Nonlinear wave propagation in strongly coupled dusty plasmas

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The nonlinear propagation of low-frequency waves in a strongly coupled dusty plasma medium is studied theoretically in the framework of the phenomenological generalized hydrodynamic (GH) model. A set of simplified model nonlinear equations are derived from the original nonlinear integrodifferential form of the GH model by employing an appropriate physical ansatz. Using standard perturbation techniques characteristic evolution equations for finite small amplitude waves are then obtained in various propagation regimes. The influence of viscoelastic properties arising from dust correlation contributions on the nature of nonlinear solutions is discussed. The modulational stability of dust acoustic waves to parallel perturbation is also examined and it is shown that dust compressibility contributions influenced by the Coulomb coupling effects introduce significant modification in the threshold and range of the instability domain.

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I. INTRODUCTION

The study of nonlinear wave propagation in dusty plasmas has seen an explosive growth in recent years motivated to a large extent by the novelty of the dusty plasma medium as well as by its potential diverse applications in space plasmas, astrophysical phenomena, and laboratory experiments on dusty plasmas [1–4]. The massive and highly charged dust grains invest the plasma with a variety of effects including a host of collective modes and associated instabilities that are not found in the usual two component electron-ion plasmas. For example, in an unmagnetized dusty plasma, there exist new low-frequency electrostatic modes—the dust acoustic (DA) wave (DAW) [5] and the dust ion acoustic (DIA) [6] waves which are distinct from the normal ion-acoustic waves of a two component plasma. For the longitudinal DAW occurring in the regime ($\omega \ll k v_{ti} \ll k v_{te}$), the dust particles provide the inertia and the Boltzmann distributed electrons and ions provide the thermal pressure effects. For the DIA waves, the ions provide the inertia while the pressure of inertialess electrons provides the restoring force for the sustainment of oscillations. A further interesting feature of dusty plasmas is that because of the large charges on the individual dust particles, the dust component of the plasma can easily be in the strongly coupled regime in which the electrostatic energy of dust particle interactions greatly exceeds the dust kinetic energy. The screened Coulomb coupling parameter $\Gamma \simeq (Z_d e)^2/aT_d \exp(-a/\lambda_d) = (Z_d e)^2/aT_d \exp(-\kappa)$ (where $Z_d e$ is dust grain charge, a is the average inter grain distance, $\kappa = a/\lambda_d$ is the measure of magnitude of the dust grain charge screened by plasma, λ_d is Debye length in plasma, and T_d is the dust grain temperature) characterizing this ratio can be of order unity or larger in such a strongly coupled dusty plasma. This strong correlation among the dust particles leads to physical effects such as formation of ordered dust crystalline patterns [7], etc., and many such effects associated with high Γ have now been experimentally [8–10] observed and several theoretical [11,12] and simulation [13,14] studies have been undertaken for their understanding.

Collective oscillations in weakly coupled dusty plasmas, such as of the DA and DIA modes, have been extensively studied both theoretically [5,6] and experimentally [15,16] and their linear properties are now fairly well understood [17]. Their nonlinear behavior have also been well charted in the usual weak amplitude limit by employing standard perturbation procedures [18–21]. For example, the nonlinear evolution of the DA mode [18,19] in certain limits has been shown to be governed by the Korteweg–de Vries (KdV) equation. As is well known the KdV equation describes the nonlinear propagation of small amplitude waves in a weakly dispersive medium and it admits special exact solutions called solitons which are stationary nonlinear structures resulting from a balance between nonlinear wave steepening effects and dispersion induced broadening. These nonlinear wave structures remain undeformed in shape and size even after collisions with other solitons and their velocities are amplitude dependent. The nonlinear evolution characteristics change significantly when dissipative effects are important in the medium and instead of the symmetric soliton structures one encounters nonlinear shock-like structures. The evolution of these structures are no longer governed by the KdV equation but by a KdV-Burger-type equation. Several theoretical studies have considered the KdV-Burger model equation in the context of dusty plasmas while discussing dissipation effects arising from viscosity, ion-dust collisions, Landau damping, etc. [22]. There has also been a recent experimental investigation of shock waves in dusty plasmas [23]. Another interesting area of nonlinear investigation has been that of the modulational instability [24] of finite amplitude DA and DIA waves. In a weakly coupled dusty plasma it has been shown that the evolution equation governing this instability is the nonlinear Schrödinger (NLS) equation.

Linear wave propagation in strongly coupled dusty plasmas has also received a fair amount of attention in recent years. A number of authors have studied the effects of strong correlations of the dust particles on the linear dispersion properties of low-frequency modes by using a variety of theoretical models [25–31]. These linear studies have revealed that strong correlations introduce modifications to the modes

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such as new dispersion corrections, additional damping effects due to enhanced viscous contributions and the possibility of sustaining a transverse shear mode even in the fluid state due to the presence of strong-coupling induced *elastic* effects. Some of these predictions have been experimentally verified including that of the existence of a transverse fluid mode [32]. In contrast to the weak-coupling regime, there have been very few studies of nonlinear wave propagation in strongly coupled dusty plasmas. The difficulty arises mostly in the treatment of the dust dynamics where an analytic treatment of nonlinear effects in the strong-coupling limit is not very straightforward. For example, if one models the dust dynamics using a generalized hydrodynamic (GH) approach [26] one has to deal with a nonlinear nonlocal equation of momentum evolution. In this paper we try to overcome this problem by deriving a simpler set of model nonlinear equations through the use of an appropriate physical ansatz on the integrodifferential form of the GH equation. The reduced equations are in the differential form with an additional time derivative representing the viscoelastic time scale and are amenable to standard perturbation analysis. We use the reductive perturbation technique to derive nonlinear evolution equations in various interesting physical regimes. In the present work we confine our attention to just the DA mode and examine its nonlinear behavior in parametric regimes that are characterized by different features of strong-coupling effects.

The paper is organized as follows. In Sec. II we present the GH model for the strongly coupled dust fluid and derive a simple extension of this model to the nonlinear regime. In Sec. III we describe a complete set of model equations to describe the propagation of dust acoustic waves in strongly coupled plasmas. In Sec. IV we use the standard reductive perturbation technique on this model and show that depending on the relative ordering of the strong-coupling induced effects the nonlinear evolution equation can take the form of either the KdV or the KdVB equations. In Sec. V we discuss the modulational instability of the DA mode in the strongly coupled regime and show that the evolution equation is now the nonlinear Schrödinger equation. We compare our results with past work carried out in weakly coupled plasmas and discuss the differences in the threshold and instability regime introduced by strong-coupling effects. A brief summary of our results and some concluding remarks are made in Sec. VI.

II. MODEL NONLINEAR DUST DYNAMIC EQUATIONS

A variety of approximate methods have been employed in past studies [33–37] for the description of strongly coupled dusty plasma dynamics. Among these various approaches, one of the most convenient and physically appealing model for investigating strong-coupling effects is the so-called GH model [38]. This approach takes account of the strong correlation effects in the dust dynamics through the introduction of model viscoelastic coefficients in the hydrodynamic equations. The phenomenological GH model has been shown to be valid over a wide range of Γ values ($1 \ll \Gamma < \Gamma_c$, where Γ_c is the critical Γ for crystallization) and has been successfully employed in a number of other strongly coupled media, e.g., liquid metals, etc. [39]. It has also been successful in predicting linear dispersive effects and the existence of transverse shear waves in a strongly coupled dusty plasma that is in the liquid state [26]. Some of these theoretical predictions of the GH model have also been experimentally established for dusty plasmas [32,40].

The application of the GH model has however been restricted mainly to the study of linear problems and its use for nonlinear problems poses certain operational and mathematical difficulties such as dealing with a nonlocal nonlinear equation. Restricting ourselves to one dimensional perturbations, the generalized nonlinear momentum equation in the GH model takes the following integrodifferential form:

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x}\right) u_d + \frac{1}{M_d n_d}\frac{\partial P}{\partial x} + \frac{Z_d e E}{M_d} = \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dx' \, \eta_d(x - x', t - t') u_d(x', t'), \quad (1)$$

where u_d denotes the dust fluid velocity and P and E are the pressure and electric field respectively. M_d, n_d, T_d are the dust particle mass, dust density, and dust temperature, respectively. The quantity η_d is identified as the nonlocal viscoelastic function which accounts for memory effects with increasing values of the parameter Γ. We rewrite Eq. (1) as

$$\mathcal{L}(x,t) = \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dx' \, \eta_d(x - x', t - t') u_d(x', t'), \quad (2)$$

where the symbol $\mathcal{L}(x,t)$ stands for the left-hand side (lhs) of Eq. (1). Taking the Fourier transform in space of Eq. (2) we get

$$\bar{\mathcal{L}}(q,t) = \int_{-\infty}^{t} dt' \, \bar{\eta}_d(q, t - t') \bar{u}_d(q, t'), \quad (3)$$

where q is the Fourier transform variable for x and the overbars indicate Fourier transformed quantities. The viscoelastic function can be characterized in terms of a generalized viscosity term and a relaxation time both of which can in general be functions of q. A model expression for this function, which has been proposed and discussed in some details in [41] and has been shown to provide a good description of the collective behavior of strongly coupled systems for both high- and low-frequency limits and for all wavelengths, is

$$\bar{\eta}_d(q,t) = \bar{\eta}_d(q) \frac{\exp\left(\frac{-t}{\tau(q)}\right)}{\tau(q)}, \quad (4)$$

where $\bar{\eta}_d(q)$ and $\tau(q)$ represent the generalized viscosity term and the relaxation time, respectively. It is worth remarking here that the above model presented by Murillo in [41] has been derived by him from a general prescription that seeks to overcome one of the fundamental weaknesses of the hydrodynamic model, namely, that it does not correctly describe dynamical phenomena at moderate frequencies and wave numbers. In his work he has extended the hydrodynamic theory to finite frequencies and wave numbers through the introduction of a nonlocal memory function $\eta(q,t)$ that governs the evolution of the correlation function. The memory function being a dynamical function itself is further assumed to satisfy a nonlocal evolution equation. The Murillo model expression for the memory function, as given in Eq. (4), has been derived by making the Markov approximation for the evolution of the memory function—in other words by assuming that there are no temporal correlations affecting the evolution of the memory function itself. As demonstrated further in his paper, neglect of the non-Markovian contributions is a good approximation as the model results compare quite well with molecular dynamics (MD) and experimental results.

A partial time derivative of Eq. (3) yields

$$\frac{\partial\bar{\mathcal{L}}(q,t)}{\partial t} = \bar{\eta}_d(q,0)\bar{u}_d(q,t) - \int_{-\infty}^{t} dt' \frac{\bar{\eta}_d(q,t-t')\bar{u}_d(q,t')}{\tau(q)}, \tag{5}$$

where we have used Eq. (4) to substitute $\partial\bar{\eta}_d(q)/\partial t = -(\bar{\eta}_d(q)/\tau(q)^2)\exp(-(t-t')/\tau(q))$. Performing the operation $[(3)+\tau(q)(5)]$, we get

$$\left(1+\tau(q)\frac{\partial}{\partial t}\right)\bar{\mathcal{L}}(q,t) = \bar{\eta}_d(q)\bar{u}_d(q,t), \tag{6}$$

where we have used Eq. (4) to substitute for $\bar{\eta}_d(q,0)$. For the model memory function (4) the above evolution equation, Eq. (6) represents the most general form for the nonlinear momentum equation. The nonlinear terms are embedded in $\bar{L}(q,t)$ whose inverse Fourier transform has the convective derivative term. The strong-coupling effects show up as an additional time derivative term (the second term on the lhs) and through the Γ dependence of various transport coefficients such as the generalized viscosity η_d and the compressibility μ_d. To recover the standard Navier-Stokes equation from Eq. (6) we need to take the limit of $\tau(q)\frac{\partial}{\partial t} \ll 1$ which is the weak-coupling limit (when the relaxation time is short compared to the wave period) and take the following form for the viscosity term $\bar{\eta}_d(q)$:

$$\bar{\eta}_d(q) = \frac{\left(\frac{4}{3}\eta+\zeta\right)q^2}{M_d n_{d0}}, \tag{7}$$

where η,ζ are the shear and bulk viscosity coefficients. To study finite τ effects one needs to know the functional form of $\tau(q)$ and solve the resultant operator form of Eq. (6). In general the wave-number dependence of the relaxation time $\tau(q)$ is most pronounced at short wavelengths. For longitudinal oscillations it can be modeled as $\tau(q)=\tau_m e^{-\alpha^2 q^2}$, where α is a constant that needs to be determined from experimental observations or MD simulation data. Equation (6) is in general quite difficult to handle with even this simple model for the relaxation time. However we can easily deduce the physical effects arising from this model term by considering the long-wavelength limit in which one can approximate $\tau(q)$ as $\tau(q)\approx\tau_m(1-\alpha^2 q^2)$. The $\alpha^2 q^2$ terms are clearly seen to result in additional dispersive corrections. For simplicity we will at present ignore these additional dispersive corrections and treat $\tau(q)=\tau_m$ as a constant. The resultant nonlinear differential equation on taking the inverse Fourier transform of Eq. (7) with the model form of $\bar{\eta}_d(q)$ given by Eq. (7) is

$$\left(1+\tau_m\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial}{\partial t}+u_d\frac{\partial}{\partial x}\right)u_d + \frac{1}{M_d n_d}\frac{\partial P}{\partial x} + \frac{Z_d e E}{M_d}\right] = \frac{\eta^*}{M_d n_{d0}}\frac{\partial^2 u_d}{\partial x^2}, \tag{8}$$

with $\eta^* = (\frac{4}{3}\eta+\zeta)$. In general the memory relaxation time τ_m and the various transport coefficients such as η,ζ, etc. in the above equation are functions of Γ and thereby introduce various strong-coupling effects in the collective properties of the system. In addition to the above generalized momentum equation, the complete GH model consists of the continuity equation and the energy equation. For wave propagation studies in a dusty plasma these need to supplemented by the dynamical equations of the electron and ion species and the Maxwell equations that couple the field quantities to dynamical physical perturbations in density, momentum, and temperature. In the next section we will consider an appropriate complete set of equations for investigating the nonlinear propagation of the dust acoustic mode and derive suitable evolution equations in various limits.

III. MODEL EQUATIONS FOR THE DUST ACOUSTIC MODE

For the low-frequency $(\omega \ll kv_{the}, kv_{thi})$ dust acoustic waves we can assume the electrons and ions to behave as light fluids compared to the dust fluid and model them by Boltzmann distributions,

$$n_e = n_{e0}\exp\left(\frac{e\phi}{k_B T_e}\right), \tag{9}$$

$$n_i = n_{i0}\exp\left(-\frac{e\phi}{k_B T_i}\right). \tag{10}$$

The dust dynamics can be modeled by the generalized momentum equation derived in the previous section and the dust continuity equation. We neglect temperature perturbations and hence do not consider the energy equation. This approximation is justified since the basic character of the dust acoustic wave is not affected by the dust temperature which is assumed to be much smaller than the electron and the ion temperatures. Furthermore, the dust dynamics is essentially adiabatic, i.e., $\omega \gg kV_{thd}$, where V_{th_d} is the dust thermal velocity. In this limit we can assume $\omega \gg k^2\bar{\kappa}$, where $\bar{\kappa}$ is the thermal conductivity of the dust component and dust temperature changes and fluctuation effects can be neglected;

$$\left[1+\tau_m\frac{\partial}{\partial t}\right]\left[\left(\frac{\partial}{\partial t}+u_d\frac{\partial}{\partial x}\right)u_d + \frac{1}{M_d n_d}\frac{\partial P}{\partial x} - \frac{Z_d e}{M_d}\frac{\partial\phi}{\partial x}\right] = \frac{\eta^*}{M_d n_{d0}}\frac{\partial^2 u_d}{\partial x^2}, \tag{11}$$

$$\frac{\partial n_d}{\partial t} + u_d\frac{\partial n_d}{\partial x} + n_d\frac{\partial u_d}{\partial x} = 0. \tag{12}$$

The set is completed by the Poisson equation,

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e[n_i - n_e - Z_d n_d]. \tag{13}$$

In the above equations, $n_j (j=e,i,d)$ denotes the number density of different species with n_{j0} being their equilibrium values, $Z_d e$ is the charge on a dust grain, k_B is the Boltzmann constant, and e is the electron charge. $T_e(T_i)$ is the electron- (ion) temperature and ϕ is the electric potential. It is to be noted that the equilibrium number densities are related by the following charge neutrality condition:

$$n_{i0} = Z_d n_{d0} + n_{e0}. \tag{14}$$

It is convenient to reduce the above set of equations to a dimensionless form. For this we introduce the following normalizations, $\bar{\phi}=e\phi/k_B T_i$, $\bar{n}=n_d/n_{d0}$, $\bar{n}_i=n_i/n_{i0}$, $\bar{n}_e=n_e/n_{e0}$, $\bar{t}=\omega_{pd} t$, $\bar{u}=u_d/\lambda_D \omega_{pd}$, $\bar{x}=x/\lambda_D$, $\bar{\tau}_m=\omega_{pd}\tau_m$, and $\bar{\eta}^* = \eta^*/M_d n_{d0} \omega_{pd} \lambda_D^2$. Here the dust plasma frequency is defined as $\omega_{pd}^2 = 4\pi (Z_d e)^2 n_{d0}/M_{d0}$, the plasma Debye length is $\lambda_D^2 = k_B T_i / 4\pi Z_d n_{d0} e^2$, $\lambda_d^2 = k_B T_d / 4\pi Z_d n_{d0} e^2$, $\sigma_i = T_i/T_e$, $\mu_e = n_{e0}/Z_d n_{d0}$, and $\mu_i = n_{i0}/Z_d n_{d0}$. The normalized equations obtained by dropping the overbar notation are then given by

$$\left[1 + \tau_m \frac{\partial}{\partial t}\right]\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)u + \left(\frac{T_d}{T_i}\right)^2 \frac{1}{z_d n_d} \mu_d \gamma_d \frac{\partial n}{\partial x} - \frac{\partial \phi}{\partial x}\right] = \eta^* \frac{\partial^2 u}{\partial x^2}, \tag{15}$$

$$\frac{\partial n}{\partial t} + u\frac{\partial n}{\partial x} + n\frac{\partial u}{\partial x} = 0, \tag{16}$$

$$\frac{\partial^2 \phi}{\partial x^2} = [n + \mu_e \exp(\sigma_i \phi) - \mu_i \exp(-\phi)]. \tag{17}$$

In writing down Eq. (15) we have expressed the pressure term in terms of the compressibility coefficient μ_d, where $\mu_d = \frac{1}{T_{d0}} \frac{\partial P}{\partial n}\big|_T$ and γ_d is the adiabatic index. A model dependence of μ_d on Γ is given by [26]

$$\mu_d = 1 + \frac{u(\Gamma)}{3} + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma}, \tag{18}$$

where the function $u(\Gamma)$, the so-called excess energy, can be written as

$$u(\Gamma) = a(\kappa)\Gamma + b(\kappa)\Gamma^{1/3} + c(\kappa) + d(\kappa)\Gamma^{-1/3} \tag{19}$$

for a Yukawa fluid [25].

The coefficients up to order κ^4 are given by

$$a(\kappa) = \frac{\kappa}{2} - 0.899 - 0.103\kappa^2 + 0.003\kappa^4,$$

$$b(\kappa) = 0.565 - 0.026\kappa^2 - 0.003\kappa^4,$$

$$c(\kappa) = -0.207 - 0.086\kappa^2 + 0.018\kappa^4,$$

$$d(\kappa) = -0.031 + 0.042\kappa^2 - 0.008\kappa^4. \tag{20}$$

The set of Eqs. (11)–(13) for $\kappa=0$ (the unscreened case) have been previously analyzed in the linear limit to study the propagation characteristics of various low-frequency waves in a strongly coupled dusty plasma [26]. For the DA mode the linear dispersion relation is given by

$$1 + \frac{1}{k^2\lambda_p^2} - \frac{1}{\omega^2 - \gamma_d \mu_d k^2 \lambda_\delta^2 + i\omega k^2 \frac{\eta^*}{1-\omega\tau_m}} = 0, \tag{21}$$

where $\lambda_p^2 = \lambda_e^2 + \lambda_i^2$ and λ_δ corresponds to dust screening length. Finite κ effects were considered by Rosenberg *et al.* [25]. Here, we analyze the general case with a finite value of κ.

It is clear from the above that strong-coupling modifications enter through the contributions of terms due to η^*, μ_d and τ_m. Solution of the linear dispersion relation shows that there are additional dispersive corrections to the DA arising through the μ_d and η^* contributions as well as damping due to the viscosity term η^* which supplements the usual damping due to dust neutral collisions. The relaxation time τ_m provides a characteristic time scale to distinguish between two classes of modes, those with $\omega\tau_m \ll 1$ called the hydrodynamic modes and modes with $\omega\tau_m \gg 1$—the so-called kinetic modes. The dispersion curves of the DA mode for various values of Γ show that in the long-wavelength limit the frequency ω typically varies linearly with k with the dispersive corrections being proportional to the cubic power of k. For a linear dispersive behavior of this kind it is well known that the weakly nonlinear behavior of such waves are likely to be governed by a KdV-type equation. The question of interest then is to investigate the effect of the additional dispersion terms (due to μ_d and η^*) and also the dissipation term (due to η^*) on the nonlinear behavior of the DAW. We carry out such an analysis in the next two sections. Another interesting strong-coupling feature of the linear dispersion relation is that in the range of $1 < \Gamma < 10$, μ_d can change sign causing the dispersion curve to turn over at a certain value of k with the group velocity going to zero and then to negative values. In this region it would be interesting to examine the strong-coupling induced effect brought about by compressibility changes on the modulational instability of the DA mode. We carry out such an analysis in Sec. V. We should like to mention here that further modifications to the linear dispersion relation of the DAW in the strongly coupled regime can occur if additional effects such as dust charge fluctuations, wave-number dependence of the relaxation time, etc. are taken into account, as has been discussed by Xie *et al.* in a series of papers [29–31]. Such modifications would then also influence the nonlinear propagation properties of the DAW by contributing to the dispersive or dissipative coefficients of the propagation equations. However since the primary objective in our present work is to present a nonlinear extension of the GH model, we have adopted a minimal description for the linear system and neglected these additional contributions. They can be the subject of interesting future explorations of the nonlinear GH model.

IV. NONLINEAR EVOLUTION OF DAW IN THE LONG-WAVELENGTH REGIME

We begin our analysis of the nonlinear set of Eqs. (15)–(17) by employing the standard reductive perturbative method of expanding the variables n, u, and ϕ in terms of a small parameter ϵ as

$$n = 1 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \epsilon^3 n^{(3)} + \cdots, \tag{22}$$

$$u = \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \epsilon^3 u^{(3)} + \cdots, \tag{23}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \tag{24}$$

and introduce the stretched variables ξ and τ such that $\xi = \epsilon^{1/2}(x - Mt)$ and $\tau = \epsilon^{3/2} t$, where M is the wave frame speed normalized to the dust acoustic speed. The operators $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial t}$, and $\partial^2/\partial x^2$ then take the following forms:

$$\frac{\partial}{\partial x} = \epsilon^{1/2}\frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial x^2} = \epsilon \frac{\partial^2}{\partial \xi^2}, \tag{25}$$

$$\frac{\partial}{\partial t} = - M\epsilon^{1/2}\frac{\partial}{\partial \xi} + \epsilon^{3/2}\frac{\partial}{\partial \tau}. \tag{26}$$

We will use Eqs. (22)–(24) in Eqs. (15)–(17) and make use of Eqs. (25) and (26) to expand them to various orders in ϵ. It is also important to fix the ordering of the various transport coefficients, e.g., μ_d and η^* as well as τ_m before proceeding on the expansion. We consider below two different limits based on such relative orderings of τ_m, the viscous contribution due to η^* and the dispersive terms.

A. $\omega\tau_m \gg 1, \eta^* \sim O(1)$

In this limit we note that in the expansion of the nonlinear momentum Eq. (15) the viscous contributions enter at the lowest order and have a dispersive nature. We will neglect the pressure contributions since their dispersive contributions are small for the long-wavelength limit. Thus to the first two lowest orders in ϵ, from each of the Eqs. (15)–(17), we obtain the following set of equations:

$$[\eta^* - M^2\tau_m]\frac{\partial^2 u^{(1)}}{\partial \xi^2} = M\tau_m \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \tag{27}$$

$$\begin{aligned}[\eta^* - M^2\tau_m]\frac{\partial^2 u^{(2)}}{\partial \xi^2} = {} & M\tau_m \frac{\partial^2 \phi^{(2)}}{\partial \xi^2} - M\tau_m u^{(1)}\frac{\partial^2 u^{(1)}}{\partial \xi^2} \\ & - M\tau_m\left(\frac{\partial u^{(1)}}{\partial \xi}\right)^2 - \tau_m \frac{\partial}{\partial \tau}\left(\frac{\partial \phi^{(1)}}{\partial \xi}\right) \\ & - 2M\tau_m \frac{\partial}{\partial \tau}\left(\frac{\partial u^{(1)}}{\partial \xi}\right),\end{aligned} \tag{28}$$

$$\frac{\partial}{\partial \xi}[u^{(1)} - Mn^{(1)}] = 0, \tag{29}$$

$$\frac{\partial n^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi}[u^{(2)} - Mn^{(2)}] + \frac{\partial}{\partial \xi}[n^{(1)}u^{(1)}] = 0, \tag{30}$$

$$[\mu_e\sigma_i + \mu_i]\phi^{(1)} + n^{(1)} = 0, \tag{31}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = n^{(2)} + (\mu_e\sigma_i + \mu_i)\phi^{(2)} + \frac{1}{2}[\sigma_i^2\mu_e - \mu_i](\phi^{(1)})^2. \tag{32}$$

Eliminating the variables $n^{(2)}$, $u^{(2)}$, and $\phi^{(2)}$ in terms of $n^{(1)}$ by making use of Eqs. (27), (29), and (31), we find that it leads to the following equation:

$$A\frac{\partial^4 n^{(1)}}{\partial \xi^4} + B\frac{\partial^2}{\partial \xi^2}(n^{(1)})^2 + C\frac{\partial}{\partial \tau}\left(\frac{\partial n^{(1)}}{\partial \xi}\right) = 0. \tag{33}$$

Integrating once in ξ, we find that Eq. (33) leads to the following evolution equation which is of the KdV form:

$$A\frac{\partial^3 n^{(1)}}{\partial \xi^3} + \frac{B}{2}n^{(1)}\frac{\partial n^{(1)}}{\partial \xi} + C\frac{\partial n^{(1)}}{\partial \tau} = 0, \tag{34}$$

where the coefficients A, B, and C of the Kdv Eq. (34) are given as follows: where $[M^2 = 1/\tau_m(\eta^* + \tau_m/(\mu_e\sigma_i + \mu_i))]$

$$A = \frac{M}{(\mu_e\sigma_i + \mu_i)}, \tag{35}$$

$$B = M + \frac{M}{2}\frac{(\sigma_i^2\mu_e - \mu_i)}{(\mu_e\sigma_i + \mu_i)^2} - \frac{M^3}{2}\left(\frac{\tau_m}{\eta^* - M^2\tau_m}\right), \tag{36}$$

$$C = \left[1 - \frac{2M^2\tau_m}{(\eta^* - M^2\tau_m)} + \frac{\tau_m}{(\eta^* - \tau_m M^2)(\mu_e\sigma_i + \mu_i)}\right]. \tag{37}$$

It should be pointed out that in the usual weak-coupling limit, assuming an ordering of $\eta^* \sim 1$, leads to a KdV-Burger-type equation and consequently the breakup of KdV solitons into nonlinear shocklike solutions [22,23]. In the present case the additional time derivative arising from the strong-coupling contribution changes the nature of the nonlinear wave propagation permitting soliton formation even when $\eta^* \sim 1$. The viscous term provides an additional dispersive contribution to the propagation. Physically the existence of a large value of τ_m (such that $\omega\tau_m \gg 1$) implies strong memory effects in the medium and therefore the predominance of elastic effects. The limit $\omega\tau_m \gg 1$, often referred to as the "kinetic regime," is particularly relevant for the very strong-coupling regime (e.g., close to crystallization) where both τ_m and η^* can be quite large. We next consider the limit of $\omega\tau_m \ll 1$, the so-called "hydrodynamic regime" and examine the effect of viscosity on nonlinear DA wave propagation.

B. $\omega\tau_m \ll 1, \eta^* \sim O(\epsilon^{1/2})$

In this limit, substitution of expansions Eqs. (22)–(24) in Eqs. (15)–(17) with the pressure term neglected in Eq. (15), leads to the following equations in the first two lowest order in ϵ for the momentum equation while those for the continuity and Poisson's equation remain the same as Eqs. (29)–(32):

$$-M\frac{\partial u^{(1)}}{\partial \xi} = \frac{\partial \phi^{(1)}}{\partial \xi}, \tag{38}$$

$$\frac{\partial u^{(1)}}{\partial \tau} - M\frac{\partial u^{(2)}}{\partial \xi} + u^{(1)}\frac{\partial u^{(1)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi} = \eta^{*}\frac{\partial^2 u^{(1)}}{\partial \xi^2}. \quad (39)$$

We eliminate the variables $u^{(2)}$, $\phi^{(2)}$, and $n^{(2)}$ from Eqs. (38), (39), and (29)–(32), similar to the previous section in terms of $n^{(1)}$ leading to the following equation: where $M^2 = 1/(\mu_e\sigma_i + mu_i)$

$$D\frac{\partial n^{(1)}}{\partial \tau} + En^{(1)}\frac{\partial n^{(1)}}{\partial \xi} + F\frac{\partial^3 n^{(1)}}{\partial \xi^3} + G\frac{\partial^2 n^{(1)}}{\partial \xi^2} = 0. \quad (40)$$

It is seen that Eq. (40) is the Kdv-Burger's equation. The coefficients of the above equation are given by

$$D = 2M, \quad (41)$$

$$E = 3M^2 + \frac{(\sigma_i^2\mu_e - \mu_i)}{(\mu_e\sigma_i + \mu_i)^2}, \quad (42)$$

$$F = \frac{1}{(\mu_e\sigma_i + \mu_i)^2}, \quad (43)$$

$$G = -\eta^{*}M. \quad (44)$$

In this limit the viscosity contribution is truly dissipative and one can expect shocklike nonlinear solutions of the dust acoustic mode. In a strongly coupled plasma viscosity is a sensitive function of the coupling parameter Γ as well as the screening parameter κ. For weakly coupled plasmas viscosity is a diminishing function of Γ. As past theoretical and MD simulations studies have shown η^* decreases as a function of Γ [42,43] and displays a broad minimum in the region of $1<\Gamma<10$. The present limit of nonlinear DAW propagation is relevant in this regime. As Γ increases further η^* begins to rise again. Very close to the crystallization point (near $\Gamma \approx \Gamma_c$) there is a sharp and very large rise in η^* which is attributed to a change in the momentum transfer mechanism in the presence of short-range order [42]. In this regime the KdV equation derived in the previous section becomes the appropriate nonlinear propagation equation. The characteristics of the nonlinear propagation of the dust acoustic wave can thus provide a means of qualitatively marking the various regimes. It should be pointed out however that the actual viscosity as opposed to the normalized viscosity can have a different scaling with Γ depending on the amount of screening in the system and can thus make a quantitative deduction of viscosity from the wave propagation characteristics somewhat difficult.

V. NONLINEAR EVOLUTION OF DAW IN THE SHORT-WAVELENGTH REGIME

It is well known that the DAW can suffer a modulational instability at short wavelengths such that a slow parallel modulation of a finite amplitude monochromatic plane wave can grow and in some limits lead to the formation of an envelope soliton pulse. The evolution equation then takes the form of a nonlinear Schrödinger equation. A number of recent studies have examined the modulational instability of the DAW in the weak-coupling regime and have considered the effect of oblique propagation. In this section we will investigate the modulational instability of the DAW in the strong-coupling limit in the hydrodynamic regime ($\omega\tau_m \ll 1$). In particular we are interested in the regime where the linear dispersion curve of DAW turns over and μ_d contributions are important. The turnover can also be influenced by dispersive contributions from viscosity as seen from the linear dispersion relation (21). We restrict our calculation to cases where the viscosity contribution is small compared to the compressibility contribution. As discussed in [26] simulation and model calculations typically show $\eta^* \sim 0.08$ for $\Gamma \sim 10$, whereas $\mu_d \sim 1$ in that region. We will also therefore neglect viscous dissipation effects by a suitable choice of ordering. We consider Eqs. (15)–(17) but now include the next order term ϕ^2 in the Poisson's equation, which in dimensionless variables is given by

$$\left[\frac{\partial^2}{\partial x^2} - \alpha\right]\phi - \beta\phi^2 = [n-1]. \quad (45)$$

The parameters α and β are given by

$$\alpha = (\mu_e\sigma_i + \mu_i), \quad (46)$$

$$\beta = \frac{1}{2}(\mu_e\sigma_i^2 - \mu_i). \quad (47)$$

We now introduce the slow space and time scales for the envelope evolution modulating a fast carrier wave through the stretched variables defined by

$$\xi = \epsilon_1(x - \lambda t), \quad (48)$$

$$\tau = \epsilon_1^2 t. \quad (49)$$

where ϵ_1 is a small parameter and λ is the group velocity of the wave along it's propagation direction. It should be noted that the smallness parameter ϵ_1 employed in this section is distinct from ϵ used in the previous section and the two expansions are valid in different parameter regimes. In Sec. IV the typical wave number $k \sim \epsilon^{1/2}$ whereas in the present section the typical wave number is given by $k \sim \epsilon_1$. Since we are considering the short-wavelength regime in this section $k_1 > k$ and hence $\epsilon_1 > \epsilon$. We then expand the variables n, v, ϕ in terms of the expansion parameter as

$$n(x,t) = 1 + \sum_{n=1}^{\infty} \epsilon_1^n \sum_{l=-\infty}^{\infty} n_l^n(\xi,\tau)\exp[il(kx - \omega t)], \quad (50)$$

$$u(x,t) = \sum_{n=1}^{\infty} \epsilon_1^n \sum_{l=-\infty}^{\infty} u_l^n(\xi,\tau)\exp[il(kx - \omega t)], \quad (51)$$

$$\phi(x,t) = \sum_{n=1}^{\infty} \epsilon_1^n \sum_{l=-\infty}^{\infty} \phi_l^n(\xi,\tau)\exp[il(kx - \omega t)]. \quad (52)$$

Using the stretched variables [Eqs. (48) and (49)] the operators $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial t}$, and $\frac{\partial^2}{\partial x^2}$ are given by

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} + \epsilon_1 \frac{\partial}{\partial \xi}, \tag{53}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \epsilon_1 \lambda \frac{\partial}{\partial \xi} + \epsilon_1^2 \frac{\partial}{\partial \tau}, \tag{54}$$

$$\frac{\partial}{\partial x^2} = \frac{\partial}{\partial x^2} + 2\epsilon_1 \frac{\partial^2}{\partial x\, \partial \xi} + \epsilon_1^2 \frac{\partial^2}{\partial \xi^2}. \tag{55}$$

Using expressions (48)–(55) we expand Eqs. (15), (16), and (45) to obtain the nth-order reduced equations as follows:

$$-il\omega u_l^{(n)} - \lambda \frac{\partial u_l^{(n-1)}}{\partial \xi} + \frac{\partial u_l^{(n-2)}}{\partial \tau} + \mathcal{A}\mu_d\gamma_d\left(ilkn_l^{(n)} + \frac{\partial n_l^{(n-1)}}{\partial \xi}\right) - \left(ilk\phi_l^{(n)} + \frac{\partial \phi_l^{(n-1)}}{\partial \xi}\right) + \frac{1}{2}\sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty} ilku_{l-l'}^{(n-n')}u_{l'}^{(n')} + \frac{1}{2}\frac{\partial}{\partial \xi}\sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty} u_{l-l'}^{(n-n'-1)}u_{l'}^{(n')} = 0. \tag{56}$$

Here assumption is taken as $[(T_d/T_i)^2(1/z_d n_d) \simeq (T_d/T_i)^2(1/z_d n_{d0}) = \mathcal{A}]$,

$$-il\omega n_l^{(n)} + ilku_l^{(n)} - \lambda \frac{\partial n_l^{(n-1)}}{\partial \xi} + \frac{\partial u_l^{(n-1)}}{\partial \xi} + \frac{\partial n_l^{(n-2)}}{\partial \tau} + \sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty} ilk(u_{l-l'}^{(n-n')}n_{l'}^{(n')}) + \sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty}\frac{\partial}{\partial \xi}(u_{(l-l')}^{(n-n'-1)}n_{l'}^{(n')}) = 0, \tag{57}$$

$$-(\alpha + l^2k^2)\phi_l^{(n)} - n_l^{(n)} + 2ilk\frac{\partial \phi_l^{(n-1)}}{\partial \xi} + \frac{\partial^2 \phi_l^{(n-2)}}{\partial \xi^2} - \sum_{n'=1}^{\infty}\sum_{l'=-\infty}^{\infty} \beta\phi_{l-l'}^{(n-n')}\phi_{l'}^{(n')} = 0. \tag{58}$$

For first order $(n, l=1)$ it is seen that we obtain the following equations:

$$-i\omega n_1^{(1)} + iku_1^{(1)} = 0, \tag{59}$$

$$-(\alpha + k^2)\phi_1^{(1)} - n_1^{(1)} = 0, \tag{60}$$

$$-i\omega u_1^{(1)} + ik\mathcal{A}\mu_d\gamma_d n_1^{(1)} - ik\phi_1^{(1)} = 0. \tag{61}$$

The linear dispersion relation for the DA mode in the strong-coupling limit using Eqs. (59)–(61) is then given as

$$\omega^2 = k^2\left[\mathcal{A}\mu_d\gamma_d + \frac{1}{\alpha + k^2}\right]. \tag{62}$$

It is seen that this dispersion relation agrees with the one obtained by Kaw and Sen [26] in the strong-coupling limit and the standard dispersion relation of DA wave in the weak-coupling limit in an unmagnetized dusty plasma as derived Rao, Shukla, and Yu [5] if we revert back to the dimensional form. From Eqs. (59)–(61) we can express the first-order quantities $u_1^{(1)}$ and $\phi_1^{(1)}$ in terms of $n_1^{(1)}$. In the second order $(n=2, l=1)$, we obtain corrections to the first-order quantities in terms of a function, $n_1^{(2)}(\xi,\tau)$ and $\partial n_1^{(1)}(\xi,\tau)/\partial \xi$. It is seen that in this order we obtain the compatibility condition, i.e.,

$$\lambda = \frac{\partial \omega}{\partial k} = \frac{k}{\omega}\left[\mathcal{A}\mu_d\gamma_d + \frac{\alpha}{(\alpha + k^2)^2}\right]. \tag{63}$$

To next order, i.e., $(n=2, l=2)$ the expansion equations are given by

$$-2i\omega n_2^{(2)} + 2iku_2^{(2)} + 2iku_1^{(1)}n_1^{(1)} = 0, \tag{64}$$

$$-2i\omega u_2^{(2)} + 2ik(\mathcal{A}\mu_d\gamma_d n_2^{(2)} - \phi_2^{(2)}) + iku_1^{(1)}u_1^{(1)} = 0, \tag{65}$$

$$-(\alpha + 4k^2)\phi_2^{(2)} - n_2^{(2)} - \beta\phi_1^{(1)}\phi_1^{(1)} = 0. \tag{66}$$

Using the Eqs. (64)–(66) we can obtain the second-harmonic mode of the carrier wave in terms of the nonlinear self interaction term $n_1^{(1)}n_1^{(1)}$. These second-harmonic quantities are given by

$$n_2^{(2)} = \left[\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)}\right]n_1^{(1)}n_1^{(1)}, \tag{67}$$

$$u_2^{(2)} = \frac{\omega}{k}\left[\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)} - 1\right]n_1^{(1)}n_1^{(1)}, \tag{68}$$

$$\phi_2^{(2)} = \left\{-\frac{3}{2}\frac{\omega^2}{k^2} - \left(\frac{\omega^2}{k^2} - \mathcal{A}\mu_d\gamma_d\right)\left[\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)}\right]\right\}n_1^{(1)}n_1^{(1)}. \tag{69}$$

The zeroth harmonic mode due to the self interaction is determined from the $l=0$ components of the $n=3$ order equations and are found to be given by

$$n_0^{(2)} = \frac{1}{(\lambda^2 - 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left[\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right]|n_1^{(1)}|^2, \tag{70}$$

$$u_0^{(2)} = \left\{\frac{\lambda}{(\lambda^2 - 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left[\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right] - \frac{2\omega}{k}\right\}|n_1^{(1)}|^2, \tag{71}$$

$$\phi_0^{(2)} = \frac{1}{\alpha}\left\{\frac{1}{(\lambda^2 - 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left[\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right] + \frac{2\beta}{(k^2 + \alpha)^2}\right\}|n_1^{(1)}|^2. \tag{72}$$

From above evaluations and $l=1$ component of the third-order equations, one obtains the following nonlinear Schrödinger equation for the first-order amplitude of the perturbed plasma density $n_1^{(1)} \equiv A$:

$$i\frac{\partial A}{\partial \tau} + P\frac{\partial^2 A}{\partial \xi^2} + Q|A|^2 A = 0. \tag{73}$$

The coefficients P and Q of the above equation are given by

$$P = \left(\frac{1}{2}\right)\left[\frac{\alpha(\alpha - 3k^2)}{\omega(\alpha + k^2)^3} + \frac{\mathcal{A}\mu_d\gamma_d}{\omega} - \frac{\lambda^2}{\omega}\right]. \tag{74}$$

The coefficient Q is the sum of the contribution from the zeroth harmonic and the second-harmonic terms for the nonlinear term $|A|^2 A$ and is given by

$$Q = \frac{k}{2}I - \frac{2k\beta}{\omega(k^2 + \alpha)}J - \frac{k^2}{2\omega}L, \tag{75}$$

where the terms I, J, and L are given by

$$I = \left[\left(\lambda + \frac{\omega}{k}\right)\frac{1}{(\lambda^2 - 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left(\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right) - \frac{3\omega}{k} + \frac{2\omega}{k}\left(\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)}\right)\right], \tag{76}$$

$$J = \frac{1}{\alpha(k^2 + \alpha)}\left[\frac{1}{(\lambda^2 - 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left(\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right) + \frac{2\beta}{(k^2 + \alpha)^2}\right] + \frac{1}{k^2 + \alpha}\left[\frac{3}{2}\frac{\omega^2}{k^2} + \left(\frac{\omega^2}{k^2} - \mathcal{A}\mu_d\gamma_d\right)\left(\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)}\right)\right], \tag{77}$$

$$L = \frac{\omega}{k}\left[\left\{\frac{\lambda}{(\lambda^2 + 1/\alpha - \mathcal{A}\mu_d\gamma_d)}\left[\frac{2\lambda\omega}{k} + \frac{2\beta}{\alpha(k^2 + \alpha)^2} + \frac{\omega^2}{k^2}\right] - \frac{2\omega}{k}\right\} + \frac{\omega}{k}\left[\frac{1}{2k^2}\frac{\omega^2}{k^2}(\alpha + k^2)(\alpha + 4k^2) + \frac{\beta}{3k^2(\alpha + k^2)} - 1\right]\right]. \tag{78}$$

In the above coefficients, it is seen that $P \equiv (1/2)\,\partial\lambda/\partial k \equiv (1/2)\partial^2\omega/\partial k^2$ and Q contains the contribution from the zeroth- and second-harmonic carrier wave. When μ_d is set to zero, it is possible to show, with a little bit of straightforward but tedious algebra, that the above expressions for P and Q reduce to those derived by Amin *et al.* [24] for the limit of parallel propagating waves (i.e., for $\theta=0$ in their expressions). Our main interest is to examine the influence of the dust thermal contribution arising through μ_d which can become important in the strongly coupled regime. As is well known, the criterion for modulational stability of the envelope wave, described by the nonlinear Schrödinger Eq. (73), is given by the sign of the product PQ. The wave is modulationally stable if $PQ < 0$. For $PQ > 0$ the wave can become modulationally unstable particularly to long-wavelength perturbations with a threshold wave number [24] given as $K_{cr} = \sqrt{2|P/Q|}|a_0|$, where a_0 is the perturbation amplitude. We have investigated the influence of the μ_d term on the modulational stability question by numerically determining the marginal curve $PQ=0$ over a range of values of μ_d and k. In evaluating the expressions for P and Q we have used typical values of $\alpha=0.8, \beta=-3.6, \mathcal{A}=1.0$, γ_d is taken to be 5/3 and ω is determined from the linear dispersion relation for a given value of k. As discussed in [26] the negative contribution of μ_d can give rise to an unphysical instability at large values of k—an artifact of the model chosen to evaluate μ_d. For each value of Γ (and corresponding μ_d), we therefore restrict ourselves to a range of k for which the linear dispersion relation gives a real value of ω. Our results are consolidated in the form of a stability diagram shown in Fig. 1, where the solid curve represents the loci of all points in μ_d and carrier wave-number k space where $PQ=0$. The region below the curve is the modulationally unstable region. We see that μ_d has a significant influence on the modulational stability domain. With decreasing values of μ_d the unstable

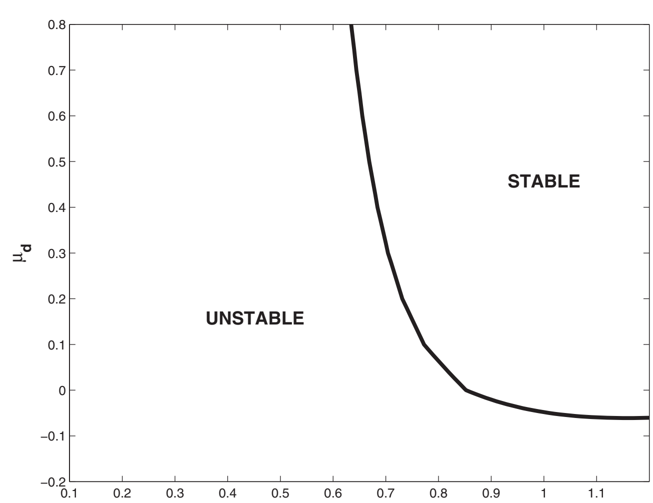

FIG. 1. Modulationally stable and unstable regions of the dust acoustic wave in the parameter space of μ_d and carrier wave number k. Decreasing μ_d implies increasing Γ and therefore stronger coupling effects.

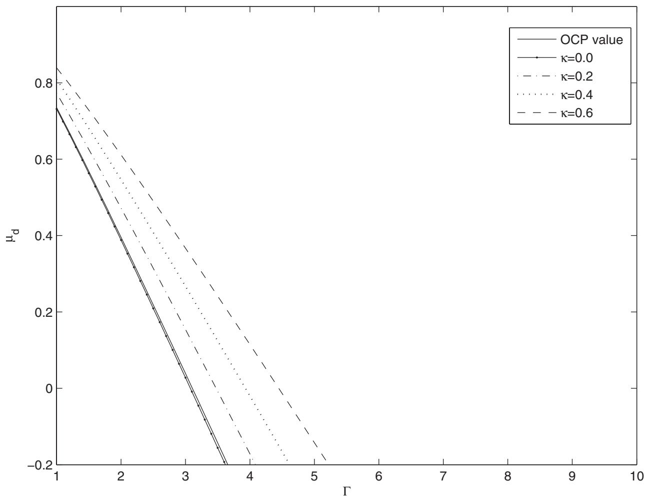

FIG. 2. Comparison of plots of variation in μ_d with Γ for the OCP model and the Yukawa model with different values of the screening parameter κ.

region expands and this trend continues into the region where μ_d becomes negative. The decrease in μ_d corresponds to increasing values of Γ as is evident from expressions (18)–(20). These expressions also show a dependence on the screening parameter κ. However the changes in μ_d due to κ for a given value of Γ are not very significant as illustrated in Fig. 2. Here we have plotted the variation in μ_d with Γ for the one component plasma (OCP) model [26] as well as for various values of κ in the Yukawa model. Thus as the dust component gets increasingly correlated due to strong-coupling effects, the nonlinear dust acoustic wave can become modulationally unstable over a wider range of carrier wave numbers. The nonlinear saturation of such waves can give rise to envelope soliton structures. As is well known [20], in the stable region a finite amplitude DAW cannot form an envelope soliton but could form a shocklike structure or a dark soliton which are both stationary solutions of the NLSE for $PQ<0$. Since μ_d can be controlled by changing Γ (e.g., by cooling or heating the dust component) and to some extent by changing κ (through a change in the surrounding plasma properties) an interesting experiment to do would be to look for changes in the characteristics of a stationary nonlinear DAW structure as a function of Γ and κ.

VI. SUMMARY AND CONCLUSIONS

We have investigated the nonlinear propagation of small amplitude dust acoustic waves in a strongly coupled dusty plasma medium. The dust medium is modeled by the phenomenological generalized hydrodynamic equations. A set of simplified nonlinear equations are derived from the original nonlinear integrodifferential form of the GH model by employing an appropriate physical ansatz. Thereafter characteristic evolution equations for finite small amplitude dust acoustic waves are obtained in various propagation regimes with the help of standard perturbation techniques. Our primary motivation has been to study the influence of dust correlations as manifested in the various transport coefficients such as η^*, μ_d, etc., on the nature of the nonlinear solutions. For this we have considered some of the familiar nonlinear solutions that have been earlier studied in the context of weakly coupled dusty plasmas. A well-known result from these earlier studies is that in the presence of any weak dissipation in the system (e.g., finite viscosity) small amplitude DAW propagation is governed by the KdVB equation which gives shocklike solutions. In a strongly coupled system this scenario can get altered due to the introduction of memory effects. If the randomizing time due to collisions is longer than the memory time τ_m then the viscous term displays elastic properties and provides additional dispersion to the wave. Consequently the nonlinear propagation equation in this case turns out to be the KdV equation which can support soliton solutions. Shocklike solutions will only exist in the regime where the memory time is short and the viscous term plays a dissipative role. Thus strong-coupling effects as represented by the relaxation time period τ_m introduce a threshold for the transition from KdV solitonlike solutions to KdVB shocklike solutions. A similar result is found for the modulational stability of dust acoustic waves to parallel perturbation where it is found that dust compressibility (μ_d) contributions under the influence of Coulomb coupling effects can introduce significant modifications in the threshold and range of the instability domain. Specifically we find that with decreasing values of μ_d the unstable region expands and this trend continues into the region where μ_d becomes negative. The decrease in μ_d corresponds to increasing values of Γ which implies that as the dust component gets increasingly correlated due to strong-coupling effects, the nonlinear dust acoustic wave can become modulationally unstable over a wider range of carrier wave numbers. The unstable and stable regions admit different nonlinear saturated solutions—the unstable region can give rise to envelope soliton structures, whereas in the modulationaly stable region one can only sustain shocklike structures or dark solitons. For a given carrier wave number the transition from one region to the other can be effected by inducing a change in the transport coefficient through a change in Γ. Our calculations in this paper have been restricted to the DAW but one can expect similar effects to occur for other low-frequency dusty plasma modes as well. Since the Coulomb coupling parameter in a dusty plasma can be easily controlled, e.g., by heating or cooling the dust component, it would be interesting to look for these "signatures" in the propagation characteristics of finite amplitude low-frequency waves in controlled laboratory experiments. Such investigations would provide an understanding of nonlinear wave propagation in strongly coupled systems. We would like to mention here that the strong-coupling induced changes in the viscosity can be masked by effects arising from a high level of dust neutral collisions. A quantitative measure of the competition between these two effects had been obtained for the linear propagation characteristics of dust acoustic waves in a strongly coupled dusty plasma [26]. An experimental confirmation of such a distinction was also subsequently demonstrated [40] by measuring the linear wave dispersion characteristics in different neutral pressure regimes. It is suggested that similar experiments on propagation of nonlinear waves be carried out with the minimal neutral density necessary to be in the strong-coupling regime but with low collisional drag effects in order to detect the effects of strong coupling on viscosity through changes in their propagation characteristics. Such experiments could provide useful insights into the scaling of viscosity with Γ and complement other recently proposed methods of direct measurement of viscosity using particle imaging methods [44].

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