Reconstruction of time-delay systems using small impulsive disturbances

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We propose a method for the reconstruction of time-delayed feedback systems from time series. The method is based on the analysis of the system response to a weak external disturbance having the form of rectangular pulses. To apply the method one must have access to the state variable of the system in order to perturb it and the time series of the driving signal and the system response having at least about one hundred points on the time interval equal to the delay time. The method is intended to recover delays in low-order time-delay systems performing periodic oscillations, but can also be applied to systems in chaotic regimes in the presence of high level of noise. We verify the method by applying it to both numerical and experimental data.

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I. INTRODUCTION

Nonlinear systems with a time-delayed feedback have attracted a lot of attention due to the wide abundance of time delays in various fields of science including optics [1–4], chemistry [5], and biology [6–8] among others. The dynamics of time-delay systems critically depends on the values of delay times. Thus, the problem of reconstruction of delay times from experimental time series is of great importance for many scientific disciplines and applications. To solve this problem a variety of methods has been proposed, which allows one to recover the delay times of time-delayed feedback systems from their chaotic time series [9–17]. However, these methods fail for periodic states [18]. But in practice many important time-delay systems operate in periodic or nearly periodic regimes [19–21]. Hence, it is important to develop methods that allow one to estimate the parameters of systems with delay-induced dynamics performing not only chaotic, but also periodic oscillations.

Several such methods have been proposed recently [22–24]. These methods are based on the analysis of the time-delay system response to external perturbations. To recover the delay time it was proposed to disturb the system by a short-correlated noisy signal [22], a periodic impulsive signal leading to the appearance of a transient process [23], and a control signal suitably designed to drive the system to a steady state [24]. All these methods [22–24] require sufficiently large amplitude of perturbations. For example, in Ref. [24] the amplitude of the signal of perturbation was by order of magnitude greater than the amplitude of unperturbed self-sustained oscillations. However, the use of strong disturbances of a time-delay system performing periodic oscillations is not always possible because it can result in undesirable qualitative change in the system behavior. In these cases the use of small disturbances for estimating the system parameters is preferable. Such technique based on the method of accumulation [25] has been proposed in Ref. [23]. However, this method exploits a complicated signal of perturbation having the form of rectangular radio pulses with linearly increasing filling frequency that hampers its use in practice. Besides, the application of the method needs long time series.

In this paper we propose a method for recovering time-delay systems based on the analysis of the system response to a weak impulsive disturbance of a simple form. The method can be applied to short time series of time-delay systems performing either periodic or chaotic oscillations.

The paper is organized as follows. In Sec. II we present the idea of the method and apply it to recover first-order time-delay systems with a single delay in periodic and chaotic regimes using both numerical and experimental data. In Sec. III the method is applied for the reconstruction of delays in scalar time-delay systems of second order and with several coexisting delays and nonscalar time-delay system. In Sec. IV we summarize our results.

II. RECOVERY OF FIRST-ORDER TIME-DELAY SYSTEMS WITH A SINGLE DELAY

Let us consider a ring time-delayed feedback system composed of a delay line, nonlinear device, and filter (Fig. 1), performing self-sustained oscillation $x(t)$. We disturb the system by an external signal $y(t)$ having the form of rectangular pulses with amplitude A, period T, and duration M. Filter parameters and the point of the external signal injection into the ring time-delay system define the form of its model equation. In the case where the filter is a simple low-frequency first-order filter and the signal $y(t)$ is added to the system between the filter and the delay line (Fig. 1), the considered system is governed by the first-order delay-differential equation

$$\varepsilon\dot{x}(t) = -x(t) + f(x(t-\tau) + y(t-\tau)), \qquad (1)$$

where τ is the delay time, the parameter ε characterizes the inertial properties of the system, and f is a nonlinear function.

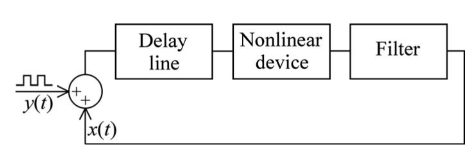

FIG. 1. Block diagram of a ring time-delayed feedback system disturbed by an external pulse signal.

The external signal $y(t)$ disturbs the system. However, as can be seen from Eq. (1), the result of $x(t)$ perturbation by $y(t)$ manifests itself only after the time τ after the beginning of perturbation. Similarly, termination of external perturbation affects the system dynamics only after the time τ after the completion of perturbation. If a disturbance has the form of rectangular pulses beginning at time moments $t=nT$ and ending at time moments $t=nT+M$, $n=1,2,\ldots$, then the trajectory $x(t)$ suffers perturbations at $t=nT+\tau$ and $t=nT+M+\tau$. At these moments of time breaks appear in the temporal realization of $x(t)$, which are practically unnoticeable in the case of small amplitudes of the disturbances. The changes in the system dynamics become more noticeable if one numerically differentiate $x(t)$ with respect to t. Time derivative $\dot{x}(t)$ shows a jump in time τ after the beginning and after the ending of a rectangular pulse. The variable best suited to analyze a time-delay system response to external perturbations by rectangular pulses is the second derivative $\ddot{x}(t)$. Its time series exhibits sharp peaks or dips in time τ after the passage of the leading and trailing edges of a rectangular pulse. These peaks and dips are pronounced even for small amplitudes of the disturbances.

Investigating correlations between a disturbance and the system response one should use signals that have undergone the same transformations. Thus, the cross-correlation function of $\ddot{x}(t)$ and the second derivative of perturbation $\ddot{y}(t)$ allows us to estimate τ. However, since $\ddot{y}(t)$ obtained from $y(t)$ using numerical differentiation takes both positive and negative values in the vicinity of pulse edge, the delay time corresponds to zero crossing of the cross-correlation function between main maximum and minimum. To recover τ it is more convenient to exploit the cross-correlation function

$$C(s) = \frac{\langle|\ddot{y}(t)||\ddot{x}(t+s)|\rangle}{\sqrt{\langle|\ddot{y}(t)|^2\rangle\langle|\ddot{x}(t)|^2\rangle}}, \tag{2}$$

where the angular brackets denote averaging over time. The magnitude $|\ddot{y}(t)|$ takes only positive values at the pulse edges and $C(s)$ has a pronounced maximum at $s=\tau$ [Fig. 2(a)]. Note that for the accurate recovery of τ the time series of $x(t)$ and $y(t)$ should be sampled at least at $\tau/100$.

It should be emphasized that the proposed method can be applied only in the case where one has an access to the state variable of the system in order to perturb it. Recently we have proposed another method for the recovery of time-delay systems in periodic regimes, which is also based on an analysis of the system response to small periodic disturbance and the use of the cross-correlation function [23]. However, the method [23] is much more complicated. First of all, it requires a complex signal of perturbation having the form of rectangular radio pulses with linearly increasing filling frequency. Second, it is necessary to filter the signals of perturbation and the system response. Then the method of accumulation [25] should be used to obtain a superposition of about 100 responses of the system to perturbations. At last, one has to define the order of the model delay-differential equation of the system. Only under fulfillment of these four points the cross-correlation function of the driving signal and the superposition of the system responses allows one to estimate the delay time.

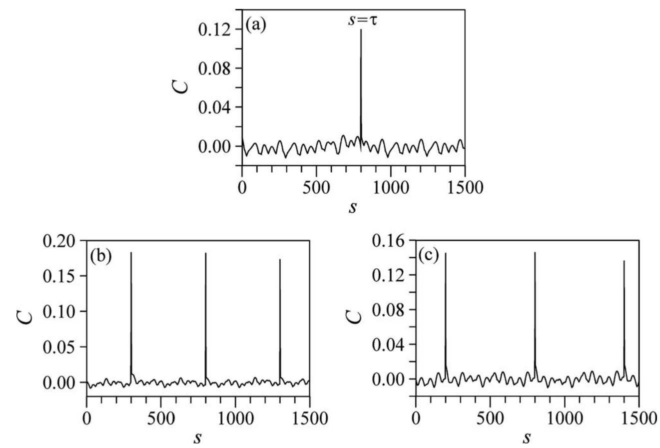

FIG. 2. The cross-correlation function (2) for the system (1) with $\tau=800$, $\varepsilon=20$, $f(x)=\lambda-x^2$, and $\lambda=1$ disturbed by rectangular pulses with $A=0.01$ and $M=T/2$. In the absence of disturbance the system (1) performs periodic oscillations. (a) $T=1900$. (b) $T=1000$. (c) $T=1200$.

The method proposed in the present paper exploits very simple external disturbance having the form of rectangular pulses. Small changes in the system dynamics caused by such pulses with low amplitude are best revealed by the second derivative of the system response. As a consequence, the cross-correlation function of the second derivatives of the driving signal and the system response becomes a very sensitive measure for detecting delay in the system. It needs neither filtration of data nor the knowledge of the system order for the recovery of τ. Since the proposed technique does not exploit the method of accumulation, it can be applied to time series by order of magnitude shorter than those required for the method in Ref. [23]. One more advantage of the present method is that it can be easily realized in the physical experiment in contrast to the method [23], which application to real data is hampered by the necessity to ensure the same initial phase of filling for all radio pulses.

We apply the proposed method to recover a delay time of system (1) with $\tau=800$, $\varepsilon=20$, $f(x)=\lambda-x^2$, and $\lambda=1$. For a given parameter of nonlinearity λ, the system (1) shows in the absence of perturbation periodic self-sustained oscillations with amplitude $A_a=1$ and period $T_a=1638$. We disturb the system by an external pulse signal with $A=0.01$, $T=1900$, and $M=T/2$. The function (2), constructed with a variation step for s equal to 1, shows the maximum at $s=\tau=800$ [Fig. 2(a)]. This maximum is located at the true value of the delay time for $A\geq 0.002$. The derivatives $\ddot{x}(t)$ and $\ddot{y}(t)$ are estimated from the time series of $x(t)$ and $y(t)$ using the simplest difference method. To construct the plot of $C(s)$ [Fig. 2(a)] we use 20 000 points, but the method can be applied to shorter time series. As the length of the time series decreases, the maximum of $C(s)$ at $s=\tau$ becomes less pronounced. For the indicated parameter values it is sufficient to take only 3500 points, i.e., the use of two pulses is sufficient for the accurate reconstruction of τ.

In general the number of peaks of $C(s)$, $s\in[0,\tau]$ is determined by the ratios T/τ and M/T. For $M=T/2$ the dis-

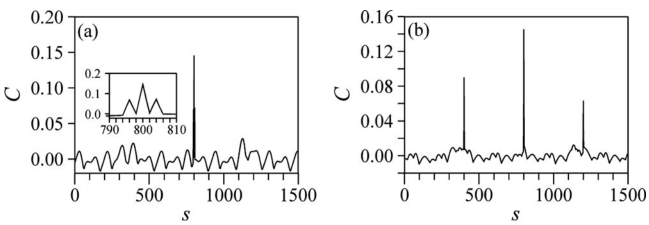

FIG. 3. The cross-correlation function (2) for the system (1) disturbed by rectangular pulses with $A=0.01$ and $T=2000$. The system (1) parameters are the same as in Fig. 2. (a) $M=4$. (b) $M=1600$. The inset in (a) is the enlarged fragment of $C(s)$ in the vicinity of $s=800$.

tance between the peaks of $C(s)$ is always equal to $T/2$. In this case the delay time corresponds to the location of the first peak of $C(s)$, if $T>2\tau$ [Fig. 2(a)], the second peak of $C(s)$, if $\tau<T\leq 2\tau$ [Fig. 2(b)], and the kth peak of $C(s)$, if $2\tau/k<T\leq 2\tau/(k-1)$, $k\geq 3$. Since the ratio T/τ is *a priori* unknown, we do not know which peak of $C(s)$ should be taken for the estimation of τ. That is why we propose to disturb the system at first by an impulsive signal with an arbitrary period $T=T_1$ and then by an impulsive signal with a period $T=T_2$ close to T_1 and to compare the functions $C(s)$ in the first and the second cases. For different T the peaks of $C(s)$ are observed at different s values. Only the location of the peak corresponding to the delay time remains fixed. If we find such a peak, we recover τ. For example, for $\tau=800$ the first peaks of $C(s)$ are located at s values equal to 300, 800, and 1300, if $T=1000$ [Fig. 2(b)] and at s values equal to 200, 800, and 1400, if $T=1200$ [Fig. 2(c)].

In the case of periodic self-sustained oscillations of a time-delay system one can easily define their period T_a. If it is known that these oscillations take place at the principal mode, for which $\tau<T_a/2$, one can choose the impulsive signal with $T\geq T_a$ ($T>2\tau$) and $M=T/2$ and recover τ as the value at which the first peak of $C(s)$ is observed. For simplicity all the subsequent figures of the cross-correlation function $C(s)$ are plotted in the paper for the case $T>2\tau$.

If $M\neq T/2$, then under fulfillment of one of the conditions: $M<\tau$ or $(T-M)<\tau$ the $C(s)$ plot exhibits an additional peak between the values of $s=0$ and $s=\tau$ (Fig. 3). This peak is located at $s=\tau-M$ in the case of $M<\tau$ [Fig. 3(a)] and at $s=\tau-(T-M)$ in the case of $(T-M)<\tau$ [Fig. 3(b)]. In both cases the second peak of $C(s)$ observed at $s=\tau$ is the maximal one. Thus, the value of M can be chosen over a wide range. The method allows one to use even very short pulses ($M<0.01T$) without their amplitude increasing. It may be useful when it is desirable to reduce the system disturbance to a minimum.

To test the method efficiency for experimental systems inevitably corrupted by noise we consider the application of the method to experimental time series gained from an electronic oscillator with time-delayed feedback perturbed by an external signal having the form of rectangular pulses (Fig. 1). As the nonlinear device we exploit an amplifier constructed using bipolar transistors and having a quadratic transfer function. The inertial properties of the oscillator are defined by a low-frequency first-order RC filter, which resistance R and capacitance C specify $\varepsilon=RC$. The oscillator dynamics is described by Eq. (1), where $x(t)$ and $x(t-\tau)$ are the delay line input and output voltages, respectively. In the absence of external perturbation the oscillator shows at $\tau=4.16$ ms and $\varepsilon=0.46$ ms periodic self-sustained oscillations with amplitude $A_a=1.5$ V and period $T_a=9.2$ ms.

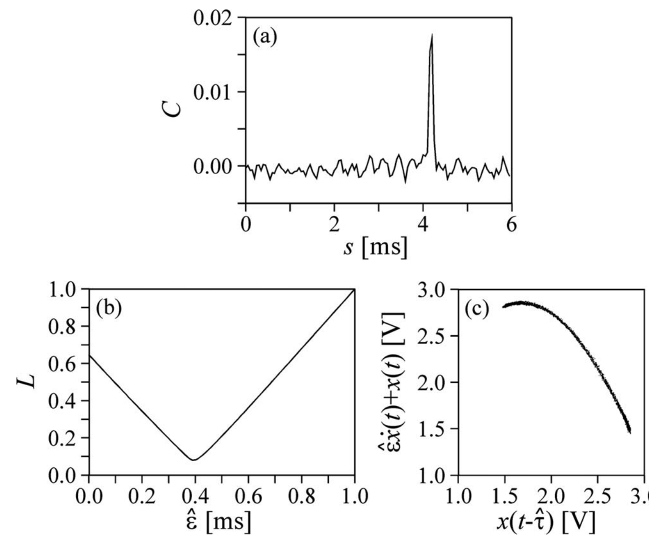

FIG. 4. Reconstruction of the electronic oscillator with delayed feedback performing periodic oscillations. (a) The cross-correlation function (2). (b) Length L of a line connecting all points ordered with respect to the abscissa in the $[x(t-\hat{\tau}),\hat{\varepsilon}\dot{x}(t)+x(t)]$ plane, as a function of $\hat{\varepsilon}$. $L(\hat{\varepsilon})$ is normalized to the most dispersed set of points. (c) The nonlinear function recovered from experimental periodic time series at $\hat{\tau}=4.2$ ms and $\hat{\varepsilon}=0.39$ ms.

Using an analog-to-digital converter with sampling frequency $f_s=20$ kHz we record the signals $x(t)$ and $y(t)$ at the pulse signal parameters $A=20$ mV, $T=11.1$ ms, and $M=T/2$. The function (2) is plotted in Fig. 4(a). For a variation step for s equal to 0.05 ms it has the maximum at $s=4.20$ ms, i.e., the delay time is recovered with high accuracy.

After estimation of τ one can reconstruct the parameter ε and the nonlinear function f of a scalar time-delay system using the method described in Ref. [16]. Following this method, we have to project the trajectory of undisturbed system (1) on the plane $[x(t-\tau),\varepsilon\dot{x}(t)+x(t)]$. As follows from Eq. (1), in the absence of perturbation the points of the projection reproduce the nonlinear function f, which can be approximated if necessary. Since the parameter ε is *a priori* unknown, one needs to plot $\hat{\varepsilon}\dot{x}(t)+x(t)$ versus $x(t-\tau)$ under variation in $\hat{\varepsilon}$, searching for a single-valued dependence in the $[x(t-\tau),\hat{\varepsilon}\dot{x}(t)+x(t)]$ plane, which is possible only for $\hat{\varepsilon}=\varepsilon$. As a quantitative criterion of single-valuedness in searching for ε we use the minimal length $L(\hat{\varepsilon})$ a line, connecting all points ordered with respect to $x(t-\tau)$ in the plane $[x(t-\tau),\hat{\varepsilon}\dot{x}(t)+x(t)]$. The minimum of $L(\hat{\varepsilon})$ gives us an estimation of ε.

The $L(\hat{\varepsilon})$ plot constructed at the recovered delay time $\hat{\tau}=4.2$ ms is shown in Fig. 4(b). A variation step for $\hat{\varepsilon}$ in Fig. 4(b) is equal to 0.01 ms. The minimum of $L(\hat{\varepsilon})$ takes place at $\hat{\varepsilon}=0.39$ ms, giving us a close estimation of $\varepsilon=0.46$ ms. Using the estimated values $\hat{\tau}$ and $\hat{\varepsilon}$ we reconstruct the nonlinear function from experimental undisturbed time series [Fig.

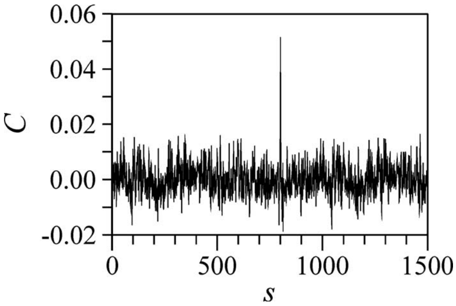

FIG. 5. The cross-correlation function (2) for the system (1) with $\tau=800$, $\varepsilon=20$, $f(x)=\lambda-x^2$, and $\lambda=1.85$ disturbed by rectangular pulses with $A=0.1$, $T=1900$, and $M=T/2$. In the absence of disturbance the system (1) performs chaotic oscillations.

4(c)]. Such a technique allows us to recover only a fragment of the function f, since the oscillations take place only in a small region of phase space because of their periodicity.

The proposed method can be applied to chaotic time series for the delay-time reconstruction. Let us consider the system (1) with $\tau=800$, $\varepsilon=20$, and $\lambda=1.85$ corresponding to a chaotic dynamics. The system is disturbed by a pulse signal with $A=0.1$, $T=1900$, and $M=T/2$ and a zero-mean Gaussian white noise with a level of 5% (the signal-to-noise ratio is about 26 dB) is added to the system dynamics. As can be seen from Fig. 5, the plot of $C(s)$ exhibits the maximum accurately at $s=800$. Similarly to Figs. 2 and 3, this figure is plotted varying s with a step of 1.

The results of the application of the method to experimental data obtained from the electronic oscillator with delayed feedback operating in chaotic regime are presented in Fig. 6. The oscillator parameters $\tau=4.16$ ms and $\varepsilon=0.46$ ms are chosen the same as in the considered above case of periodic oscillations (Fig. 4). To obtain chaotic oscillations in the system we increase the amplifier gain. The oscillator is disturbed by rectangular pulses with $A=50$ mV, $T=11.1$ ms, and $M=T/2$. The cross-correlation function (2), constructed varying s with a step of 0.05 ms, exhibits the maximum at $s=4.20$ ms [Fig. 6(a)]. The $L(\hat{\varepsilon})$ plot, constructed with $\hat{\tau}=4.2$ ms and a variation step for $\hat{\varepsilon}$ equal to 0.01 ms, shows the minimum at $\hat{\varepsilon}=0.39$ ms [Fig. 6(b)]. The nonlinear function recovered at $\hat{\tau}=4.2$ ms and $\hat{\varepsilon}=0.39$ ms [Fig. 6(c)] coincides closely with the true transfer function f of the amplifier.

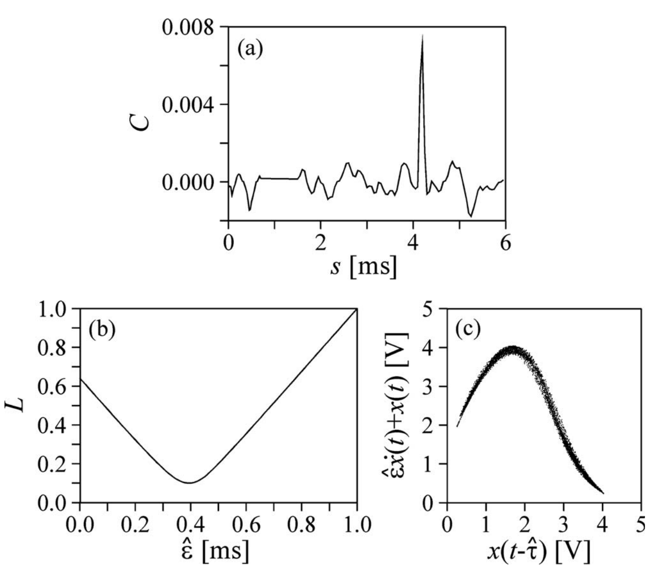

FIG. 6. Reconstruction of the electronic oscillator with delayed feedback performing chaotic oscillations. (a) The cross-correlation function (2). (b) The $L(\hat{\varepsilon})$ plot. $L(\hat{\varepsilon})$ is normalized to the most dispersed set of points. (c) The nonlinear function recovered from experimental chaotic time series at $\hat{\tau}=4.2$ ms and $\hat{\varepsilon}=0.39$ ms.

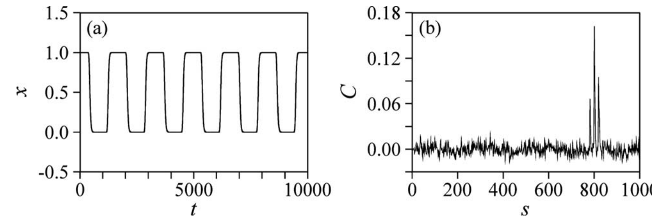

FIG. 7. (a) The time series of periodic self-sustained oscillations of the system (3) in the absence of perturbation and noise. (b) The cross-correlation function (2).

III. DELAY ESTIMATION IN SCALAR TIME-DELAY SYSTEMS OF SECOND ORDER AND WITH MULTIPLE DELAYS AND NONSCALAR TIME-DELAY SYSTEM

The proposed method can be applied for the reconstruction of time-delay systems of higher order than system (1). Let us consider a time-delay system described by the second-order delay-differential equation in the presence of dynamical noise $\xi(t)$,

$$\varepsilon_2\ddot{x}(t)+\varepsilon_1\dot{x}(t)=-x(t)+f(x(t-\tau)+y(t-\tau))+\xi(t). \quad (3)$$

Equation (3) governs the system depicted in Fig. 1 if the filter is composed of two in-series low-frequency RC filters. In this case $\varepsilon_1=R_1C_1+R_2C_2$ and $\varepsilon_2=R_1C_1R_2C_2$, where R_1, R_2, C_1, and C_2 are respectively the resistances and capacitances of the first and the second filters. At $\tau=800$, $\varepsilon_1=25$, $\varepsilon_2=100$, $f(x)=\lambda-x^2$, and $\lambda=1$ in the absence of perturbation and noise the system (3) exhibits periodic oscillations. Part of the time series is shown in Fig. 7(a).

We disturb the system by a very short pulse signal with $A=0.05$, $T=1900$, and $M=0.01T=19$ and add to the system dynamics a zero-mean Gaussian white noise $\xi(t)$ with a standard deviation of 3% of the standard deviation of data without noise (the signal-to-noise ratio is about 30 dB). The cross-correlation function (2) shows the maximum at $s=801$ giving a close estimation of τ [Fig. 7(b)]. It should be noted that the error of τ estimation increases with the increase in the system order. The appearance of an additional peak to the left of the main maximum in Fig. 7(b) is the result of $M\neq T/2$ (see Sec. II). In the case of chaotic behavior of the system (3) the function (2) has a shape qualitatively similar to the one presented in Fig. 7(b).

The method can be extended to time-delay systems with several coexisting delays. We demonstrate the efficiency of the proposed technique with a generalized Ikeda equation obtained by introducing a further delay in the presence of dynamical noise,

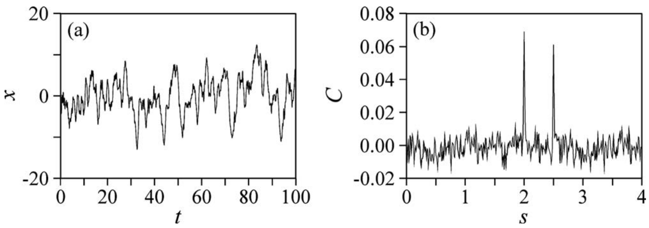

FIG. 8. (a) The time series of chaotic oscillations of the system (4) in the absence of perturbation and noise. (b) The cross-correlation function (2).

$$\dot{x}(t) = -x(t) + \mu[\sin(x(t-\tau_1) - x_{01} + y(t-\tau_1)) + \sin(x(t-\tau_2) - x_{02} + y(t-\tau_2))] + \xi(t). \quad (4)$$

The Ikeda equation describes a phase lag x of the electrical field across the optical resonator. The parameter μ characterizes the laser power intensity injected into the system, τ_1 and τ_2 are the delay times, and x_{01} and x_{02} are the constant phase lags. At $\mu=10$, $\tau_1=2$, $\tau_2=2.5$, and $x_{01}=x_{02}=\pi/3$ in the absence of perturbation and noise the system (4) exhibits chaotic oscillations [Fig. 8(a)].

Figure 8(b) shows the function (2) plot for the case where the system (4) is disturbed by a pulse signal $y(t)$ with $A=0.5$, $T=5.2$, and $M=T/2$ and corrupted by a zero-mean Gaussian white noise $\xi(t)$ with a standard deviation of 20% of the data standard deviation (the signal-to-noise ratio is about 14 dB). For a variation step for s equal to 0.01 the first two peaks of $C(s)$ are located at $s=2.00$ and $s=2.50$. In spite of the high level of noise, the delay times are recovered accurately. For the indicated parameter values the method provides accurate reconstruction of τ_1 and τ_2 for noise levels up to 40% (the signal-to-noise ratio is about 8 dB). Being applied to the system (4) in periodic regimes the method gives the similar results. Thus, the proposed method is more tolerant to noise than the method [24], which is efficient only for very small levels of noise.

Let us consider the application of the method to nonscalar time-delay system in periodic regime. For a demonstration on numerically generated data we use a system of two coupled nonlinear delayed equations introduced in [26] and disturb the variable $x(t)$ by an external signal $y(t)$ similarly to the way used in Eqs. (1), (3), and (4),

$$\dot{x}(t) = rx(t) - \mu\{[x(t-\tau) + y(t-\tau)]^2 + cz^2(t-\tau)\}x(t),$$

$$\dot{z}(t) = rz(t) - \mu[z^2(t-\tau) + cx^2(t-\tau)]z(t). \quad (5)$$

We choose the parameters to be $r=4$, $\mu=4$, $c=0.5$, and $\tau=0.35$. As it was shown in [26], at these parameter values the system (5) shows periodic oscillations in the absence of perturbation. Part of the time series is shown in Fig. 9(a). The period of self-sustained oscillations is $T_a=3.38$.

We disturb the system by rectangular pulses with a very small amplitude $A=0.01$, $T=4$, and $M=T/2$. For a variation step for s equal to 5×10^{-4} the maximum of $C(s)$ is observed at $s=0.3500$, giving an accurate recovery of τ [Fig. 9(b)].

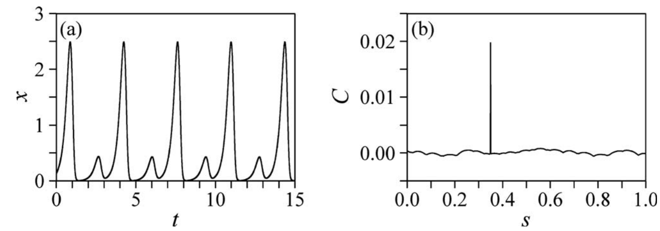

FIG. 9. (a) The time series of periodic self-sustained oscillations of the system (5) in the absence of perturbation. (b) The cross-correlation function (2).

IV. CONCLUSION

We have proposed the method for the reconstruction of low-order time-delayed feedback systems based on the analysis of the system response to an external disturbance having the form of rectangular pulses. To implement this method one must have access to the state variable of the system in order to perturb it. The method allows one to use very short and low-amplitude pulses. It can be successfully applied to short time series and data heavily corrupted by noise. However, the time series of the driving signal and the system response must have at least about 100 points on the time interval equal to the delay time. In contrast to the method proposed by us in Ref. [23], the considered technique can be used for the recovery of delays in time-delay systems with multiple delays and nonscalar time-delay systems and is easily realized in practice. The method can be applied also for the reconstruction of high-order time-delay systems, but the error of the delay-time estimation increases with the increase in the system order.

The proposed method is oriented to the recovery of time-delay systems performing periodic oscillations. However, it can be applied to systems with delay-induced dynamics performing chaotic oscillations. A limitation of the proposed technique in comparison with other methods of time-delay system reconstruction from chaotic time series is the necessity of disturbing the system dynamics. On the other hand, the advantage of the considered method over other ones proposed for chaotic time series is its efficiency for higher levels of noise.

We verified the method by applying it to periodic and chaotic time series of various model delay-differential equations, including those corrupted by noise, and an experimental time series acquired from an electronic oscillator with delayed feedback, disturbed by rectangular pulses.

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