Inverse bremsstrahlung absorption with nonlinear effects of high laser intensity and non-Maxwellian distribution

Su-Ming Weng,[1,2] Zheng-Ming Sheng,[3,1,*] and Jie Zhang[3,1]
[1]*Beijing National Laboratory of Condensed Matter Physics, Institute of Physics, CAS, Beijing 100190, China*
[2]*Theoretical Quantum Electronics (TQE), Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[3]*Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China*
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Inverse bremsstrahlung (IB) absorption and evolution of the electron distribution function (EDF) in a wide laser intensity range (10^{12}–10^{17} W/cm^2) have been studied systematically by a two velocity-dimension Fokker-Planck code. It is found that Langdon's IB operator overestimates the absorption rate at high laser intensity, consequently with an overdistorted non-Maxwellian EDF. According to the small anisotropy of EDF in the oscillation frame, we introduce an IB operator which is similar to Langdon's but without the low laser intensity limit. This operator is appropriate for self-consistently tackling the nonlinear effects of high laser intensity as well as non-Maxwellian EDF. Particularly, our operator is capable of treating IB absorption properly in the indirect and direct-drive inertial confinement fusion schemes with the National Ignition Facility and Laser MegaJoule laser parameters at focused laser intensity beyond 10^{15} W/cm^2.

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I. INTRODUCTION

As a basic energy absorption mechanism in laser plasmas, inverse bremsstrahlung (IB) has been intensively studied for decades and still attracts significant attention with the development of high-energy lasers such as the National Ignition Facility (NIF) and Laser MegaJoule (LMJ) [1–4]. Analytic expressions for IB absorption rate have been derived from classical approaches [5–8] and quantum-mechanical approaches [9–14]. However, these analytic approaches are all based on prearranged electron distribution functions (EDFs), which are usually assumed to be Maxwellian. Therefore, it is difficult for them to update the absorption rates with the evolution of EDFs.

The celebrated Langdon's IB operator [15] was introduced to treat IB absorption consistently with the evolution of EDF. It could be conveniently integrated into multispatial dimensional Fokker-Planck codes for studying the nonlocal heat transport [16], laser filamentation [17], laser-solid interaction [18], ion-acoustic waves [19], magnetic field generation [20], magnetized heat transport [2,3,21], magnetic cavitation [1], etc. However, Langdon's IB operator is valid only for $u_0 \ll v_e$ as stated by himself [15] ($u_0 = eE/m_e\omega$ is the peak electron oscillating velocity and v_e is the electron thermal velocity), and it overestimates the absorption rate at moderate and high intensity [8]. While IB absorption at moderate laser intensity is particularly relevant to the indirect and direct-drive inertial confinement fusion (ICF) scheme with the completion of the NIF. In this facility, 192 beams of NIF are divided into sets of four, and each four overlapped high-energy beams can form a 3.5-ns-long square pulse with intensity of 2×10^{15} W/cm^2 [4]. For this intensity, $\lambda^2 I < 10^{17}$ μm^2 W/cm^2 is still fulfilled and IB absorption dominates the laser energy deposition [22–24]. However, at these laser intensities $u_0 \ll v_e$ often fails, especially in early and middle stage of heating. Unfortunately, Langdon's operator is still stiffly used to treat IB absorption in such cases [1]. Therefore, it is an urgent issue to properly treat IB absorption consistently with the evolution of EDF at high laser intensity.

In this paper we use a two-velocity-dimension Fokker-Planck (2VFP) code to investigate IB absorption and the evolution of EDF at laser intensity roundly from 10^{12} to 10^{17} W/cm^2, with u_0 varying from $0.2v_e$ to $65v_e$. Following the form of Fokker-Planck equation in the oscillating frame, we present an IB operator to deal with the nonlinear IB absorption consistently with the evolution of EDF in a wide range of laser intensity. The absorption rates and the EDFs obtained from different models are compared in the laser intensity interval 10^{12}–10^{17} W/cm^2. We find that Langdon's IB operator overestimates the absorption, hence resulting an overdistorted non-Maxwellian EDF at moderate and high intensity. At sufficiently large laser intensity our IB operator shows that the absorption rate decreases slowly with increasing laser intensity and that the resulting EDF will come back to be Maxwellian, in good agreement with the results from 2VFP code. Particularly, numerical simulation shows that it is of great importance to treat IB absorption properly in the indirect drive ICF scheme by our IB operator.

II. INVERSE BREMSSTRAHLUNG ABSORPTION AT HIGH LASER INTENSITY

In the presence of the laser electric field **E** the evolution of EDF in a homogeneous fully ionized plasma can be described by the Fokker-Planck equation [15,25]

$$\frac{\partial f}{\partial t} = \frac{e\mathbf{E}}{m_e} \cdot \nabla_{\mathbf{v}} f + C_{ei}(f) + C_{ee}(f). \quad (1)$$

Thereby **E** is assumed to be linearly polarized along the z direction, and $C_{ei}(f)$ and $C_{ee}(f)$ are the electron-ion (e-i) and electron-electron (e-e) collision operators, respectively.

*zmsheng@sjtu.edu.cn

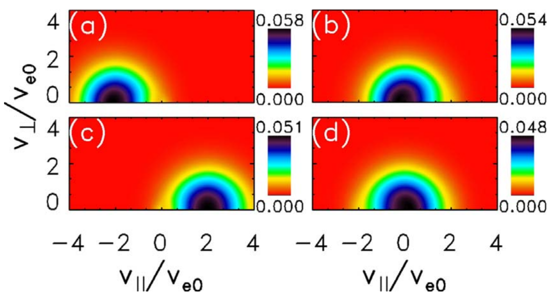

FIG. 1. (Color online) Snapshots of the EDFs after (a) 1/4, (b) 1/2, (c) 3/4, and (d) 1.0 laser cycle. The EDF is in unit of n_e/v_{e0}^3, and v_{e0} is the initial thermal velocity. The plasmas and laser parameters are: electron density $n_e=10^{20}$ cm^{-3}, initial temperature $T_e=10$ eV, ionization state $Z_i=1$, laser wavelength $\lambda=1.06$ μm, and intensity $I=10^{14}$ W/cm^2 with $v_0\simeq 2.0v_e$. For convenience of visualization, the EDFs are drawn in cylindrical coordinate ($v_\parallel=v\cos\theta$, $v_\perp=v\sin\theta$).

As the detailed knowledge of EDF will be helpful for studying processes in plasma [26], in Fig. 1 we draw the EDFs that are obtained from the Fokker-Planck code with complete two velocity dimensions [25] at four representative times in an intense laser. Note that the EDF in an intense laser oscillates with amplitude u_0, and the assumption of small anisotropy could only be well satisfied in the frame with oscillating velocity $\mathbf{u}=-u_0\sin(\omega t)\mathbf{e}_z$ rather than in the rest frame. With the transformation $\mathbf{v}'=\mathbf{v}-\mathbf{u}$, we write Eq. (1) in the oscillating coordinate system (v',θ') as

$$\frac{\partial f'}{\partial t}=C'_{ei}(f')+C'_{ee}(f'), \qquad (2)$$

where $C'_{ee}(f')$ is identical to $C_{ee}(f)$ due to the invariance of the self-collision term under coordinate transformation; the e-i collision operator is given by

$$C'_{ei}(f')=\frac{\nu_{ei}(u,v',\theta')}{2}\Bigg[\frac{u^2+2uv'\cos\theta'+u^2\cos^2\theta'}{v'}\frac{\partial f'}{\partial v'}+\frac{v'^2+2uv'\cos\theta'+u^2\cos 2\theta'}{v'^2\tan\theta'}\frac{\partial f'}{\partial\theta'}+(u\sin\theta')^2\frac{\partial^2 f'}{\partial v'^2}+\left(\frac{v'+u\cos\theta'}{v'}\right)^2\frac{\partial^2 f'}{\partial\theta^2}+\frac{2u\sin\theta'(v'+u\cos\theta')}{v'}\frac{\partial^2 f'}{\partial v'\,\partial\theta'}\Bigg], \qquad (3)$$

with

$$\nu_{ei}(u,v',\theta')=\frac{Z_i\Gamma^{e|e}}{(u^2+2uv'\cos\theta'+v'^2)^{3/2}}, \qquad (4)$$

in addition, $(v_e^2+u_0^2/4)^{1/2}$ instead of v_e is suggested to be used in calculating the Coulomb logarithm in $\Gamma^{e|e}=n_e e^4\ln\Lambda^{e|e}/4\pi\epsilon_0^2 m_e^2$ at high intensity [27,28].

Since the fast oscillation has been transformed away into the coordinate system, Eq. (2) behaves friendly to the numerical treatment, especially to avoiding the vigorous runaway of electrons from the computational domain when $u_0\gg v_e$. Therefore, basing on Eq. (2) rather than Eq. (1), we rebuild a two-velocity-dimension Fokker-Planck code with the similar numerical scheme as the former one [25].

A. Inverse bremsstrahlung operator

In the oscillating frame the EDF in an intense laser appears to be nearly isotropic, so it will be a good approximation to decompose the EDF into Legendre polynomials $P_l(\cos\theta')$ and truncate it as

$$f'(v',\theta')\simeq f'_0(v')+f'_1(v')\cos\theta'. \qquad (5)$$

If we multiply Eq. (2) by $P_0=1$ and substitute Eq. (5) into it, then integrate it over θ', with the help of the Legendre polynomials' orthogonality we finally get

$$\frac{\partial f'_0}{\partial t}\simeq\frac{u^2}{3v'^2}\frac{\partial}{\partial v'}\left[v'^2\nu'_{ei}\left(\frac{\partial f_0}{\partial v'}-\frac{f_1}{u}\right)\right]+C'_0, \qquad (6)$$

with C'_0 of the e-e collision term evaluated only using f'_0 and $\nu'_{ei}=Z_i\Gamma^{e|e}/(u^2+v'^2)^{3/2}$. Here we assume that $\nu_{ei}(u,v',\theta')$ depends on $\mathbf{v}'$ weakly and replace it by ν'_{ei} in the integration since this is a good approximation when $u_0\gg v_e$; and if at the first step we multiply Eq. (2) by $P_1=\cos\theta'$ instead, similarly we can get

$$\frac{\partial f'_1}{\partial t}\simeq\nu'_{ei}\left(u\frac{\partial f_0}{\partial v'}-\frac{2u^2+5v'^2}{5v'^2}f_1\right), \qquad (7)$$

in which we have dropped the terms relevant to $\partial f'_1/\partial v'$ and the e-e collision term relevant to f'_1. Note that in Eq. (2) only the e-i collision term $C'_{ei}(f')$, which varies with the fast frequency of laser ω, provides for the perturbation of EDF. Thus we can assume $\partial f'_1/\partial t\simeq -i\omega f'_1$ and solve f'_1 from Eq. (7) as

$$f'_1=\frac{b(\nu'_{ei}/\omega)^2}{1+b^2(\nu'_{ei}/\omega)^2}u\frac{\partial f_0}{\partial v'} \qquad (8)$$

with $b=(2u^2+5v'^2)/5v'^2$. After substituting Eq. (8) into Eq. (6) this becomes

$$\frac{\partial f'_0}{\partial t}\simeq\frac{u^2}{3v'^2}\frac{\partial}{\partial v'}\left[v'^2\nu'_{ei}g(\nu'_{ei})\frac{\partial f_0}{\partial v'}\right]+C'_0, \qquad (9)$$

with $g(\nu'_{ei})=1-b(\nu'_{ei}/\omega)^2/[1+b^2(\nu'_{ei}/\omega)^2]$. Although Eq. (9) is deduced according to the properties of EDF in an intense laser, we are delighted to find that the time average of Eq. (9) with $\nu'_{ei}\simeq Z_i\Gamma^{e|e}/v'^3$ and $b\simeq 1$ at low laser intensity perfectly reproduces Langdon's model. However, it is difficult to analytically integrate Eq. (9) over time for arbitrary ratios u_0/v_e. By numerical simulation, we find that the following equation:

$$\frac{\partial f'_0}{\partial t}\simeq\frac{u_0^2}{6v'^2}\frac{\partial}{\partial v'}\left[v'^2\nu'_{eff}g_0(\nu'_{eff})\frac{\partial f_0}{\partial v'}\right]+C'_0 \qquad (10)$$

generates the absorption rate in good quantitative agreement with that obtained from 2VFP code for $0.2\le u_0/v_e\le 65$ as shown in Fig. 2, where

$$g_0(\nu'_{eff})=1-b_0(\nu'_{eff}/\omega)^2/[1+b_0^2(\nu'_{eff}/\omega)^2], \qquad (11)$$

with

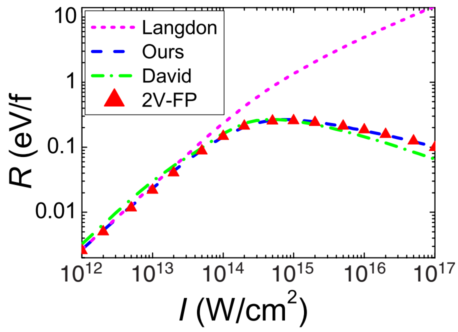

FIG. 2. (Color online) The absorption rates R averaged over the first four laser cycles as functions of laser intensity obtained from 2VFP code (triangle), Langdon's IB operator (short dashed line), our IB operator (dashed line), and David's fitted formula (6) in Ref. [29] from molecular-dynamic method (dash-dotted line). The plasma and other laser parameters are as in Fig. 1.

$$b_0 = (u_0^2 + 5v'^2)/5v'^2, \tag{12}$$

and

$$\nu'_{eff} = \frac{Z_i \Gamma^{e|e}}{(v'^2 + u_0^2/\zeta)^{3/2}}, \tag{13}$$

the coefficient ζ is numerically fitted as

$$\zeta = 3.84 + \frac{142.59 - 65.48 u_0/v_e}{27.3 u_0/v_e + (u_0/v_e)^2}. \tag{14}$$

B. Absorption rate

In Fig. 2, we compare the absorption rates calculated from our 2VFP code, Langdon's IB operator [15], our IB operator, and from a molecular-dynamic method [29]. The absorption rate R is defined as the rate of increase in electron temperature, and all absorption rates in Fig. 2 are averaged over the first four laser cycles as David *et al.* did [29]. The molecular-dynamic method is chosen for comparison because it avoids most of the assumptions used in other methods and thus provides reliable tests. In addition, it has been compared and shown to be in good agreement [29] with the classical [8] and the quantum-mechanical approaches [12]. It is clear that at low intensity the absorption rates from all methods agree well with each other and increase linearly with the intensity as predicted [8,10]. However, Langdon's IB operator has already obviously overestimated the absorption rate when $I=10^{14}$ W/cm^2 with $u_0 \simeq 2v_e$, and this deviation grows dramatically to several orders of magnitude with increasing intensity, while the absorption rate calculated from our IB operator at non-low intensities $I > 10^{14}$ W/cm^2 decreases slowly with the intensity and still shows a very good quantitative agreement with 2VFP code (relative error is below 4%) and the molecular-dynamic method. This illustrates that the absorption rate will decrease with the reduced effective e-i collision frequency owing to the increase in effective electron velocity at high intensity [7,14]. Taking account of the nonlinear high intensity effect in Eq. (13) for ν'_{eff}, our IB operator can also produce the proper absorption rate at high laser intensity. Therefore, it can be considered as the generalized version of Langdon's IB operator to be conveniently integrated into multispatial dimensional Fokker-Planck codes for a variety of practical applications [1–3,16–21].

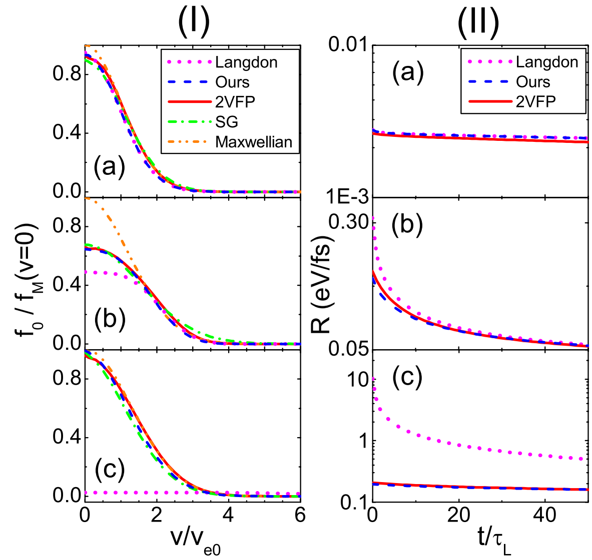

FIG. 3. (Color online) Column (i): EDFs obtained from 2VFP code (solid line), Langdon's IB operator (short dashed line), and our IB operator (dashed line) after $10\tau_L$ at three representative laser intensities (a) 10^{12} W/cm^2, (b) 10^{14} W/cm^2, and (c) 10^{16} W/cm^2. The fitted super-Gaussian EDFs from Eq. (15) (dash-dotted line) and the corresponding Maxwellian EDFs (dash-dot-dotted line) with the temperature obtained from our 2VFP code are also drawn for comparison. Column (II): the corresponding time evolution of absorption rates obtained from our 2VFP code (solid line), Langdon's IB operator (short dashed line), and our IB operator (dashed line) at these intensities. The plasma and other laser parameters are as in Fig. 1.

C. Evolution of electron distributions

All absorption rates shown in Fig. 2 are obtained on the basis of an initial Maxwellian distribution. However, Langdon has shown that IB absorption may produce a non-Maxwellian distribution which will reduce the absorption rate significantly [15]. Actually, the fact that in Fig. 2 the absorption rate from Langdon's operator increases a little slower than linear with increasing intensity implies that the effect of non-Maxwellian distribution has already arisen in these first four laser cycles. Therefore, we compare the obtained EDFs in detail at three representative intensities in Fig. 3(I) and trace the time evolution of absorption rates at these intensities in Fig. 3(II).

At low intensity $I=10^{12}$ W/cm^2, the EDFs obtained from all models are close to a Maxwellian distribution as shown in Fig. 3(Ia), and the corresponding absorption rates in Fig. 3(IIa) decrease gradually through the whole heating process.

At intensity $I=10^{14}$ W/cm^2, the initial Maxwellian distribution responds to efficient IB heating and will quickly deviate from the Maxwellian toward a non-Maxwellian self-similar state. As shown in Fig. 3(Ib), the resulted EDF, except for the high-energy tail [25,30], is well fitted by the super-Gaussian EDF

$$f_m = C_m \exp\left[-\left(\frac{a_m v^2}{2v_e^2}\right)^{m/2}\right], \tag{15}$$

with

$$a_m = \frac{2\Gamma(5/m)}{3\Gamma(3/m)} \quad \text{and} \quad C_m = \frac{n_e m}{4\pi\Gamma(3/m)}\left(\frac{a_m}{2v_e^2}\right)^{3/2} \tag{16}$$

as defined in Ref. [31], but the formula for m is required to be modified at increased intensity as

$$m = 2 + 3/(1 + 0.62/\alpha_{eff}), \tag{17}$$

with $\alpha_{eff}=Z_i v_0^2 v_e^2/(v_0^2+v_e^2)^2$. In addition, we find that Langdon's operator results an overdeviated non-Maxwellian EDF since it already obviously overestimates the absorption rate at this intensity. Due to the fast deviation from a Maxwellian distribution, the absorption rates decrease sharply at the first few laser cycles as shown in Fig. 3(IIb) and then decreases slowly with increasing electron temperature after entering a self-similar state as explained by Langdon [15].

As shown in Fig. 3(Ic), at higher intensity $I=10^{16}$ W/cm^2, we confirm that the EDF will revert to the Maxwellian [6,8]. It means that the effect of non-Maxwellian distribution will be suppressed by the nonlinear effect of high laser intensity that limits the absorption at high intensity. However, as shown in Fig. 3(Ic) at this intensity Langdon's IB operator results in a much more deformed non-Maxwellian EDF since its absorption rate is one order of magnitude higher as shown in Fig. 3(IIc). So it can be concluded that the nonlinear effect of non-Maxwellian distribution is tightly related to the nonlinear effect of increased intensity. With the growing laser intensity, the self-similar distribution departs from the Maxwellian to non-Maxwellian and finally reverts to the Maxwellian when $u_0 \gg v_e$. Our IB operator treats these two nonlinear effects self-consistently.

III. APPLICATION RELEVANT TO ICF

As an example, we simulate the IB heating process of Ref. [1], which is used for discussing the relationship between magnetic cavitation and nonlocal heat transport in indirect drive ICF scheme. For convenience, we simplify the simulation condition as that a laser with intensity of 6.3×10^{14} W/cm^2 and wavelength of 1.054 μm propagates in a homogeneous non-magnetic plasma with electron density of 1.5×10^{19} cm^{-3}, ionization state of $Z_i=7$, and initial temperature of $T_e=284$ eV, which in Ref. [1] is the peak temperature in the non-magnetic case at 440 ps. For $T_e=284$ eV, v_0/v_t is about 0.96, and it will be larger for other lower temperatures. Therefore, Langdon's IB operator, which is valid for $v_0/v_t \ll 1$, may be no longer suitable for treating IB absorption in this case. In Fig. 4, we compare the evolution of absorption rates and plasma temperatures obtained from different models. It is found that our IB operator accurately estimates the absorption rate during the heating process, the averaged relative error with the 2VFP code is below 1.25%. However, Langdon's operator results in an

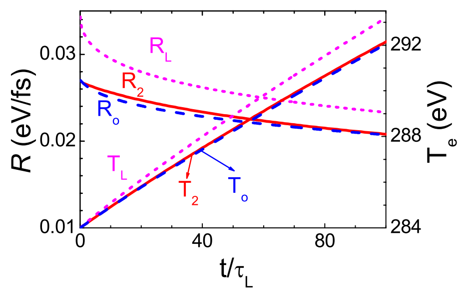

FIG. 4. (Color online) The absorption rates R and plasma temperatures T_e updated by 2VFP code (solid line), Langdon's IB operator (short dashed line), and our IB operator (dashed line). The plasma and laser parameters are electron density of 1.5×10^{19} cm^{-3}, initial temperature of 284 eV, ionization state of $Z_i=7$, laser wavelength of 1.054 μm, and intensity of 6.3×10^{14} W/cm^2.

overestimated absorption rate, and the averaged relative error with 2VFP code is about 12.9%. As a result, the plasma is also shown to be overheated by Langdon's operator in Fig. 4, which may affect the heat transport and subsequent processes in the indirect drive ICF scheme [1–3]. Therefore, it is more suitable to treat IB absorption accurately in the indirect drive ICF scheme by our IB operator instead of Langdon's.

IV. CONCLUSION

In this paper we have calculated the IB absorption rate and shown the evolution of EDF at a wide range of laser intensity with u_0/v_e varying from 0.2 to 65. It is found that if $u_0 > v_e$ the absorption will be inhibited with the increasing intensity and the EDF will revert to satisfy the (oscillating) Maxwellian rather than the super-Gaussian, i.e., the effect of non-Maxwellian distribution is closely related to the nonlinear effect of high intensity. Considering the EDF nearly isotropic in oscillation frame, we obtain an IB operator to treat these nonlinear effects self-consistently without the low laser intensity limit, and it can be integrated into large Fokker-Planck codes for practical applications [1–3,16–21] as conveniently as Langdon's IB operator [15]. In particular, the numerical simulation relevant to indirect drive ICF scheme shows that our IB operator is more suitable that Langdon's to handle the IB absorption accurately with nonlinear effects at this intensity.

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