors difficult to model for each individual’s behavior, the interactions are frequently easier to specify, characterize, and model.

The purpose of this paper is to understand the factors at work in setting up and maintaining the large scale structure of societies from the point of view of an abstract model. Other models [17] have analyzed the stability and transitions of an established form of social order. In this paper, we will instead seek to explain how social order emerges from an unstructured state due to collective interactions between individual agents. We will consider the optimum networks that could emerge, without treating the dynamics of the network evolution or active networks [18].

The emergence of networks of preferred interactions between agents has been observed in [19,20]. The resultant structure of agents is heterogeneous—a state emerges in which some subset of the population (the leaders) extracts maximal benefit. There is however no explicit flow of information from the leaders to the other agents. We posit that the structure of information exchange in the system is a key element to the form of political organization it possesses. We

would like to differentiate between the agent with the greatest payoff and the agent whose decisions hold maximal weight in influencing the decisions of others. In our model, we observe the development of a division of labor from simple selfish behavior and communication between the members of the system. The role of active information is central to achieving this heterogeneous population structure. This mechanism is not unique as far as ways in which the division of labor might emerge [21,22]. Any system that encourages specialization and provides some way for the proceeds of labor to be redistributed may very well produce division of labor, and there are a number of proposals for how this might come about.

The mechanism studied in the present paper is differentiated from earlier work because the method of redistribution (information exchange) has the additional consequence that networks of behavioral control can emerge from the population. They are not mandatory consequences of the dynamics but only arise under certain conditions specified by parameters in the model. Thus our model does not function as a zero-sum game in which there is exchange of a variety of resources. Information exchange has been studied in various other models. However, in such models it is usually a passive variable, for instance in voting and opinion formation models.

The role of active information—information used to make a decision with either positive or negative consequence—is less well known. In [23], active information played the role of a diffusive field in a spatial prisoner’s dilemma model, and in [24], information was given to a subset of members of a swarm to see how informed decisions would propagate to determine the swarm direction. In these cases, the agents had no way of evaluating the quality of the information they received—whether it had in the past led to a good or bad decision. This dynamics leads to information acting primarily as a homogenizing agent: it determines the average behavior in [23] and directs the average swarm direction in [24]. On the other hand, in our model, each agent determines the optimal degree of trust to place in information received from another. This “trust,” in other contexts such as a political system or organizational structure, could be any way in which control over an agent’s behavior is surrendered to one or more external agents. By giving each agent the ability to tune its trust in the other members of the system, it is pos-

sible for clusters to form in which the members of the cluster have voluntarily given over the reins of their decision making to a leader of their choice.

The optimization that we ask the agents to perform with respect to their communication and resource usage corresponds to the classic problem of exploration versus exploitation [25,26]. This problem is usually considered in light of some centralized, existing structure (e.g., the determination of corporate policy with regards to allocating funding between research and production). In essence we test if the agents in our model will build such a centralized structure through their own individual optimizations or if other types of organization may occur.

This organization, in its simplest form, arises from uniform information exchange between the individuals in the system, resulting in a homogeneous shared information pool. This corresponds to communal decision making by majority vote. In a system in which different agents are better or worse at making decisions, one would expect the emergence of a system of weighting by reputation simply as a tool to optimize the decision-making process. If, however, resources can be allocated toward making better decisions, it becomes possible for a subset of the individuals to specialize in being an information source. At this point, the majority of agents in the system will be following instructions provided by a minority of agents, without a significant information flow in the reverse direction. These two phases—unstructured and structured, respectively—are distinct forms of political organization. Which is achieved depends on the costs and benefits associated with information generation.

A requirement for stability in the structured phase is that the agents which are acting as an information source must either gain from producing information or lose if they fail to produce information as they dedicate their own resources into providing this information. In modern governments, systems of taxation subsidize the decision makers, but the emergence of such structures is difficult without a heterogeneous system already being in place. Our results show that in certain circumstances, the decision-making structure of a population may become heterogeneous even without the inclusion of subsidies or resource exchange, due to a collective effect where the refusal to generate information by the majority of the population forces the agents that are the last to act to take on the decision-making role simply to preserve their own benefit. From this phase, the introduction of a resource subsidy would improve the efficiency of the system and could be done in a continuous manner. A schematic phase diagram that qualitatively exhibits the nature of the phases and transitions between them is illustrated in Fig. 1.

II. MODEL

We propose the following model to capture the dynamics of information exchange. The system consists of a set of agents, each of which chooses one out of a set of O different actions to perform during each iteration of the game. Each agent must choose how to allocate its time between producing resources or producing information about the environmental state (“thinking”). In this model, there is a single

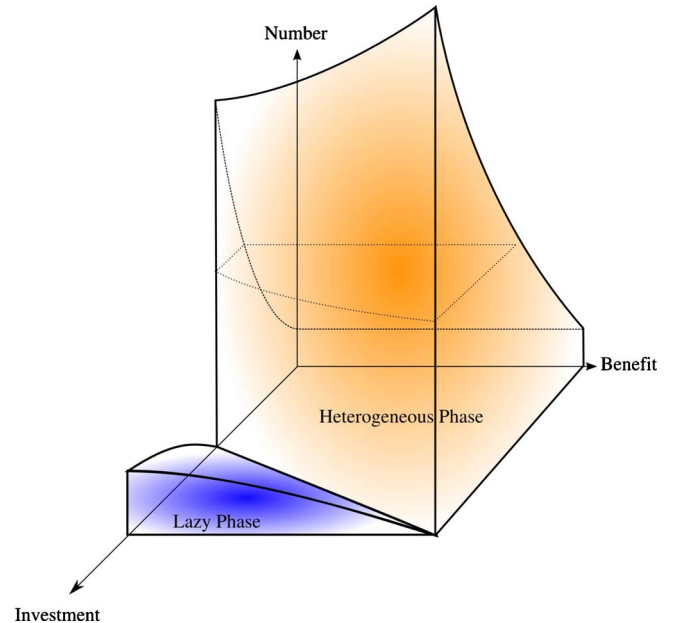


FIG. 1. (Color online) Schematic phase diagram for our model. The Investment axis is the degree that a large initial investment of resources is needed to see an improvement in accuracy: this corresponds to the nonlinearity α in the model. The benefit axis is the total difference in accuracy between random guessing and perfect knowledge, which corresponds to the variable O in our model. In the lazy phase, random guessing is the optimal behavior. In the heterogeneous phase, a subset of agents dedicate their resources to thinking whereas the rest of the agents dedicate their resources to working (division of labor). In the homogeneous phase, all the agents dedicate the same nonzero amount of resources to thinking.

correct action which changes during each iteration of the game. This represents an environmental state that in some sense needs to be responded to in order for resource production to be successful. For simplicity, in this model resource production is either successful or it is not although one could conceive of variations of the model with a best choice, second best choice, etc. If the agent guesses the environmental state correctly, it produces a number of resources proportional to the fraction of its time it allocated to production.

Before the actual resource production occurs, each agent can observe the guesses made by all of the other agents in the system. This is sensible for a small, highly connected community of agents (such as a small business, a set of neighboring farmers, or an insect hive). It also makes sense as a first-order approximation for a large system with some sort of information aggregation, where an agent’s ability to see the choices of other agents comes through some intermediary reporting system such as public opinion polls although depending on the biases inherent in the aggregator there would be higher order effects not considered here.

Each agent makes its choice by combining the information it observes from the other agents in some weighted fashion with its own generated information. This set of weights forms a trust network—the weight of each link measures how much that agent trusts that the choice made by another is the correct one. In principle this network emerges through observation of past performance or some other dynamical

process. We consider the infinite time limit so that each agent has perfect knowledge of each other agent's prior performance and thus their accuracy.

We assume in this first part that each agent has a number of degrees of freedom (how to combine information from other agents and how much time to allocate toward producing resources) which are adjusted in order to maximize its average score. The immediate consequence of this is that we may determine what the optimal trust network should be and thus determine our trust-network order parameters in terms of the distribution of thinking values—the agent with the highest thinking value will have the most trust directed at it, and if the thinking values are distributed homogeneously then trust will also be distributed homogeneously. This treatment neglects dynamical effects and fluctuations. Later, we will analyze the effect of fluctuations and dynamics on the stability of the various phases. We will also examine the consequences of allowing agents to make a centralized decision to exchange resources with each other, in essence subsidizing agents who generate useful information in order to support a locally suboptimal but globally optimal choice.

The base accuracy—that due to the agent's own production of information—is a nonlinear function of the fraction of time dedicated toward information production T . A successful guess then produces one resource per unit time spent on resource generation. This results in a total production of $1 - T$ resources from a successful guess, or zero from a failed guess. If we take the average performance over many such trials, then we can derive a score function that the system may try to optimize.

We must now determine how the accuracy depends on the amount of time spent upon thinking. The choice of functional form must satisfy a number of constraints. The accuracy should monotonically increase with the fraction of time dedicated toward it. Additionally, it is bounded above by 1 and below by $1/O$ (the accuracy of a random guess). In principle one could have agents that perform worse than random although the proper way to model this so as to be general is not obvious. However, if we consider the limit of $O \rightarrow \infty$, then a random choice is the same as a zero accuracy. This means that the effect of agents performing worse than random will be to effectively increase the value of O above the actual number of options present in the system. Given these constraints, we may choose any function of the form $A = [1/O + (1 - 1/O)f(T)]$, where $f(T)$ is a monotonically increasing function that maps the interval $[0, 1]$ to itself. Given an accuracy, one can calculate the average score of an agent operating only on its own information (the score function S):

$$S(T) = (1 - T)A(T) = (1 - T) \left[\frac{1}{O} + \left(1 - \frac{1}{O}\right)f(T) \right]. \quad (1)$$

The key character of our choice of function will be the range of values of the other parameters for which the score function has a local maximum in the interval $[0, 1]$. The point at which this maximum appears or disappears will control part of the resultant phase diagram. If $f(T)$ is monotonically increasing, then the larger O is, the more likely there is for there to be a maximum, and the less concave up f is, the more likely there is to be a maximum. This can be seen by

calculating the concavity of the score function at the location of its extremum in terms of an arbitrary f :

$$\text{sgn}(S'') = \text{sgn}[(1 - T)f'' - 2f']. \quad (2)$$

Consequently, the specific details of $f(T)$ should not strongly influence the results. Its local derivatives in the vicinity of the extremum of S are the only relevant properties. If $f(T)$ is concave up, then specialization is favored. If concave down, then there are diminishing returns and even an infinitesimal amount of time dedicated toward producing information will be beneficial. While we could in principle combine an arbitrary number of concave up and concave down regions in order to create a series of optima in S , it is hard to justify that arbitrary complication.

A simple choice of function that allows us to smoothly vary between concave up and concave down behavior with a single parameter is $A = [1/O + (1 - 1/O)T^\alpha]$, where α is a parameter of the model controlling the disposition of the social problem toward specialization or generalization. If $\alpha > 1$ then the function is concave up, and specialization is favored. With this basis, we can discuss a number of possible system configurations and evaluate their average score for optimal choices of T .

While one could argue that we have put in the possibility of the existence of an optimum value of T by hand, it is an allowed possibility given the most arbitrary choice of $f(T)$. We are then exploring the consequences to the phase diagram of political organization that result from the existence of this optimum choice, rather than saying that we have shown that fundamentally that optimum must exist in real social systems. It is clear that in many cases such as social insects there is such an optimum because in those systems specialization is favored over generalization.

A. Disconnected homogeneous phase

In the case that no agent in the system uses information from any other agent, there is an optimal value of T to maximize an agent's score. The average score in this phase S_{DH}^- is performance independent of N .

$$S_{DH}^-(T) = \frac{1}{O}(1 - T)[1 + (O - 1)T^\alpha]. \quad (3)$$

The optimal value of T satisfies

$$T^{\alpha-1}[\alpha - (\alpha + 1)T] = \frac{1}{O - 1}. \quad (4)$$

If $\alpha = 1$, then this value of T is always less than zero, so $T = 0$ is the optimal choice. At larger values of α a local maximum appears in the curve at a finite value of O and then becomes a global maximum as O increases. The value of O at which the maximum value of the score is equal to the value at $T = 0$ is $O = 1 + \alpha^\alpha / (\alpha - 1)^{\alpha-1}$. In the limit of large α , this becomes $O \approx e\alpha + (1 - e/2)$. So in effect, for values of $\alpha > 1$ (representing a nonlinear reward for dedicating resources to thinking) there is a first-order transition between a guessing phase and a thinking phase, where the more options there are, the more valuable a resource spent on 'thinking' is.

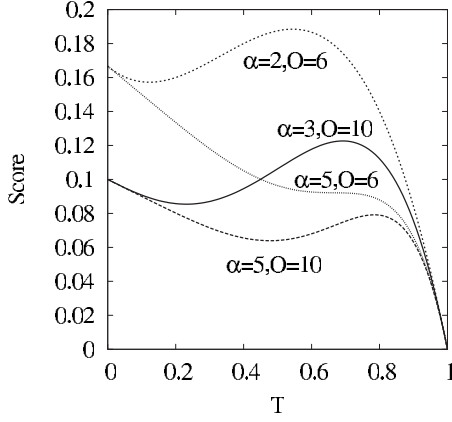


FIG. 2. Score functions in the Isolated Phase as a function of thinking time T , for four values of O and α . In the Isolated Phase, each agent does not receive information from other agents.

The larger α is, the larger O must be for a nonzero thinking phase to be optimal. The score function for various values of α and O is plotted in Fig. 2.

An additional consideration is the effect of fluctuations on this phase. If each agent may only specify their actual thinking value to within some standard deviation, then the resulting average score is lower than if fluctuations had been absent. Near the limits of the range of the thinking variable fluctuations are constrained such that they may not take it outside of the range. For fluctuations of magnitude σ around an optimal value of T , we expect that the average score will change by

$$\Delta S = \sigma^2 \frac{d^2 S}{dT^2} = \sigma^2 \alpha T^{\alpha-2} [(\alpha-1) - (\alpha+1)T] \frac{O-1}{O}. \quad (5)$$

For fixed α , as O becomes large the optimal value of T approaches $\alpha/(\alpha+1)$ and so the decrease in the score approaches

$$\Delta S = -\alpha \left(\frac{\alpha}{\alpha+1} \right)^{\alpha-2} \sigma^2. \quad (6)$$

When the optimal solution is $T=0$, however, the first derivative is nonzero and so fluctuations have a linear effect. The effect of this is that $\Delta S = \sigma \frac{dS}{dT} / \sqrt{\pi}$ assuming Gaussian fluctuations. The slope of the score function around $T=0$ is

$$\frac{dS_{DH}^-}{dT} = -1/O. \quad (7)$$

So we expect $\Delta S = -\sigma/(O\sqrt{\pi})$ to be the leading effect at this point. The consequence of this is that sufficiently large fluctuations will favor the $T=0$ phase.

B. Connected homogeneous phase

If communication between agents is permitted but no resource reallocation takes place, then the resulting accuracy is higher than any of the individual accuracies in the system (so this phase is always favored over the disconnected phase for permitted values of T). The optimal trust network in this

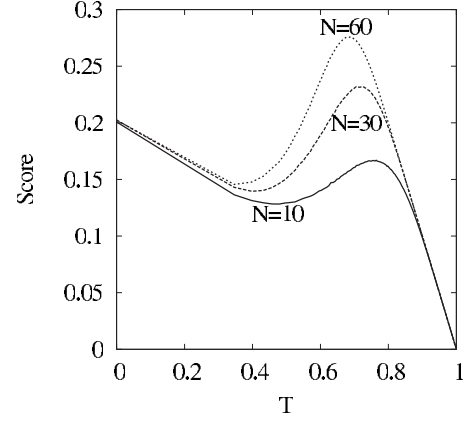


FIG. 3. Score functions in the Homogeneous Phase as a function of thinking time T , for different numbers of agents N , with $O=5$ and $\alpha=5$. In the Homogeneous Phase all agents share information and have the same parameters.

phase must be homogenous, as the agents are indistinguishable from each other, and turns out to be a fully connected graph where all agents equally trust the decisions of each other and themselves.

For $O=2$, the effective accuracy can be solved for in the large N limit. If the initial accuracy is A , then the total number of agents that pick the correct option C_0 is $C_0 = \sum_i \eta_i$, where η_i is either 1 (with chance A) or 0 (with chance $1-A$). In the large N limit, C_0 is described by a Gaussian distribution with mean NA and standard deviation $A(1-A)\sqrt{N}$.

The probability that the system picks the correct option is thus the probability that $C_0 > N/2$. As the range of permitted values is not infinite, care must be taken to compute the correct normalization factor. So

$$A_{eff} = \frac{\int_{N/2}^N \exp\left[-\left(\frac{x-AN}{2NA(1-A)}\right)^2\right]}{\int_0^N \exp\left[-\left(\frac{x-AN}{2NA(1-A)}\right)^2\right]}, \quad (8)$$

which evaluates to

$$A_{eff} = \frac{\operatorname{erf}\left[\frac{(1-A)\sqrt{N}}{\sqrt{2A(1-A)}}\right] - \operatorname{erf}\left[\frac{(1/2-A)\sqrt{N}}{\sqrt{2A(1-A)}}\right]}{\operatorname{erf}\left[\frac{(1-A)\sqrt{N}}{\sqrt{2A(1-A)}}\right] - \operatorname{erf}\left[-\frac{A\sqrt{N}}{\sqrt{2A(1-A)}}\right]}, \quad (9)$$

where $A(T) = (1+T^\alpha)/2$ in this case.

For $O=2$ and $\alpha=2$, benefits from a nonzero value of T do not appear until around $N > 35$. Figure 3 shows the score function for the homogeneous, connected phase compared with the isolated phase.

The effects of fluctuations are less obvious in this case because they must be considered each agent independently, whereas this analysis is done for all agents behaving in the same fashion. In the case of this model, fluctuations may actually increase the effective score, as a fluctuation to higher thinking rate in one agent benefits the guesses of all

other agents. Similarly, a decrease in thinking rate in one agent will not significantly decrease his accuracy but may increase his yield. This is a hint that this particular phase is unstable to an inhomogeneous phase.

A rough estimate would suggest that when adding together the effects of fluctuations on each of the individual agents, the effective size of fluctuations is reduced from σ to $\sigma' = \sigma/\sqrt{N}$. This has the consequence that the connected, homogeneous phase is less sensitive to fluctuations than the disconnected phase.

C. Connected inhomogeneous phase

If the agents become inhomogeneous and divide their labor between thinking and working, then structures in which there is a directional information flow become possible. Given perfect communication and no fluctuations, the optimal configuration will be that of a single agent with high accuracy (T) and $N-1$ agents with minimum accuracy but always picking the action of the “leader” agent. Because the leader is also trying to optimize their own score, the leader does not have the highest possible accuracy ($T=1$) but instead has the optimal accuracy for a completely isolated agent. The trust network in this phase is a directed graph where every agent trusts the leader but no agents use information from each other. The average score for this phase is simply

$$S_{CI} = \frac{(N-1) + (1-T)[1/O + (1-1/O)T^\alpha]}{N}. \quad (10)$$

This phase in static conditions scores far better than the homogeneous phases, but it is very susceptible to fluctuations lowering the score compared to the connected homogeneous phase. The result of this is that neither the pure homogeneous nor heterogeneous phases are realized. In a fully connected population with some form of noise, the system produces a number of leaders L which scales with the population size.

The inhomogeneous phase with a number of leaders can always have a higher population average score than the homogeneous phase, but is not generally stable when the individual scores are examined. Each leader agent can improve their score by decreasing the portion of resources they dedicate to thinking to the optimal value for the disconnected phase. When the disconnected phase optimal value is T is greater than zero, the inhomogeneous phase may still occur. This occurs for small α , large O , and small N . If α is too large, the height of the secondary maximum is decreased below that of the $T=0$ score function maximum and a homogeneous $T=0$ phase occurs (Figs. 4 and 5). If N is sufficiently large, the homogeneous connected phase with non-zero T can outperform a phase consisting of a single “selfish leader.” So there are first-order phase transitions in the space of O , α , and N between three phases: $T=0$, leader, and homogeneous connected (or “communal” phase).

D. Resource subsidy

We have so far shown that for certain values of the parameters, the inhomogeneous leader phase is stable even

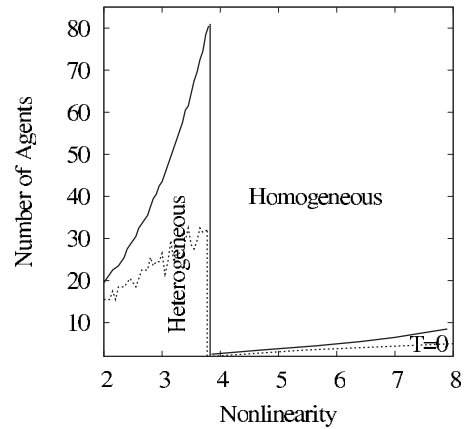


FIG. 4. Phase diagram for $O=10$ in the space of the nonlinearity α and number of agents N . The phase transition from the heterogeneous phase as N increases is due to the communal phase being more efficient than a selfish leader phase. The phase transition as α increases is due to the transition of the isolated phase to a $T=0$ phase. The dotted lines show the phase boundaries when Gaussian fluctuations with a standard deviation of 0.1 are added to the T value of each agent.

without the leaders being subsidized. The system has not maximized its resource production in this phase—rather, the limit on resource production is set by the cost to the leader agent, in that even though it might produce a large amount of resources for others by changing its behavior, doing so would decrease its own resource production.

If we allow agents to exchange resources as well as information, then starting from the connected inhomogeneous phase it is possible to improve or keep constant the scores of all agents. If we have a phase with a single leader agent, then for that agent to dedicate more than the disconnected optimal fraction of resources to thinking, it must be reimbursed by at least the same amount of resources as it loses to increase the resources it spends on thinking. This resource cost may then be absorbed by the remaining $N-1$ agents. In effect, the criterion of selfish optimization becomes one of global opti-

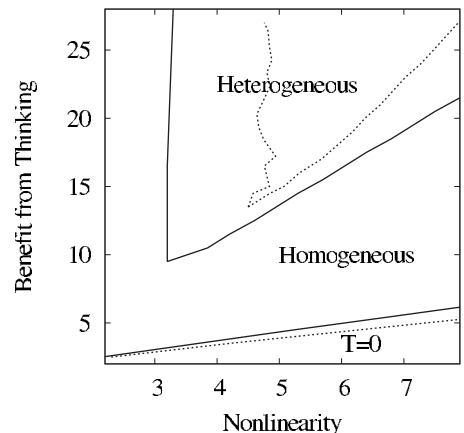


FIG. 5. Phase diagram for $N=50$ in the space of the nonlinearity α and thinking benefit O . The dotted lines show the phase boundaries when Gaussian fluctuations with a standard deviation of 0.1 are added to the T value of each agent.

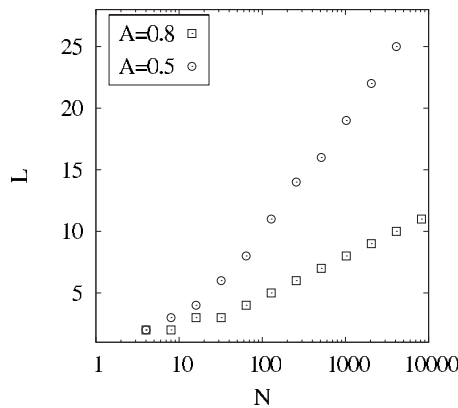


FIG. 6. Optimal number of leaders as a function of total number of agents for a system with $O=5$ and two different fluctuation strengths. In this case, the fluctuations are parameterized by the resultant average accuracy A of a leader with $T=1$.

mization. The globally optimal phase in the absence of fluctuations is that with a single leader agent.

This need not be the case in general, as one may posit the existence of cheaters: agents which do not give resources toward the subsidy but still gain its benefits. A system with multiple types of resource or multiple agendas, such as in [27] might also retain a more detailed structure.

When fluctuations are added, it becomes beneficial to have multiple leaders in order to reduce the impact of fluctuations but retain the benefit of increased efficiency. We use the connected, homogeneous solution for L agents to determine the accuracy of the remaining $N-L$ given a known accuracy of the leaders. For simplicity, we will assume that the leader agents have $T=1$, which the optimal choice converges to as $N \gg L$. For a given level of fluctuations, each leader will have an effective adjusted accuracy. We evaluate the score function numerically as a function of L and find the location of the maximum as a function of N . The results are plotted in Fig. 6. At large N , the optimal number of leaders approaches $L \propto \log(N)$.

For a spatially distributed system or one in which there is not total connectivity, it is expected that such effects will require a larger number of leaders to cover the system extent. For example, in a two-dimensional system in which agents can only communicate within a radius R , a number of leaders proportional to $\sqrt{N}/\pi R^2$ would be expected to ensure total coverage.

III. APPLICATIONS

The abstract model of emergent political systems that we have outlined is capable of providing a framework in which to analyze real social systems, and in this section we briefly indicate some examples. It is important to emphasize that our model is not required to model all situations in which division of labor occurs—a simpler model with only a nonlinear benefit to specialization and some form of exchange of services would be sufficient to enable division of labor to emerge. Such a process would not need to involve information sharing as a core element. On the other hand, our work

shows that the emergence of the leader phase (which corresponds to the occurrence of a division of labor in other pictures) is primarily a consequence of the special property of information when compared to other resources that, once created, it can be duplicated with a much smaller additional cost than the cost to first generate it. This process of information amplification makes the leader phase described here distinct from other scenarios that produce division of labor.

We must also be careful to understand the nature of the relevant optimization being implicitly performed when considering a given system. In human economic and political behavior, one considers that each individual tries to maximize its personal benefit in the context of the greater system. In other systems, such as foraging insects, the net benefit to the colony as a whole is what is likely maximized—this corresponds to the case where resources may be redistributed, which in our model means that the leader phase is always optimal for all parameter values.

Even with these caveats, there are several systems which could potentially be understood in the context of our model: the behavior of social and hierarchical insects compared to asocial insects [22,28,29], the distribution of information in swarms [30], and innovation sharing in unicellular organisms via horizontal gene transfer [31,32]. All of these cases involve some piece of information being discovered by a single individual—a randomly chosen one of a set of similar individuals in the case of swarm behavior (corresponding to the homogeneous phase) or via directed searching by a specialized subset of the population, as is the case in some foraging insects (corresponding to the leader phase). We will now briefly discuss each case.

Different species of insects are socialized to different degrees. On one extreme, there are insects such as the solitary wasps [28], which do not share resources or information. On the other extreme, eusocial insects such as bees, ants, and certain kinds of wasps have highly structured communication channels and vehicles of information discovery. Foragers and scouts use various means to communicate the location of food supplies or nesting sites. The distinction here seems to be that bees and ants reproduce centrally via a queen, and so maximizing their interest corresponds to maximizing the interest of the queen. As a result, resources can be redistributed freely, and so we expect the system to emerge in the leader phase—this is equivalent to a system of resource subsidy as discussed earlier. Trust, here, is embodied in the genetically programmed behaviors of the individuals in following signals sent by other insects.

In the case of honey bees, the various scouts return with information about potential food locations, after which the swarm comes to a unified decision about which site to pursue. The method of decision making seems to be a weighted average [30], similar to what we use to model the decision making of our agents. Each scout has a certain chance of finding the best site within a given distance—even if they spend 100% of their time searching, they have a limited maximum accuracy. This corresponds to the fluctuating case in our model, so, as the swarm size grows, we can predict that the optimal number of scouts should scale logarithmically with the swarm size.

Microbial organisms [33,34] and even multicellular eukaryotes [35–38] have the ability to swap genetic material

and integrate it into their genomes via several pathways mediated often by mobile genetic elements such as viruses and plasmids. There is cellular machinery associated with this process, which can be active or inactive in a given cell. In microbes, the state in which such an organism is receptive to external DNA is called genetic competence. The regulatory network associated with competence has been shown to generate a distribution of cells with differing levels of competence [31]. A small subset of the cells at any given time end up being receptive to this information exchange, whereas the rest remain closed. The competent subset changes with time, so eventually, all cells will at some time be able to accept foreign genetic material. This competence mechanism is then the microbial analog of trust in our model. This dynamic may be analogous to the leader phase in our model. Here, the information amplification takes place when a subset of cells exchange material and either live or die as a result. The surviving exchanges are then passed on to the local population, amplifying the induced information. An analogous process to taxation (e.g., resource subsidy) may occur via a form of symbiosis or biofilm formation, such that nutrient resources are shared.

IV. DISCUSSION

We have shown that a model of communicating agents that divide their time between information generation and information usage has three distinct phases of organization corresponding to structures identifiable in human political systems. The flow of information between agents in the system is critical to this phase structure. If agents can exchange resources in a way that does not permit cheating, then the optimal structure is to have a small number of leaders that scales logarithmically with the system size, and a larger number of workers. Fluctuations in the reliability of agents tend to emphasize the communal phase over the leader phase.

The social networks that emerge in these phases are all simple structures, as we are looking at the optimum networks in a homogeneous environment without considerations of spatial or temporal organization. In essence, this is a zero-temperature model—only the optimal network is considered. It may be possible to devise a social entropy measure for this model that would allow one to talk about statistical distribu-

tions of trust networks around these optima, which might give rise to more the complicated structures that one sees in real social networks [10,11,39,40]. Even something as simple as assigning a cost to the number of trusted links may give rise to more nontrivial network topologies in the homogeneous phase although it would likely also stabilize and extend the leader phase.

The addition of dynamics is another interesting potential area of expansion of this model. There is already an indication that dynamical elements are important to the leader phase as the last agent to optimize their thinking value becomes the one stuck in a local maximum that it cannot escape. The actual occurrence of this is a dynamical process. With a reasonable ansatz for the decision-making process of the agents of the model, the approach to the leader state and its stability could be studied.

The phase transitions predicted by this model are all first order in nature. As such, in a situation in which the agents are approaching equilibrium dynamically, the various phases can coexist over much of the parameter space. This makes sense when one looks at the diversity of actual political systems in existence, on both the local and national scales. The transition to the leader phase from a communal phase takes the form of an inhomogeneous decay in the levels of decision making of the agents in the system, leaving one agent in charge by default. In a dynamical version of this model in which the distribution of agents changes with time, the transition between different leader agents could be studied.

This model has a relatively simple phase structure, as only the thinking value and trust levels are allowed to vary. The addition of spatial considerations, information exchange costs, lying, resource exchange with cheating, or other such factors could vastly increase the diversity of phases exhibited by the model.

ACKNOWLEDGMENTS

We would like to thank Ira Carmen for discussion on political systems and his interest in this work, and Tom Butler for useful discussions on the mathematics of the multiple-option connected phase. We acknowledge partial support from the National Science Foundation through Grant No. NSF-EF-0526747. N.G. was partially supported by the University of Illinois.

-
- [1] E. Bonabeau, *Proc. Natl. Acad. Sci. U.S.A.* **99**, 7280 (2002).
 - [2] M. Granovetter, *Am. J. Sociol.* **83**, 1420 (1978).
 - [3] C. McPhail and R. Wohlstein, *Annu. Rev. Sociol.* **9**, 579 (1983).
 - [4] D. Helbing, I. Farkas, P. Molnar, and T. Vicsek, *Pedestrian and Evacuation Dynamics*, edited by M. Schreckenberg and S. D. Sharma (Springer, Berlin, 2002), pp. 21–58.
 - [5] G. Robins, P. Pattison, and P. Elliott, *Psychometrika* **66**, 161 (2001).
 - [6] S. Huet and G. Deffuant, *J. Artif. Soc. Soc. Simul.* **11**, 10 (2008).
 - [7] C. Nardini, B. Kozma, and A. Barrat, *Phys. Rev. Lett.* **100**, 158701 (2008).
 - [8] W. Zachary, *J. Anthropol. Res.* **33**, 452 (1977).
 - [9] B. Skyrms and R. Pemantle, *Proc. Natl. Acad. Sci. U.S.A.* **97**, 9340 (2000).
 - [10] E. M. Jin, M. Girvan, and M. E. J. Newman, *Phys. Rev. E* **64**, 046132 (2001).
 - [11] M. Newman, D. Watts, and S. Strogatz, *Proc. Natl. Acad. Sci. U.S.A.* **99**, 2566 (2002).
 - [12] N. L. Geard and S. G. Bullock, in *The Eleventh International Conference on the Simulation and Synthesis of Living Systems*

- (*Artificial Life XI*), edited by S. Bullock, J. Noble, R. A. Watson, and M. A. Bedau (MIT Press, Cambridge, MA, 2008), pp. 197–203.
- [13] J. M. Pacheco, A. Traulsen, and M. A. Nowak, *Phys. Rev. Lett.* **97**, 258103 (2006).
- [14] Y. Shoham and M. Tennenholtz, *Artif. Intell.* **94**, 139 (1997).
- [15] G. Boella and L. van der Torre, *Lect. Notes Comput. Sci.* **3913**, 198 (2006).
- [16] A. Kirou, B. Rusczycki, M. Walser, and N. Johnson, *Lect. Notes Comput. Sci.* **5101**, 33 (2008).
- [17] J. Gandhi and A. Przeworski, *Econ. Polit.* **18**, 1 (2006).
- [18] T. Gross and B. Blasius, *J. R. Soc., Interface* **5**, 259 (2008).
- [19] M. G. Zimmermann and V. M. Eguíluz, *Phys. Rev. E* **72**, 056118 (2005).
- [20] V. Eguíluz, M. Zimmermann, C. Cela-Conde, and M. Miguel, *Am. J. Sociol.* **110**, 977 (2005).
- [21] P. Romer, *Am. Econ. Rev.* **77**, 56 (1987).
- [22] G. Robinson, *Annu. Rev. Entomol.* **37**, 637 (1992).
- [23] F. Schweitzer, J. Zimmermann, and H. Mühlenbein, *Physica A* **303**, 189 (2002).
- [24] I. Couzin, J. Krause, N. Franks, and S. Levin, *Nature (London)* **433**, 513 (2005).
- [25] M. Crossan, H. Lane, and R. White, *Acad. Manage. Rev.* **24**, 522 (1999).
- [26] N. Meuleau and P. Bourguine, *Mach. Learn.* **35**, 117 (1999).
- [27] I. Trofimova and N. Mitin, *Nonlinear Dyn. Psychol. Life Sci.* **6**, 351 (2002).
- [28] H. Evans, *Annu. Rev. Entomol.* **11**, 123 (1966).
- [29] M. Richter, *Annu. Rev. Entomol.* **45**, 121 (2000).
- [30] T. Seeley and S. Buhrman, *Behav. Ecol. Sociobiol.* **45**, 19 (1999).
- [31] I. Hecht, E. Ben-Jacob, and H. Levine, *Phys. Rev. E* **76**, 040901(R) (2007).
- [32] K. Vetsigian, C. Woese, and N. Goldenfeld, *Proc. Natl. Acad. Sci. U.S.A.* **103**, 10696 (2006).
- [33] H. Ochman, J. Lawrence, E. Groisman *et al.*, *Nature (London)* **405**, 299 (2000).
- [34] A. Babic, A. B. Lindner, M. Vulic, E. J. Stewart, and M. Radman, *Science* **319**, 1533 (2008).
- [35] J. Hotopp *et al.*, *Science* **317**, 1753 (2007).
- [36] E. Gladyshev, M. Meselson, and I. Arkhipova, *Science* **320**, 1210 (2008).
- [37] P. Keeling and J. Palmer, *Nat. Rev. Genet.* **9**, 605 (2008).
- [38] J. Pace, C. Gilbert, M. Clark, and C. Feschotte, *Proc. Natl. Acad. Sci. U.S.A.* **105**, 17023 (2008).
- [39] M. Newman, *Proc. Natl. Acad. Sci.* **98** 404 (2001).
- [40] M. Girvan and M. Newman, *Proc. Natl. Acad. Sci. U.S.A.* **99**, 7821 (2002).