Characterization of intermittency in renewal processes: Application to earthquakes

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We construct a one-dimensional piecewise linear intermittent map from the interevent time distribution for a given renewal process. Then, we characterize intermittency by the asymptotic behavior near the indifferent fixed point in the piecewise linear intermittent map. Thus, we provide a framework to understand a unified characterization of intermittency and also present the Lyapunov exponent for renewal processes. This method is applied to the occurrence of earthquakes using the Japan Meteorological Agency and the National Earthquake Information Center catalog. By analyzing the return map of interevent times, we find that interevent times are not independent and identically distributed random variables but that the conditional probability distribution functions in the tail obey the Weibull distribution.

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I. INTRODUCTION

Recently, intermittent phenomena, characterized by a power law of the laminar state, have attracted interest in nonequilibrium statistical physics as well as biological and atomic physics. Examples of intermittent phenomena are the fluorescence of quantum dots [1] and nanocrystals [2], and ion channel gating [3]. Earthquakes can be recognized as an intermittent phenomenon. Actually, a $1/f^{\delta}$ spectrum is observed in the *P*-wave and *S*-wave velocities in function of depth [4]. Also, the residence time distribution of the laminar state, in which the number of earthquakes per unit time is lower than a threshold, obeys a power law [5], and the intermittency for the occurrence of earthquakes in Irpinia, Italy, has been quantified using the correlation codimension [6]. Moreover, intermittency appears in the stick-slip model of earthquakes [7]. In such non-Poisson processes, a long-time tail and an aging have been clearly observed [8,9]. Nonhyperbolic dynamical systems, which have at least one indifferent fixed point, also show intermittent behavior such as a long-time tail and nonstationarity [10,11]. By applying renewal theory to symbolic dynamics generated by the coarse graining of an orbit, it was also shown that nonhyperbolic dynamical systems generate a $1/f$ spectrum [10,12].

An outstanding problem in intermittent phenomena is the incompleteness of the usual statistical descriptors such as mean and variance because of the divergence of the mean interevent time [13]. It is remarkable that infinite measure preserving dynamical systems, which are closely related to intermittent phenomena, exhibit intrinsic nonstationarity [14]. Even when the mean interevent time is finite, the long tail of distribution makes it difficult to characterize the interevent time by the mean. Usually, intermittency is characterized by the exponent of a power law. However, this characterization does not include a long tail distribution heavier or not heavier than a power law. In the present paper, we characterize intermittency to estimate the difficulty of forecasting rare events. Moreover, the degree of activity of events is studied from the viewpoint of a nonhyperbolic dynamical system.

Renewal processes have been drawing attention not only in mathematics but also in physics and are useful to analyze intermittent phenomena [15]. In fact, intermittent phenomena can be rewritten as renewal processes by focusing attention on the residence time distribution of laminar state. In renewal processes, it is assumed that the interevent times between renewals are independent and identically distributed (iid) random variables. For a given renewal process we construct a one-dimensional dynamical system to characterize intermittency in renewal processes. Then, we develop a concept of intermittency based on dynamical systems. One of our results is a unified characterization of intermittency in renewal processes, where we classify renewal processes into five different regimes according to difficulty to forecast the next event: (i) *nonstationary essential singular intermittency*, (ii) *nonstationary very strong intermittency*, (iii) *stationary strong intermittency*, (iv) *stationary weak intermittency*, and (v) *stationary nonintermittent chaos*.

The paper is organized as follows. In Sec. II, we construct one-dimensional piecewise linear intermittent maps from renewal processes with any distribution, and then intermittency is characterized by the asymptotic behavior near the indifferent fixed point. Using the constructed map, we can calculate the Lyapunov exponent of a renewal process. In Sec. III, we study the occurrence of earthquakes. To apply our method to the occurrence of earthquakes in Japan and in the world, we verify whether the occurrence of earthquakes is a renewal process or not. As a result, we find that the occurrence of earthquakes is not a renewal process but that the tail of conditional probability distribution function of interevent times obeys the *Weibull distribution*. Then, our method is applied to the occurrence of earthquakes using the conditional probability distribution function. Conclusions are given at the end of the paper.

II. UNDERLYING DYNAMICAL SYSTEMS IN RENEWAL PROCESSES

A. Construction of one-dimensional maps

To characterize intermittency, we construct one-dimensional maps from discrete time renewal processes. Let

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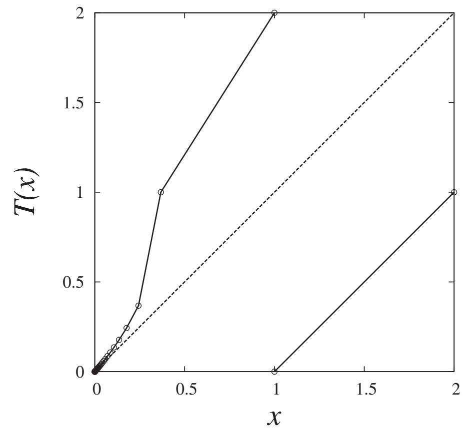

FIG. 1. Piecewise linear map $T(x)$ for the Weibull distribution $\{\mathcal{F}(m)=\exp[-(m/5)^{0.5}]\}$ and $c=2$. Circles indicate the end points of the straight-line segments.

$f(m)$ be the probability distribution function of a random variable m $(m=1,2,\ldots)$, and $F(m)=\Sigma_{k=1}^{m}f(k)$ and $\mathcal{F}(m)=1-F(m)$, where $F(0)=0$ and $\mathcal{F}(0)=1$. Then, we can obtain the following relationship:

$$f(m)=\mathcal{F}(m-1)-\mathcal{F}(m). \tag{1}$$

Using this relation, we can construct a one-dimensional map on $[0,c]$ in which the residence times in $[0,1]$ are iid random variables with probability $f(m)$. Concretely, the map is given by a piecewise linear map $T:[0,c]\to[0,c]$ defined by

$$x_{n+1}=T(x_n)=\begin{cases}\dfrac{a_{k-1}-a_k}{a_k-a_{k+1}}(x_n-a_k)+a_{k-1}, & x_n\in[a_{k+1},a_k),\\[2mm] \dfrac{x_n-1}{c-1}, & x_n\in[1,c],\end{cases} \tag{2}$$

where a sequence a_k is given by $a_k=\mathcal{F}(k)$ and $\mathcal{F}(-1)=c$. Actually, a point in the interval $[a_m,a_{m-1}]$ is mapped into the interval $[1,c]$ by m iterations, and then the probability of the residence time m in the interval $[0,1]$ is given by a_m-a_{m-1} (see Fig. 1).

Next, we focus on the constructed map near the fixed point $(x=0)$. The derivative of the map is given by

$$T'(x)|_{x\in[a_n,a_{n-1})}=\frac{a_{n-1}-a_n}{a_n-a_{n+1}}=\frac{f(n)}{f(n+1)}. \tag{3}$$

Considering the maps near the fixed point, i.e., $x\cong a_n=\mathcal{F}(n)$, $a_{n-1}-a_n\cong 0$, we obtain the asymptotic form,

$$T'(x)\sim\frac{f(\mathcal{F}^{-1}(x))}{f(\mathcal{F}^{-1}(x)+1)} \quad \text{as } x\to 0. \tag{4}$$

In particular, the asymptotic behavior for a power-law distribution $(\mathcal{F}(m)\sim m^{-\beta})$ is given by

$$T'(x)-1\propto x^{1/\beta} \quad \text{as } x\to 0. \tag{5}$$

This map is the same as the Pomeau-Manneville map [16], which is a typical example of intermittent maps, and the piecewise linear version is also well studied as an intermittent map [17,18]. The asymptotic behavior is given by

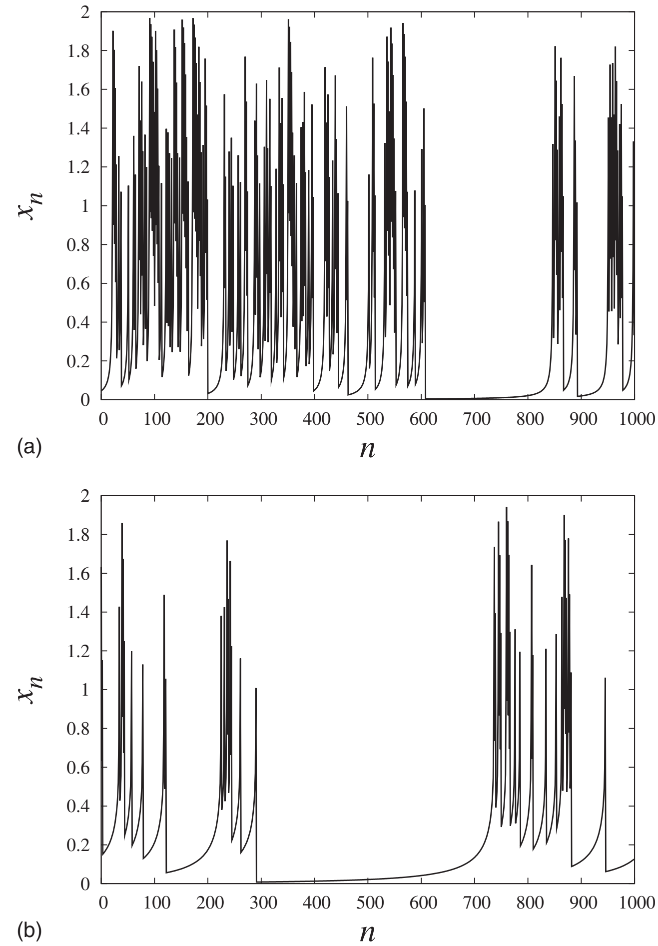

FIG. 2. Time series of piecewise linear intermittent maps. The map in the upper figure is constructed using the power-law distribution $(\mathcal{F}(m)=m^{-1})$. The map in the lower figure is constructed using the Weibull distribution $\{\mathcal{F}(m)=\exp[-(m/5)^{0.35}]\}$.

$$T'(x)\sim e^{1/\tau} \quad \text{as } x\to 0 \quad \text{for } \mathcal{F}(m)\sim e^{-m/\tau}, \tag{6}$$

and

$$T'(x)-1\propto(-\log x)^{(a-1)/a} \quad \text{as } x\to 0 \tag{7}$$

for the Weibull distribution $(\mathcal{F}(m)\sim e^{-(m/\tau)^a})$ $(a\neq 1)$. In the case of the log-Weibull distribution $(\mathcal{F}(m)\sim e^{-(\log m/\tau)^b})$, the asymptotic behavior is given by

$$T'(x)-1\propto\exp[-\tau(-\log x)^{1/b}] \quad \text{as } x\to 0. \tag{8}$$

We refer to Eqs. (7) and (8) as the Weibull map and the log-Weibull map, respectively. Note that the Weibull map with exponent $a<1$ and the log-Weibull map have the indifferent fixed point $[T'(0)=1]$.

B. Characterization of intermittency

In intermittent chaos, an orbit stagnates near indifferent fixed points for an extremely long time (laminar state), and then irregular chaotic motion occurs (see Fig. 2). The residence time distribution of the laminar state is determined by the structure of a map near the indifferent fixed points. Here, we characterize intermittency from the asymptotic behavior of the derivative at the indifferent fixed point ($x=0$),

$$T'(x) - 1 \propto L(x)x^{\alpha} \quad \text{as} \quad x \to 0, \tag{9}$$

where $T(x)$ is the one-dimensional map constructed by a renewal process and the function $L(x)$ is slowly varying at 0 [19]. The degree of intermittency is classified into five types: (i) $\alpha=\infty$, i.e., $T'(x)-1 \propto e^{-x^{-\alpha'}}$ as $x\to 0$ ($\alpha'>0$): *nonstationary essential singular intermittency*; (ii) $1 \leq \alpha < \infty$: *nonstationary very strong intermittency*; (iii) $0<\alpha<1$: *stationary strong intermittency*; (iv) $\alpha=0$: *stationary weak intermittency*; and (v) $T'(x)-1>0$ as $x\to 0$: *stationary nonintermittent chaos*.

The intensity of intermittency can be quantified by the exponent α. The larger α is, the more difficult to forecast the next event becomes. This is because slightly different reinjections near the fixed point make the residence time in $[0,1]$ completely different in the case of large α. With regard to type (iv), to be more precise,

$$T'(x) - 1 = O(\exp[-(-\log x)^{\gamma}]) \quad \text{as} \quad x \to 0, \tag{10}$$

we can quantify the intensity of intermittency by the exponent γ, and for

$$T'(x) - 1 = O\left(\frac{1}{(-\log x)^{\eta}}\right) \quad \text{as} \quad x \to 0, \tag{11}$$

the intensity of intermittency is determined by the exponent η. Exponents α, γ, and η represent the degree of the difficulty to forecast events in renewal processes because the sensitive dependence of the interevent time on reinjection points is determined by these exponents.

Note that the renewal function does not increase linearly with time for long times; that is, the occurrence of renewals is not stationary, when the mean of the interevent times is not finite [20]. Accordingly, the occurrence of renewals becomes nonstationary in the case of $\gamma \geq 1$. For $\gamma>1$, intermittency is classified into the nonstationary essential singular intermittent regime.

C. Lyapunov exponent

We can calculate the Lyapunov exponent in renewal processes because we can construct one-dimensional maps from renewal processes. In general, the Lyapunov exponent λ is defined by

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \ln|T'(x_k)|, \tag{12}$$

where $T(x)$ is a one-dimensional map constructed from a renewal process. The slope of the piecewise linear map on $[a_1, a_0)$ and $[1,c]$ is given by $(c-1)/(1-a_1)$ and $1/(c-1)$, respectively. Therefore, the Lyapunov exponent does not depend on c because $\ln(c-1)/(1-a_1)+\ln 1/(c-1)=-\ln(1-a_1)$. However, the Lyapunov exponent strongly depends upon the unit of time in renewal processes. Physical meaning of the Lyapunov exponent is the degree of activity of events. In other words, a large Lyapunov exponent implies high activity of an underlying dynamical system.

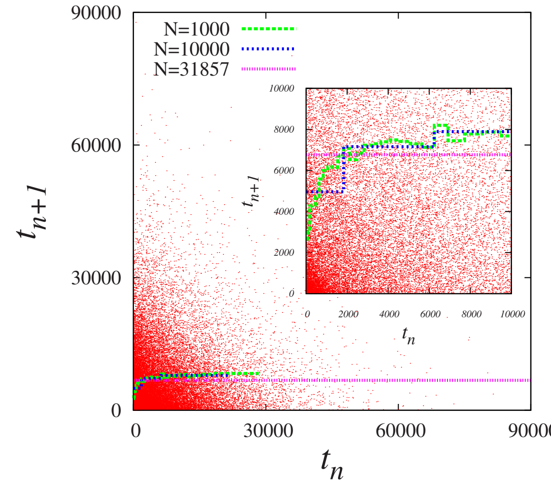

FIG. 3. (Color online) Return map of interevent times in Japan (M_c=3.0). The dotted lines are averages of the interevent time, where averages are taken in three different bins (N=1000, 10 000, and 31 857). Inset figure is a blowup of return map (M_c=3.0).

III. APPLICATION TO THE OCCURRENCE OF EARTHQUAKES

We apply this method to the occurrence of earthquakes using the Japan Meteorological Agency (JMA) catalog [21] and the National Earthquake Information Center (NEIC) catalog [22]. The area of JMA catalog is enclosed within 25–50°N latitude and 125–150°E longitude with magnitude $M \geq 2$ and that of NEIC is the world. We are interested in using the interevent time distribution for a tail part to construct a one-dimensional piecewise linear map. Similar to previous studies [23–25], we consider earthquakes with magnitude above a certain threshold M_c. In other words, we study the interevent time statistics in which magnitude is greater than M_c. Here, we use the earthquake data for JMA and NEIC from January 1, 2001 to October 31, 2007 and from January 1, 1973 to December 31, 2007, respectively.

To verify the hypothesis that the occurrence of earthquakes is a renewal process, we use the return map (t_n, t_{n+1}) of interevent times, where t_n is the nth interevent time. Then, to analyze the dependence of the distribution of interevent times $F(t_{n+1})$ on the previous one t_n in detail, we sorted data in order

$$\{t_1, \ldots, t_M\} \to \{t'_1, \ldots, t'_M\},$$

where $t'_i \leq t'_{i+1}$ for $i=1,2,\ldots,M-1$, and we made ordered data sets, $\mathcal{T}_{1,N}, \ldots, \mathcal{T}_{L,N}$, from them:

$$\mathcal{T}_{1,N} = \{\underbrace{t'_1, \ldots, t'_N}_{N\ times}\}, \ldots, \mathcal{T}_{L,N} = \{\underbrace{t'_{NL+1}, \ldots, t'_{(L+1)N}}_{N\ times}\},$$

where $N(L+1)<M$ and $N(L+2)>M$. To avoid statistical error, the number of data N in each ordered data set must be more than 10^3. As shown in Fig. 3, the return map does not yield a smooth curve. This is unlike the experiment of the

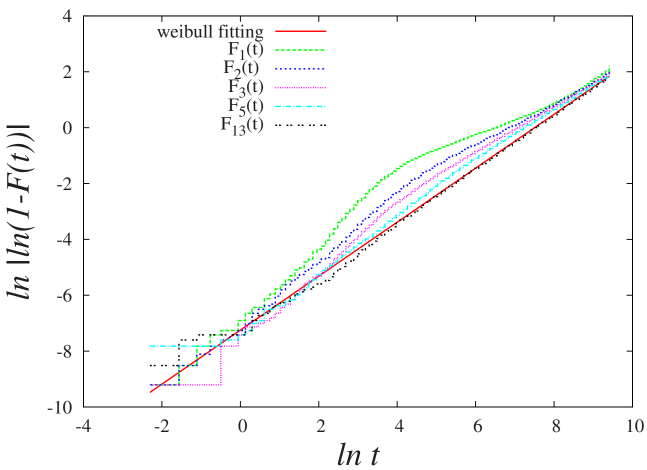

FIG. 4. (Color online) Weibull plot of the conditional probability distributions of interevent times of earthquakes in Japan (M_c =2.0 and $N=10^4$).

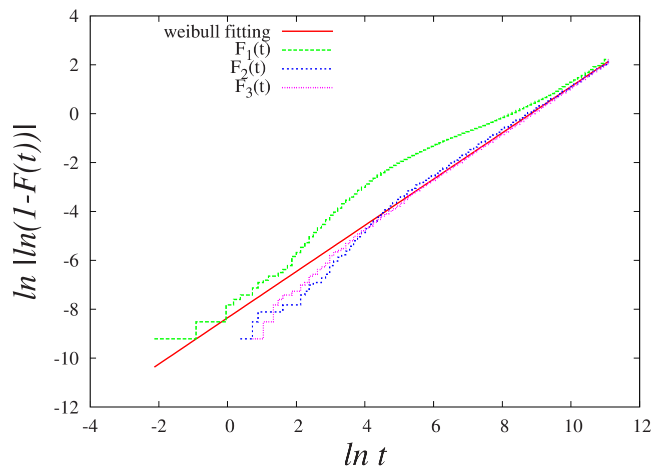

FIG. 5. (Color online) Weibull plot of the conditional probability distributions of interevent times of earthquakes in Japan (M_c =3.0 and $N=10^4$).

dripping water faucet, which did yield a smooth curve [26]. Further, interevent times appear to be random. However, the conditional average of an interevent time will be small if the previous interevent time is small. In Fig. 3, the conditional averages are taken in each ordered data set: the conditional average on ordered data set $\mathcal{T}_{i,N}$ is given by

$$\mu_{i,N} = \sum_{t_k \in \mathcal{T}_{i,N}} \frac{t_{k+1}}{N}.$$

Thus, an interevent time depends clearly on the one preceding it. However, the average of an interevent time is constant when the previous one is relatively large, which suggests that the interevent times approach iid random variables in this case.

We consider the interevent times conditioned on that the previous interevent time is in the subset $\mathcal{T}_{i,N}$, i.e., $\mathcal{T}'_{i,N} = \{t_{k+1} : t_k \in \mathcal{T}_{i,N}, k=1,\ldots,M\}$. Analyzing the conditional probability distribution functions, $F_1(t),\ldots,F_L(t)$, of interevent times for $\mathcal{T}'_{1,N},\ldots,\mathcal{T}'_{L,N}$, we find that they change systematically. Moreover, we find that the conditional probability distribution function $F_L(t)$ obeys the *Weibull distribution* in almost the entire region, where the Weibull distribution $F(t)$ is defined by

$$F(t) = 1 - \exp[-(t/\tau)^a] \tag{13}$$

(see Figs. 4 and 5). It is remarkable that all conditional probability distribution functions $F_1(t),\ldots,F_L(t)$ obey the *Weibull distribution* in the tail region and that $F_i(t)$ fits a Weibull distribution asymptotically as i increases. Therefore, the intermittency of the occurrence of earthquakes is *stationary weak intermittency* because the conditional probability distribution functions $F_1(t),\ldots,F_L(t)$ of interevent times in the tail are invariant and described by the Weibull distribution.

Analyzing the distribution for different M_c, we find that the conditional probability distribution function $F_L(t)$ in almost the entire region and the tails of all conditional probability distribution functions obey the *Weibull distribution*. Piecewise linear maps for $F_1(t)$ and $F_3(t)$ are demonstrated in Fig. 6, where we coarse grain the interevent time t by considering $t \in [2000(n-1), 2000n]$ as the discrete interevent time n. The asymptotic behavior of piecewise linear maps near the origin is the same because the conditional probability distributions in the tail region are invariant. As a result, we classify the intermittency of the occurrence of earthquakes as *stationary weak intermittency*. The Weibull exponent a, the root mean-square value, the intensity of intermittency η, and the Lyapunov exponent λ are summarized in Table I. In this table, the Weibull exponent is obtained by the Weibull fitting for the tail part of the conditional probability distribution $F_L(t)$, where bins of interevent times are set to be equal in logarithmic scale. The root mean-square value is defined as

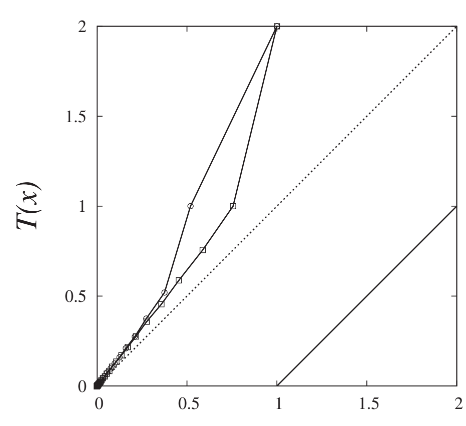

FIG. 6. Piecewise linear map for interevent times of the occurrence of earthquakes in Japan ($c=2$). Circles and squares indicate the end points of the piecewise linear map for $F_1(t)$ and $F_3(t)$, respectively (M_c=3.0 and $N=10^4$).

TABLE I. Weibull exponent a, the root mean square (rms), the intensity of intermittency η, and the Lyapunov exponent λ in Japan.

M_c	a	rms	η	λ	Number of earthquakes
2.0	0.966 ± 0.001	0.013	0.035 ± 0.001	4.6×10^{-3}	130243
2.5	0.961 ± 0.001	0.009	0.040 ± 0.001	2.7×10^{-3}	67912
3.0	0.946 ± 0.001	0.020	0.057 ± 0.001	1.4×10^{-3}	31857

$$\text{rms} = \sqrt{\frac{\sum_{i=k}^{n'+k} [F_i - F(l_i)]^2}{n' - 2}}, \tag{14}$$

where F_i is an experimental value of the conditional probability distribution, l_i is an interevent time of ith bin and where we carry out the Weibull fitting on from kth to $n' + k$th bin. In the calculation of the Lyapunov exponent, one-dimensional maps are constructed from the renewal processes with the Weibull distribution obtained by the above Weibull fitting. The time unit is 1 s in the calculation of the Lyapunov exponent.

We perform the same analysis for the occurrence of earthquakes in the world to investigate the universality of the results in Japan. Surprisingly, the conditional probability distribution changes systematically, $F_i(t)$ fits a *Weibull distribution* asymptotically as i increases, and the conditional probability distributions for the tail part obey the *Weibull distribution* in the same manner as in the preceding analysis (see Figs. 7 and 8). The Weibull exponents are 0.93 and 0.92 for M_c=4.5 and 5.0, respectively, indicating the view that the intermittency of the occurrence of earthquakes in the world is classified into *stationary weak intermittency*.

IV. CONCLUSIONS

Intermittency of renewal processes is studied by constructing one-dimensional piecewise linear maps from renewal processes. As a result, characterization of intermittent phenomena is extended to extremely heavy tail and stretched exponential relaxation phenomena.

Analyzing the occurrence of earthquakes, we found that the occurrence of earthquakes is not a renewal process, which is in agreement with [27]. However, interevent times approach iid. random variables when the previous interevent time is relatively large, and the conditional probability distribution functions in the tail are described by the *Weibull distribution*. Corral showed that interevent times of earthquakes clearly depend on the preceding interevent times [28]. Compared with this study, our study gives the detailed analysis of the conditional probability distribution.

It has been known that the distribution of interevent times of earthquakes shows stretched exponential decay [23,24], which is consistent with our result in that it has the same classification of intermittency. Exponential distribution or the gamma distribution can be well fitted because the exponent of the Weibull distribution is close to one. We note that the Weibull plot is the most useful to determine the decay of the distribution function. Focusing on the distribution function in the tail part, we conclude that the Weibull exponent is smaller than one. Therefore we characterize the intermittency of the occurrence of earthquakes as *stationary weak intermittency*. The intensity of intermittency of earthquakes depends on the threshold M_c. In particular, the intensity of intermittency increases monotonically with the threshold of magnitude, indicating the view that it is difficult to forecast the occurrence of large earthquakes. We also estimate the Lyapunov exponent of the renewal process with the Weibull distribution $F(t)$, which measures the activity of earthquakes

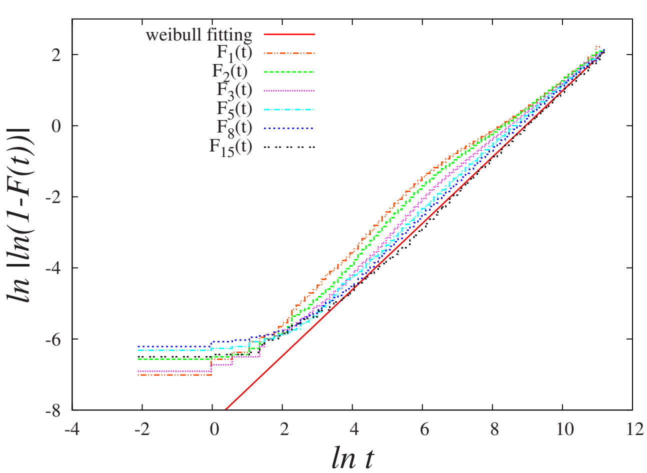

FIG. 7. (Color online) Weibull plot of the conditional probability distributions of interevent times of earthquakes in the world (M_c=4.5 and $N=10^4$).

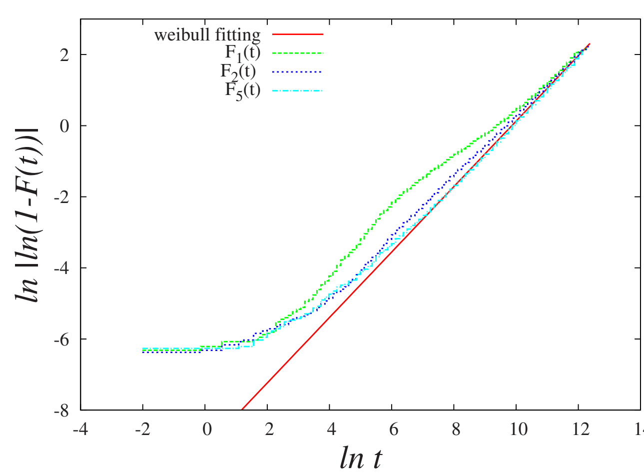

FIG. 8. (Color online) Weibull plot of the conditional probability distributions of interevent times of earthquakes in the world (M_c=5.0 and $N=10^4$).

when the previous interevent time is relatively large. Generally, the occurrence of earthquakes is considered as a *Markov renewal process* [29]. We can calculate the Lyapunov exponent of the occurrence of earthquakes by extending one-dimensional piecewise linear maps to three dimensional dynamical systems modeling Markov renewal processes if we know the form of conditional probability distributions in the entire region. For example, the dynamical system modeling the occurrence of earthquakes is given as

$$T(x,n,i) = \begin{cases} (T_i x, n+1, i), & x < 1, \\ (T_i x, 0, l), & x \in [1,c] \text{ and } n \in [m_l, m_{l+1}), \end{cases} \tag{15}$$

where $m_l = \min\{t : t \in \mathcal{T}_{l,N}\}$, $m_1 = 1$ and

$$T_i(x) = \begin{cases} \dfrac{a^i_{k-1} - a^i_k}{a^i_k - a^i_{k+1}}(x_n - a^i_k) + a^i_{k-1}, & x_n \in [a_{k+1}, a_k), \\ \dfrac{x_n - 1}{c - 1}, & x_n \in [1, c], \end{cases} \tag{16}$$

where a sequence a^i_k is obtained from the conditional probability distribution $F_i(t)$.

To quantify intermittency in point processes, Bickel proposed a clear estimation of intermittency using the correlation codimension [30]. Although we assume that interevent times are iid. random variables, which does not always hold in point processes, we can study underlying dynamics of intermittent phenomena with the aid of one-dimensional maps. If we assume the integrate-and-fire model [31], the orbit x_n in dynamical systems is considered to be the integrated value of a signal that is behind intermittent phenomena. In the occurrence of earthquakes, the integrated value would be the accumulated energy on the crust. Therefore the piecewise linear maps constructed would be related to dynamics of the energy accumulation and release in earthquakes.

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