Social tolerance allows cooperation to prevail in an adaptive environment

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In real situations, individuals often have moderate tolerance toward ambient cooperative environment in which they tend to avoid unfavorable interactions and search for favorable ones. How such social tolerance affects the evolution of cooperation and the resulting cooperative networks remains to be answered. To address this issue, here we present an effective model of co-evolutionary prisoner's dilemma by introducing cooperative environment and social tolerance for networked players. An individual's level of cooperative environment characterizes the cooperativity and sustainability of its interaction environment centered on itself. In our model, for paired individuals we assume that the one in better cooperative environment has a certain tolerance threshold to the opponent. If the opponent's cooperative environment level is beyond the tolerance threshold, the one in better cooperative environment cuts unilaterally the link, and rewires to others. Otherwise, the link is not severed, and meanwhile an inhomogeneous strategy imitation process between them is considered. Moreover, a player's cooperative environment is adjusted in response to the strategy choices in the neighborhood. Interestingly, we find that there exists a moderate tolerance threshold warranting the best promotion of cooperation. We explain the nontrivial results by investigating the time ratio of strategy (network) updating during the whole process and properties in emerging networks. Furthermore, we investigate the effect of memory-dependent discounting of individuals' cooperative environment on the evolution of cooperation. We also demonstrate the robustness of our results by considering two other modified co-evolutionary rules. Our results highlight the importance of appropriate tolerance threshold for the evolution of cooperation during the entangled co-evolution of strategy and structure.

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I. INTRODUCTION

Understanding the flourishing cooperative behavior in biological and social systems remains one of the most exciting and fundamental challenges to date [1]. Evolutionary game theory has been considered an important approach to investigating the cooperative behavior in systems consisting of selfish individuals. In particular, the prisoner's dilemma game (PDG) is one of the most commonly employed games for this purpose [2]. Originally, two players simultaneously decide whether to cooperate (C) or to defect (D). The cooperator pays a cost c, for another individual to receive a benefit b; while the defector pays no cost and does not distribute any benefits. In any mixed group, cooperative individuals would quickly be supplanted by selfish ones having higher payoffs (reproductive fitness), thus leading to an enduring social dilemma of cooperation [2].

To understand the conundrum of cooperation, different cooperation-promoting mechanisms have been proposed in recent years [3]. Most notably, it has been well recognized that population structure plays a decisive role in the evolution of cooperation [4–8] (see Ref. [9] for a recent review). In particular, it is reported that scale-free networks provide a unifying framework for the emergences of cooperation [5], because of strong heterogeneity in numbers of interaction partners [7,8]. Moreover, it is worth noting that some other structural properties, including average degree [10,11], degree-mixing pattern [12], and clustering [13], have nontrivial effects on the surviving of cooperators in networked populations. These network topological features can be helpful in unveiling in depth microscopic organization of cooperation [8].

Furthermore, as an important extension, it has been demonstrated that the entangled co-evolution of an individual's strategy and connection constitutes a key mechanism for the sustainability of cooperation [14–27]. Some approaches, e.g., dissatisfied-rewire-process [14–16], active liking rule [17], myopic cost-benefit comparison [18], and evolutionary preferential attachment [19], have been developed as potential promoters of cooperation, with noticeable success.

It should be noted that, among most of these aforementioned studies, adaptive dynamics of social ties depends mainly on individuals' strategy and payoff information. However, indeed individuals' other feature information coupled with game dynamics can be also incorporated, as a choice criterion, into co-evolutionary dynamics. For example, reputation information [3,28] can be used for partner choice; that is, individuals' partner switching is inclined to dump to the ones having low reputations, and prefers to choose the ones having good reputation [20]. In addition, an individual's level of cooperation can be used as a selection parameter, and individuals terminate future interactions with those partners who are not cooperative enough [29]. Enlightened by these studies [14–29], presently we aim to explore

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individuals' limited tolerance toward their surrounding interaction environment and its role in the evolution of cooperation.

We notice that, for PDG on graphs, individuals engage in pairwise interactions with their social partners (i.e., neighbors) [9]. The assortment between cooperators (i.e., clusters of cooperators) is crucial for the maintenance of cooperation in spatial games [4]. In other words, the fraction of a cooperator's mutual-cooperation (C-C) interactions with its neighbors well characterizes the level of its cooperative environment (abbreviated as CE hereafter). Cooperators can survive and even prevail when they have benign CE around. Whereas, a defector exploiting its neighboring cooperators is the victim of its own succuss. This is because that those cooperators either choose to terminate partnership with it (adaptation of social partners) or learn to defect in subsequent interactions (adjustment of strategy) in response to its defection. Therefore, an individual's CE level should be related not only to its cooperative partners but also to its strategy choice, reflecting the overall cooperativity of it and its surrounding neighbors.

In reality, there exists a CE centered on each player. Generally speaking, CE information reveals an important quality of networked players, and indicates the localization of cooperators within the network. A player's CE level should depend on the number of pairwise cooperative interactions in the neighborhood. Different individuals have different numbers of pairwise interactions in real graph-structured populations [5,6,10], thus the effects of degree heterogeneity on players' CE should be weakened. In other words, each player's CE level depends on the number of neighbors with whom it cooperates and the number of neighbors that cooperate with it. In fact, these two factors have been used to define general cooperative and defective agents in the situation where players take different actions with different neighbors [30]. But here they are used to distinguish networked players' different local cooperative environments in the situation where players take the same action with all neighbors. In addition, in view of the above points a player's CE level should be adjusted in response to the pairwise cooperative interactions in the evolutionary PDG.

Like the feature information used in previous studies [20,29], the introduced CE information can be used as a choice criterion. This is because that for partner choice individuals are not only interested in the type of the focal partner, but also interested in the partner's surrounding players', which could shape the interaction environment [27] and would indirectly influence their choices in the evolutionary PDG. This may correspond to the phenomena in human society that, for example, when a woman chooses a man to be her boyfriend or husband her selection would not only depend largely on the man's own qualifications, but also depend on his social surrounding, e.g., his family background. In general, players would always tend to form social ties with the ones in good CE. Indeed this could be a long-term good choice, since players not only directly interact with new potential partners but also easily make new potential acquaintances who are friends of friends after establishing connections [31]. However, because of limited eye reach or capacity of humans as well as only possessing local information, individuals tend to establish a connection as long as the opponent possesses better CE than themselves. They naively believe that such connection is potential to them. Moreover, due to social tolerance of humans and animals [32,33], individuals would keep the links to the neighbors with smaller CE level. But they generally have a certain tolerance threshold, and tend to cut unilaterally the links to the neighbors with worse CE level beyond the threshold. In this sense, individuals can construct or adjust their local interactions by combining with the tolerance threshold and CE information. In fact, such adjustment idea is similar to the one in Ref. [29], and thus can be justified.

Furthermore, although the individual would keep the link to the one whose smaller CE level is within tolerance threshold, there should be inhomogeneous influences on social learning between them. According to logical reasoning, the individual with higher CE level has a certain advantage of partner switching, and should have stronger influence to the one with smaller CE level. Such asymmetric and different influence could be involved into the imitation activity when the two players keep connecting with each other [34–36]. In general, the less influential individual is more inclined to learn from the other party under the rule "If you live with a lame person you will learn to limp."

Presently, we consider the above factors into the coevolutionary PDG. We assume that for each pair of individuals, if the individual's lower CE level is within the opponent's tolerance threshold, the link is maintained. Meanwhile, only the individual with smaller CE level learns from the opponent (here we consider a simple and extreme situation for strategy imitation between paired individuals). Otherwise, the individual in better CE cuts unilaterally the link, and tends to rewire to a new "suitable" partner. Here, we consider that the new tie can only be established with the mutual consent of both parties [18]. In general, in the real world individuals wish to make decisions based on the information they have. But usually they do not have the information about all others [16,20]. Therefore, if two unpaired players have each other's CE information, each player only tends to connect with the opponent whose CE level is within the tolerance threshold. In other words, they can connect to each other only when their CE levels are within each other's tolerance threshold. According to such establishment way, they natively believe that they can connect to a new satisfied partner, and do not suffer much loss. However, if two unpaired players do not possess the opponent's CE information, they will believe that they could both obtain a new potential beneficial interaction by creating the new link. Therefore, in this situation they would not reject the offer from the opponent and connect with each other.

By means of Monte Carlo simulations, we study the effects of tolerance threshold on cooperative behavior in this adaptive environment. Interestingly, we find that there is a moderate tolerance threshold resulting in the best promotion of cooperation. In the rest of this paper, we will describe in detail this computational model, and present main findings as well as corresponding explanations.

II. MODEL

Following previous works [37,38], we adopt the rescaled payoff matrix depending on one single parameter for PDG

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1+r & r \end{pmatrix} \end{array}, \tag{1}$$

where $r=c/b$ represents the ratio of cost to benefit of the altruistic behavior. Initially, each player x is designated to play either C or D with equal probability, and occupies one site of a random regular graph [39]. Let us denote by S_x the strategy of player x, where $S_x=1$ corresponds to play C, and $S_x=0$ corresponds to play D.

Using strategy S_x, player x engages in pairwise interactions with all its current adjacent neighbors, and then can collect its payoff P_x based on the payoff matrix

$$P_x = S_x k_c + (1-S_x)[k_c(1+r) + (k_x - k_c)r], \tag{2}$$

where k_x is the degree of player x, and k_c is the number of cooperators among player x's neighbors.

After interacting with its immediate neighbors, the CE level of player x needs to be assessed. Here, we calculate its CE level as the fraction of pairwise cooperative interactions in its neighborhood, that is,

$$R_x = S_x \frac{k_c}{k_x}. \tag{3}$$

In this model, each individual's CE level varies between 0 and 1. It only has local information about its nearest and next-nearest neighbors' CE. In addition, individuals have a tolerance threshold to the partners in worse CE. For simplicity, we assume that each individual has the same tolerance threshold, denoted by a parameter h $(0<h<1)$.

The updating of individuals' strategies and connections runs as follows. At each generation, we first randomly choose a link that connects a pair of players x and y. Correspondingly we can obtain P_x, P_y, R_x, and R_y. Then, players x and y will compare their current CE levels. Without loss of generality, we assume that player x's CE level is not less than player y's.

If

$$R_x - h > R_y, \tag{4}$$

player x will unilaterally dismiss the link to y, and then rewire to a new partner z. It will preferentially redirect the new link to another individual z randomly chosen from the neighbors of its neighbors (ordered rewiring), whose CE level satisfies the following condition:

$$|R_x - R_z| \leq h. \tag{5}$$

Occasionally, there are instances where the CE level of every agent among the neighbors of individual x's neighbors cannot satisfy the above condition. Where this occurs, player x will rewire the new link to another individual randomly chosen from the whole population except its nearest and next-nearest neighbors (random rewiring).

Otherwise (i.e., $R_x - h \leq R_y$), player x will keep this link to player y, and meanwhile player y will imitate player x's current strategy [34–36], with a probability depending on the payoff difference [40]. If $P_x < P_y$, no strategy imitation occurs. If $P_x \geq P_y$, the imitation probability is given by

$$W = \frac{P_x - P_y}{(r+1)k_>}, \tag{6}$$

where $k_> = \max\{k_x, k_y\}$. In this model, we propose the co-evolutionary rule mainly based on the parameter h, and in what follows we will focus on the effects of h on co-evolution of strategy and structure.

III. RESULTS

We start from a random regular graph [39], where all individuals have the same number of links and half of the total edges are randomly swapped. Initially, 50% of cooperators are randomly placed in the population with $N=10^3$ individuals. Notice that in this model the numbers of individuals and links remain unchanged during the co-evolutionary process. Hence the average degree z is conserved, and we set $z=8$ in this study. We impose that nodes connected by a single edge cannot lose this edge, and ensure that the population structure keeps connected at all times.

We implement this model with asynchronous update [41], and compute the cooperation level by averaging over the last 5×10^3 generations of the entire 5×10^6 generations. We also compute properties of emerging networks including the degree of heterogeneity, clustering coefficient, and assortativity coefficient. The degree of heterogeneity of the graph $D = N^{-1}\sum_k k^2 N_k - z^2$, where N_k gives the number of vertices with k edges. The clustering coefficient of a node i, C_i, can be computed as $C_i = 2e_i/[k_i(k_i-1)]$, where e_i denotes the actual number of edges in the subgraph induced by the neighbors of node i, and k_i is the degree of node i [42]. The mean clustering coefficient of the graph is then given by the average of C_i over all the nodes in the network. Suggested by Newman [43], the assortativity coefficient θ can be given as

$$\theta = \frac{M^{-1}\sum_l j_l k_l - \left[M^{-1}\sum_l \frac{1}{2}(j_l + k_l)\right]^2}{M^{-1}\sum_l \frac{1}{2}(j_l^2 + k_l^2) - \left[M^{-1}\sum_l \frac{1}{2}(j_l + k_l)\right]^2},$$

where j_l and k_l are the degrees of the nodes at the ends of the l edges, with $l=1,\ldots,M$ (M is the total number of edges in the population structure).

First, we study the cooperation level as a function of r for four different values of h, as shown in Fig. 1(a). We can see that when $r<0.5$ strikingly better results for ρ_c are obtained at moderate $h=0.7$. When $r>0.5$ the cooperation level cannot be greatly promoted for each value of h, but an optimal cooperation level still exhibits at moderate $h=0.7$. Moreover, when $r>0.5$ the cooperation level varying with r displays little changes, for each value of h. In order to examine the impact of h more precisely, we present the dependence of ρ_c on h for four different values of r in Fig. 1(b). We can find that ρ_c first slightly decreases as h increases gradually. But ρ_c starts to increase when h reaches around 0.5. For smaller r, then ρ_c can reach a high level at $h\approx0.65$. While for higher r, the cooperation level reaches a plateau, where ρ_c keeps at about 0.5. These results suggest that there exists a moderate tolerance threshold warranting the best promotion of cooperation.

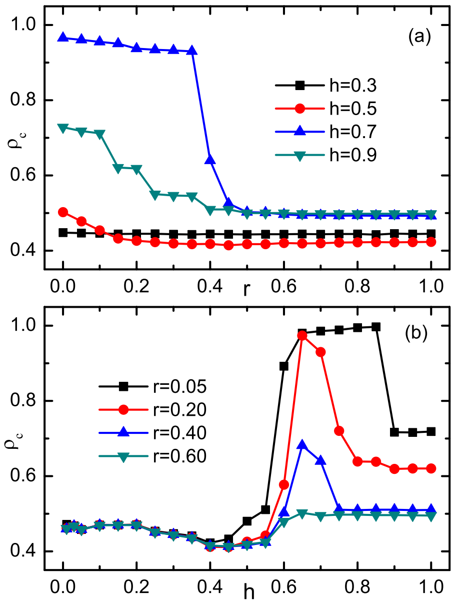

FIG. 1. (Color online) Evolution of cooperation. (a) The cooperation level as a function of r for different values of h. (b) The cooperation level as a function of h for different values of r. Here, each data point is obtained by averaging over 200 independent realizations of initial conditions.

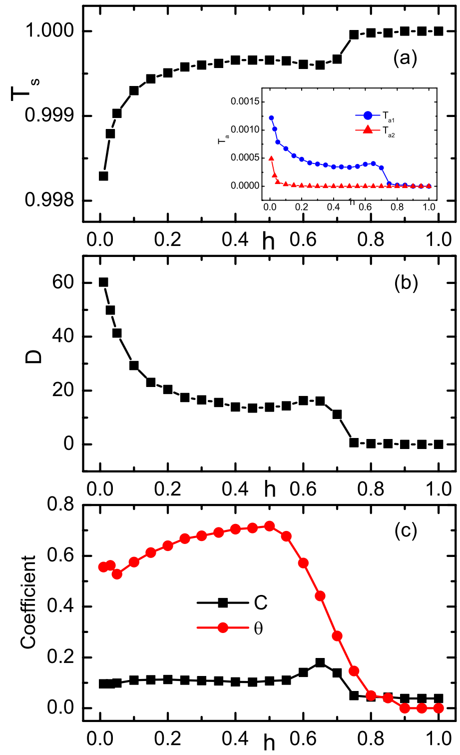

FIG. 2. (Color online) Co-evolutionary dynamics and the resulting partner network. (a) The time ratio of strategy updating during the whole co-evolutionary process as a function of h for $r=0.2$. The inset shows the corresponding time ratio of structural evolution. Here, we define T_s as the time ratio of strategy updating, and T_{a1} (T_{a2}) means the time ratio of ordered (random) rewiring. (b) The degree of heterogeneity of the graph as a function of h for $r=0.2$. (c) The clustering and assortativity coefficient as a function of h for $r=0.2$. Here, we would like to point out that the assortativity coefficient for the initial random regular graph is zero. Each data point is obtained by averaging over 200 independent realizations of initial conditions.

To gain further insights into such co-evolutionary dynamics, let us focus on how the social tolerance h affects the time scale ratio of individuals' strategy updating versus adjustment of social partners, and hence the emerging partner network topologies. In Fig. 2(a) we show the time ratios of strategy updating and structure updating during the co-evolutionary process. We can see that the system much less frequently implements structural evolution for each value of h. Furthermore, ordered rewiring is mainly implemented during the stage of structural evolution [see the inset of Fig. 2(a)]. Such ordered rewiring could be favorable for the increment of degree heterogeneity [see Fig. 2(b)], and can enhance the density of triangle, i.e., the clustering coefficient in emerging networks [13,42]. In particular, at $h=0.65$ ordered rewiring is much more frequently implemented. Correspondingly, the value of D reaches a peak, in comparison with the ones nearby [see Fig. 2(b)]. Meanwhile, the clustering coefficient reaches the maximal value at this moment [see Fig. 2(c)]. Moreover, we find that the emerging graph has positive assortativity coefficient for most h, and the assortativity coefficient reaches the maximal value around $h=0.5$ [see Fig. 2(c)]. In general, the emerging networks, built through sophisticated relationships in terms of humans' feature information [43], should have positive assortativity coefficient.

In combination with the above investigations, let us now explain the nontrivial dependence of ρ_c on h. For large h, individuals mainly carry out strategy updating [see Fig. 2(a)]. Although generally cooperators have better CE than defectors, they cannot easily wipe out defectors in the initial graph, especially for large r (as the payoff of cooperators is less than that of those periphery defectors) [39]. On the other hand, for small h strategy imitation only occurs between paired players who have very similar CE. In other words, there are mainly connected cooperators or defectors for strategy updating. Thus, the cooperation level displays little changes in this case. Combining the discussion for large and small h's, the evolution of cooperation should be different in the middle range of h.

For small intermediate h, e.g., $h=0.50$, influential cooperators cut the links to the defective partners and rewire to new partners by ordered rewiring, rather than let their defective partners learn from them. Hence cooperative action does not easily spread into defective players in the emerging graph with high assortativity coefficient [12,44–46]. Moreover, some less influential cooperators keep connecting to the defective partners. These cooperators have less potential partners, and cannot obtain high payoffs. Ultimately they are exploited by the defective partners because a negative feedback effect works on their CE levels [see Fig. 3(a)]. For large intermediate h, e.g., $h=0.70$, most CD links can be permitted to exist at first [see Fig. 3(b)]. Some cooperators have more potential partners, and can gain higher payoffs. They can easily turn some defective partners having smaller CE into cooperators in strategy dynamics. This results in a positive feedback effect on CE for the focal and surrounding players.

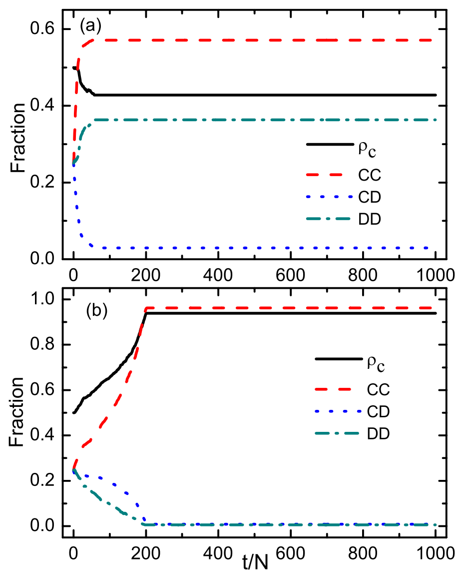

FIG. 3. (Color online) Time evolution of the cooperation level and fractions of CC/CD/DD links with $r=0.2$ and different values of h: (a) $h=0.5$ and (b) $h=0.7$. The data shown here are obtained in one realization.

As a result, the surrounding cooperators can be favored to resist the exploitation by defectors, or to seek new partners. In the latter case, the clustering coefficient of the graph can increase by ordered rewiring [see Fig. 2(c)], which plays a positive role in the survival of cooperators [13]. In addition, some cooperators may connect with more potential partners by such ordered rewiring, although there are not highly heterogeneous connections in the population structure [46,47]. Thus, being protected from the exploitation by defectors, cooperators can spread their states into defective agents by strategy updating for these intermediate values of h [see Fig. 3(b)].

Finally, we take into account some interesting and reasonable extensions after presenting main results of the original model. Similar to the definitions of trust in Ref. [18] and reputation in Ref. [20], we first consider a memory effect onto players' CE updating (memory-dependent discounting of individuals' CE level). More precisely, the CE level of player x at time t is defined as

$$R_x(t) = (1-\alpha)R_x(t-1) + \alpha S_x \frac{k_c}{k_x}, \qquad (7)$$

where each individual is initially assigned a CE level randomly chosen in the interval [0,1], and α $(0<\alpha<1)$ is a weighting factor. For $\alpha \to 0$, individuals' randomly assigned initial CE levels stay frozen. For $\alpha \to 1$, it recovers our original model. Figure 4 shows the cooperation level as a function of h for different values of α under this modified definition of the CE level. Noticeably, we see that there still exists an intermediate h, resulting in the optimal cooperation level, for each value of α. But when α is small, the cooperation level is lowered for small or large h. In fact, for small α players' CE is mainly depends on the historical behavior choices in the neighborhood, and there is a long memory effect. In other

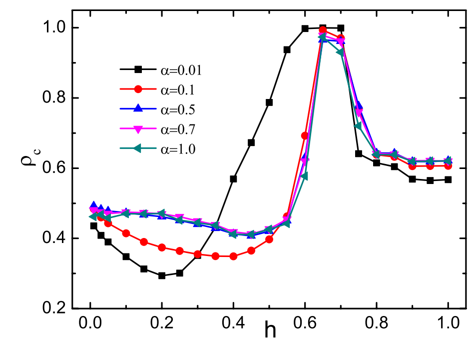

FIG. 4. (Color online) The cooperation level as a function of h for $r=0.2$ and different values of α. Each data point is obtained by averaging over 200 independent realizations of initial conditions.

words, a slow feedback mechanism is at work between individuals' current pairwise cooperative interactions and CE adjustment. Thus, not all cooperators' CE is better than defectors', in the case of initial random distribution. Some more cooperators will keep connecting with their defective partners, resulting in a worse environment for the evolution of cooperation. But the optimal intermediate h can still provide a positive effect on the enhancement of cooperation. In addition, when α is large, generally cooperators' CE is better than defectors' because of a rapid feedback effect. In this situation, the cooperation levels display very small changes for different values of α.

In what follows, we test two other modified coevolutionary rules. We notice that in the real society, individuals with little influence are generally restricted to preferentially carry out partner switching. Thus, in the modified co-evolutionary rule we consider that only the individuals whose CE level is larger than a constant threshold R_H (e.g., $R_H=0.3$) have the ability to adjust their connections. Noticeably, we find that the cooperation levels for different h increase correspondingly, and the maximal value of ρ_c can still emerge at moderate h. Under this alternative rule, some less influential cooperators are restrained to cut the link to defectors. But most defectors are inhibited to rewire to less influential cooperators, since they generally have no ability to adjust initiatively their partnerships. As a result, the exploitation by defectors will be weakened in this situation.

In addition, players tend to make full use of CE information that they possess, when choosing a candidate to rewire. Thus, for ordered rewiring we assume that the individual will rewire to the new partner who has the highest CE level among the satisfied candidates from its neighbors' neighbors. Interestingly, we find that the cooperation level does not decrease with increasing h to the optimal moderate value, and the maximal value of ρ_c increases correspondingly at the moderate optimal h. In fact, such modification helps individuals tend to connect with those players in good CE, and the impact of players in good CE on their neighborhood is enhanced [20]. As a result, the influential cooperators can easily spread their behaviors into their new partners, ultimately promoting cooperation. It is worth nothing that these altered rules are all proposed in realistic manners, and these results can further demonstrate social tolerance' nontrivial effects on cooperative behavior in the real society.

IV. DISCUSSION AND CONCLUSION

Let us further discuss the differences between our model and some relevant previous works [14–18]. In Refs. [14,15], the unsatisfied defector (the defector who has not the highest payoff among the neighbors) breaks the link to defective neighbor with probability p, and rewire with other randomly chosen agent uniformly from the network. In Ref. [16] if the player is dissatisfied with the interaction, then it competes with the partner to rewire the link. The rewiring is attempted to a random neighbor's neighbor with certain probability described by the Fermi function. In Ref. [17] the active linking rule is proposed for the network updating, which incorporates decisions of individuals when establishing new links or giving up existing links. In Ref. [18] players selectively create and/or severe ties with other players based on a myopic cost-benefit comparison. They are able to sever ties unilaterally, and new ties can only be created with the mutual consent of both parties—whereas in our model, players cut unilaterally the link to the neighbor whose lower CE level is out of tolerance threshold, and preferentially carry out ordered rewiring. If ordered rewiring fails, random rewiring is considered. Furthermore, an inhomogeneous strategy adoption occurs if the individual's lower CE level is within the opponent's tolerance threshold. Therefore, our co-evolutionary rule works in a different manner from these aforementioned ones.

It should be emphasized that our co-evolutionary rule is proposed based on players' CE information and social tolerance threshold. Players' CE depends on the strategy choices in the neighborhood, which is different from the pure strategies. Under the assessment rule, it can differentiate the players in the population. Based on the CE information, social tolerance threshold is introduced to decide to adjust connection or to update strategy for players. Furthermore, it can govern the time scales associated with strategy and structure updating in a different stochastic way from previous works, where the time scales are independent of feature information coupled with game dynamics [16,17,20]. From this perspective, our model—considering realistic spatial, social, and cognitive restraints—is somewhat complicate yet effective, and can be justified from the viewpoint of real society [31].

Recently, "tag-based cooperation" [48–51] has been suggested and established itself as an important research line in evolutionary game theory. In these models, a generic system of phenotypic (heritable) tags is used to indicate similarity between individuals. Evolutionary dynamics of cooperation is based on these tags. Cooperation is triggered only when two randomly chosen individuals have similar tags. In our model, an individual's CE can be seen as a kind of tag that is not heritable but adjustable. Here we demonstrate that cooperation can be maximized for a certain tolerance threshold. It is worth noting that these previous tag-based [48–51] studies were not investigated in the spirit of evolutionary graph theory [10], namely, the underlying topology of partner network was not well scrutinized. In this sense, our work complements these previous results by studying the evolution of cooperation on dynamic graphs. Our investigations further show that the emerging network is assortatively mixed, which well mirrors the real-life social networks [43]. Taken together, the present study is helpful to understand how tag-based cooperative dynamics shape the evolution of partnerships, and how the adjusted partner networks in turn affect the evolution of cooperation.

We believe that the parameter h can indeed characterize individuals' rationality in partner choice. Since for $h \to 0$, individuals always cut the connections to the neighbors with worse CE than themselves, and only tend to link to the neighbors with better CE than themselves; for $h \to 1$, individuals always maintain their original partnerships even the CE level is higher than the neighbors'. It is worth nothing that individuals' rationality has been considered in the Fermi function, which is usually used as a stochastic strategy updating rule [9,52–54]. Although the Fermi function has been adopted to implement structural evolution in Ref. [16], the rationality parameter is not used as a choice criterion. Here, we capture individuals' bounded rationality about choosing partners, and use it as a selection parameter in the co-evolutionary rule. Interestingly, we find that moderate rationality level can result in the optimal cooperation level, consistent with previous results [9,52–54]. The result may enrich the knowledge of rationality's effects on the evolution of cooperation in evolving networks.

In summary, we have presented a model of co-evolutionary prisoner's dilemma in combination with individuals' CE information and tolerance threshold. We have shown that moderate tolerance thresholds can result in the optimal cooperation level. Furthermore, when a memory effect is considered onto players' CE updating, we found that the evolution of cooperation depends on the weighting factor α, but there still exists a moderate tolerance threshold maximizing the cooperation level for each value of α. We also demonstrated that the nonmonotonous dependence of cooperation level on tolerance threshold displays in the two other modified co-evolutionary rules. Moreover, we compared our present model with other related ones, and pointed out that the tolerance threshold can characterize individuals' rationality in choosing partners. Our work may enhance the understanding of the evolution of cooperation in realistic systems, and provide an alternative way to study the entangled co-evolution of strategy and structure.

In future work, it would be interesting to allow individuals' tolerance thresholds to be mutable or diversely distributed, i.e., to study the effects of the diversity [9,24,55–57] of individuals' tolerance thresholds on the evolution of cooperation. Work along these lines is in progress.

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