Emergence of acoustic waves from vorticity fluctuations: Impact of non-normality

Joseph George and R. I. Sujith*
Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai 600036, India
(Received 14 April 2009; published 28 October 2009)

Chagelishvili *et al.* [Phys. Rev. Lett. **79**, 3178 (1997)] discovered a linear mechanism of acoustic wave emergence from vorticity fluctuations in shear flows. This paper illustrates how this "nonresonant" phenomenon is related to the non-normality of the operator governing the linear dynamics of disturbances in shear flows. The non-self-adjoint nature of the governing operator causes the emergent acoustic wave to interact strongly with the vorticity disturbance. Analytical expressions are obtained for the nondivergent vorticity perturbation. A discontinuity in the x component of the velocity field corresponding to the vorticity disturbance was originally identified to be the cause of acoustic wave emergence. However, a different mechanism is proposed in this paper. The correct "acoustic source" is identified and the reason for the abrupt nature of wave emergence is explained. The impact of viscous damping is also discussed.

DOI: 10.1103/PhysRevE.80.046321 PACS number(s): 47.50.Gj, 47.35.Rs

I. INTRODUCTION

A linear mechanism of acoustic wave emergence from vorticity fluctuations in an unbounded shear flow has been proposed by Chagelishvili *et al.* [1]. This paper illustrates how this phenomenon is related to the non-normality of the operator governing the linear dynamics of disturbances in shear flows. Transient growth is known to occur even in dynamical systems which are stable according to the canonical linear theory due to the non-normality of the underlying operator. The non-self-adjoint nature of an operator causes the eigenvectors to become nonorthogonal. It is well known that when the eigenvectors are not orthogonal, the resultant can show transient growth even when all the individual modes decay. This transient growth can cause the amplitudes to rise to significantly high values from where nonlinearities may cause further amplification and take the system to limit cycle oscillations. This phenomenon has been well understood in plane Couette, plane Poiseuille, and pipe Poiseuille flows [2,3], coupled-mode flutter [4], combustion instability [5,6], etc. When the operator that describes the linear dynamics of a system is non-normal, its eigenvalues are not the appropriate tools for analyzing the system as they characterize only its asymptotic behavior. The usefulness of singular value decomposition in analyzing non-normal systems has been illustrated [7,8].

An approach which has received considerable attention is the Kelvin's method wherein the problem is analyzed from a frame of reference that is fixed to the mean flow and the temporal evolution of the spatial Fourier harmonics of the disturbances is studied. In other words, the spatial inhomogeneity which arises due to shear is converted into a temporal inhomogeneity by a suitable transform. This nonmodal approach has been driving great advances in the study of evolution of acoustic and vorticity modes in shear flows [9–15], MHD waves [16,17], coupling and transformation of waves in shear and hydromagnetic flows [18,19], and stability of compressible plane Couette flow [20,21] and has helped to form a new conjuncture of transition to turbulence [22,23]. The generality of Kelvin's solutions and the possibility of extending it to systems exhibiting complex spatial inhomogeneities other than convective ones have been explained [24].

The primary objective of this paper is to demonstrate how the linear mechanism of acoustic wave emergence from vorticity disturbances is related to non-normality. The system exhibits rich physics which include interaction between acoustic waves and vorticity disturbances and transient growth by large factors even in the presence of viscosity under which condition, it shows asymptotic stability. The paper is organized as follows. In Sec. II, analytical expressions are obtained for the vorticity perturbation. It is shown that the emergent acoustic wave interacts strongly with the vorticity mode. In Sec. III, the non-normality of the nonautonomous operator obtained by performing the Lagrangian transformation is quantified by calculating the Henrici index of the operator as a function of time. The correct "acoustic source" is identified in Sec. IV. The physics of the mechanisms is explained in Sec. V. The impact of viscous damping is discussed in Sec. VI. The concluding remarks are presented in Sec. VII.

II. MATHEMATICAL FORMULATION

Consider an unbounded, inviscid, planar flow with constant density ρ_0 and constant velocity shear U_0 $(Ay, 0)$. The parameter A is assumed to be positive. Now the continuity and momentum conservation equations are linearized and separated into steady and unsteady components to obtain the following sets of equations [1]:

$$(\partial_t + Ay\partial_x)\rho' + \rho_0(\partial_x u_x + \partial_y u_y) = 0, \quad (1)$$

$$(\partial_t + Ay\partial_x)\rho_0 u_x + A\rho_0 u_y = -\partial_x p', \quad (2)$$

$$(\partial_t + Ay\partial_x)\rho_0 u_y = -\partial_y p', \quad (3)$$

$$p' = c_s^2 \rho', \quad (4)$$

*Corresponding author; sujith@iitm.ac.in

where c_s is the speed of sound. In deriving Eqs. (1)–(4), each flow quantity $q\ (r,t)$ has been assumed to consist of a steady mean flow quantity $q_0\ (r)$ and an unsteady flow quantity $q'\ (r,t)$, which represents a small perturbation about $q_0\ (r)$. The unsteady flow quantities are assumed to be small enough so that terms involving their products are negligible. The unsteady flow quantities include contributions from acoustic and vorticity disturbances. The transformation $x_1=x-Ayt$, $y_1=y$, $t_1=t$, transforms the nonhomogeneity in space to time. In the new coordinate frame, the spatial Fourier harmonic's (SFH) wave numbers depend on time [12]. A SFH of the form $\Psi(0)=\tilde{\Psi}\ [k_x,k_y(0)]\exp[ik_x x+ik_y(0)y]$ at $t=0$, where $\Psi=(v_x,v_y,\rho',p')$ evolves into

$$\Psi(t)=\tilde{\Psi}[k_x,k_y(t),t]\exp[ik_x x+ik_y(t)y], \tag{5}$$

$$k_y(t)=k_y(0)-k_x At. \tag{6}$$

From this point forward, all the quantities in this paper are in the wave-number space and for simplicity, the "tilde" sign is omitted. Substitution of Eqs. (5) and (6) into Eqs. (1)–(3) and nondimensionalization of physical quantities and parameters

$$d=i\rho'/\rho_0, \quad v=u/c_s, \quad R=A/c_s k_x, \quad \tau=c_s k_x t,$$
$$\tau^*=k_y(0)/Rk_x, \quad \beta(\tau)=k_y(\tau)/k_x=R(\tau^*-\tau) \tag{7}$$

gives the following set of ordinary differential equations in time, which can be solved numerically [1]:

$$\partial_\tau d=v_x+\beta(\tau)v_y, \tag{8}$$

$$\partial_\tau v_x=-Rv_y-d, \tag{9}$$

$$\partial_\tau v_y=-\beta(\tau)d. \tag{10}$$

The instantaneous vorticity $\Omega(\tau)$ in the z direction is given by

$$\Omega(\tau)=i[k_x v_y(\tau)-k_y(\tau)v_x(\tau)]. \tag{11}$$

Differentiating Eq. (11) with respect to τ gives

$$\partial_\tau\Omega=iRk_x\partial_\tau d. \tag{12}$$

The solution to Eq. (12) is given by $\Omega=\Omega_a+\Omega_v$, where Ω_a (the acoustic component) satisfies $\partial_\tau\Omega_a=iRk_x\partial_\tau d$ and Ω_v (the vortical component) satisfies $\partial_\tau\Omega_v=0$. Thus, Ω_v is time-invariant whereas Ω_a evolves with time as $\Omega_a(\tau)=iRk_x d(\tau)+C_1$, where C_1 is a constant of integration. The fact that the vorticity disturbance is nondivergent has been used to reach this conclusion. The explanation for the time invariance of $\Omega_v(\tau)$ is as follows. We know that vorticity mode is convected by the mean flow. Hence for an observer moving with the flow, it appears to be a frozen pattern in space. From Eq. (12), we get

$$\Omega(\tau)=iRk_x d(\tau)+C, \tag{13}$$

where C is the constant of integration. If the initial disturbance is purely vortical, then C equals $\Omega_v(0)$. The first term on the right-hand side of Eq. (13) corresponds to $\Omega_a(\tau)$, an acoustic component. The second term which is time-invariant corresponds to $\Omega_v(\tau)$, a vortical component

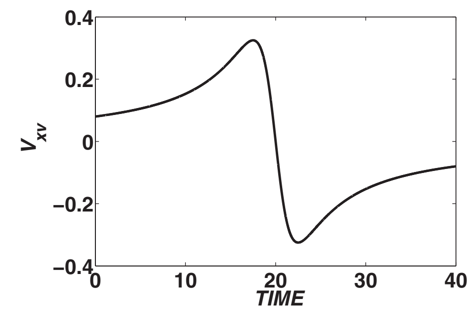

FIG. 1. Evolution of v_{xv} with τ for initial conditions of $v_{xv}(0)=0.08$, $v_{yv}(0)=-0.01$, $d(0)=0$, $v_{xa}(0)=0$, $v_{ya}(0)=0$, $R=0.4$, $\beta(0)=8$, and $\tau^*=20$.

$$\Omega_v(\tau)=\Omega_v(0)=ik_x v_{yv}(0)-ik_y(0)v_{xv}(0). \tag{14}$$

The velocity field corresponding to the vorticity mode is divergence free, which implies that

$$k_x v_{xv}(\tau)+k_y(\tau)v_{yv}(\tau)=0. \tag{15}$$

Using Eqs. (14) and (15), the velocity fluctuations corresponding to the vorticity mode can be analytically obtained as

$$v_{xv}(\tau)=[\beta(0)v_{xv}(0)-v_{yv}(0)]\{\beta(\tau)/[1+\beta(\tau)^2]\}, \tag{16}$$

$$v_{yv}(\tau)=-[1/\beta(\tau)]v_{xv}(\tau). \tag{17}$$

The time evolutions of v_{xv} and v_{yv} are shown in Figs. 1 and 2. From Eq. (16), it is clear that v_{xv} is continuous at $\tau=\tau^*$ as opposed to the observation made in [1] that v_{xv} is discontinuous. Apart from that, the analytical solution shows excellent agreement with the numerical solution presented in [1]. From Eq. (16), we also see that when τ is close to τ^*, the time evolution of v_{xv} is approximately linear with a sharp gradient which seems to have been interpreted as a discontinuity earlier [1].

The total energy of the SFH in **k** space (wave-number space) is defined as

$$E_{total}=\tfrac{1}{2}(|v_x|^2+|v_y|^2+|d|^2),$$

where $v_x=v_{xv}+v_{xa}$, $v_y=v_{yv}+v_{ya}$, $d=d_a$. The subscripts "v" and "a" represent vorticity and acoustic disturbances, respectively.

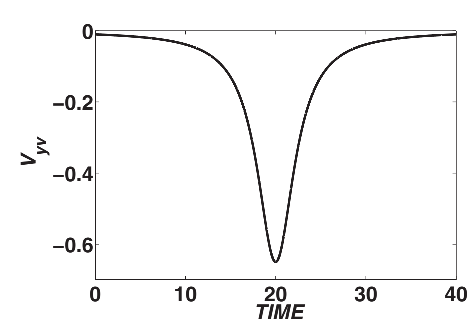

FIG. 2. Evolution of v_{yv} with τ (same initial conditions as Fig. 1).

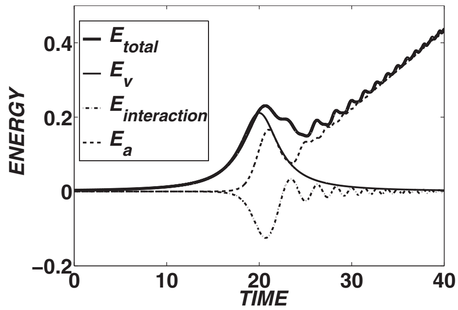

FIG. 3. Evolution of energy with time (same initial conditions as Fig. 1).

The total energy is given by $E_{total}=E_a+E_v+E_{interaction}$, where $E_a=\frac{1}{2}(v_{xa}^2+v_{ya}^2+d_a^2)$, $E_v=\frac{1}{2}(v_{xv}^2+v_{yv}^2)$, and $E_{interaction}=v_{xv}v_{xa}+v_{yv}v_{ya}$ accounts for the interaction between the acoustic and vorticity modes. If the acoustic and vorticity disturbance vectors are orthogonal, then $E_{interaction}$ equals zero. If the base flow is uniform, then the nondivergent vorticity mode and the irrotational acoustic mode are orthogonal eigenmodes of the system. However, in the presence of shear, the acoustic mode can develop rotational character [9,25]. This is also evident from Eq. (13). It is important to note that in the presence of shear, the acoustic and vorticity modes are not the eigenmodes and hence their evolution is coupled [25,26]. Once the acoustic mode develops rotational character, it is not orthogonal to the vorticity mode and hence they interact. This would mean that the total energy is not equal to the sum of the energies of the individual modes. A plot of the energies corresponding to the individual modes, interaction term, and the total energy is plotted as a function of time in Fig. 3. At the initial stages of acoustic wave emergence, the acoustic energy and the interaction term exactly cancel out each other, making no net contribution to the total energy. At later stages, the energies of the vorticity mode and the interaction term die down, making the total energy almost equal to the acoustic energy.

III. ROLE OF NON-NORMALITY

The Henrici index characterizes the amount of non-normality of an operator. The Henrici index of an operator A is defined as

$$He(A) = V(A)/\|A^2\|,$$

$$V(A) = \|AA^{\dagger} - A^{\dagger}A\|.$$

Equations (8)–(10) can be written in the matrix form as

$$\partial_\tau X = A(\tau)X,$$

where $X=[d\ \ v_x\ \ v_y]^T$,

$$A(\tau) = \begin{bmatrix} 0 & 1 & \beta(\tau) \\ -1 & 0 & -R \\ -\beta(\tau) & 0 & 0 \end{bmatrix}.$$

The Henrici index of the nonautonomous operator governing the linear dynamics of the system of interest is plotted as a function of time in Fig. 4. It can be clearly seen that the system is almost self-adjoint initially. The degree of non-normality grows, reaches a maximum value, and then decays again. The phenomenon of emergence of acoustic wave from vorticity perturbation takes place at the time when the non-normality of the governing operator hits a peak. Henrici index is a good measure of the amount of non-normality of an operator as long as the dimension of the operator does not change with time. The angle between the acoustic and vorticity vectors is plotted as a function of time in Fig. 5. It is seen that the angle is greater than $\Pi/2$ at the moment the acoustic wave emerges and it eventually decays to $\Pi/2$. This explains why the interaction is strongest near $\tau=\tau^*$ and falls to zero at large times.

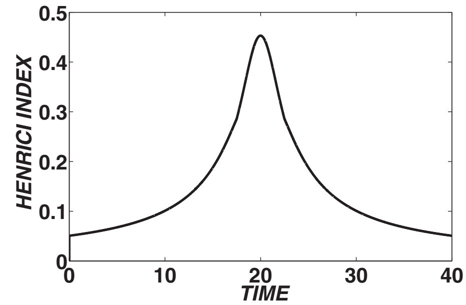

FIG. 4. Variation of Henrici index of the operator with time (same initial conditions as Fig. 1).

IV. ACOUSTIC SOURCE

The discontinuity in v_{xv} was originally identified to be the cause of emergence of acoustic waves [1]. However, from Sec. II, it is clear that v_{xv} is continuous at $\tau=\tau^*$. From Eqs. (8)–(10), we can easily derive the following second-order differential equation:

$$\partial_\tau^2 d + [1 + \beta(\tau)^2]d = -2Rv_y(\tau). \tag{18}$$

Equation (18) can be written in matrix form as

$$\partial_\tau Y = B(\tau)Y + C(\tau),$$

where $Y=[d\ \ \partial_\tau d]^T$,

$$B(\tau) = \begin{bmatrix} 0 & 1 \\ -(1+\beta(\tau)^2) & 0 \end{bmatrix},$$

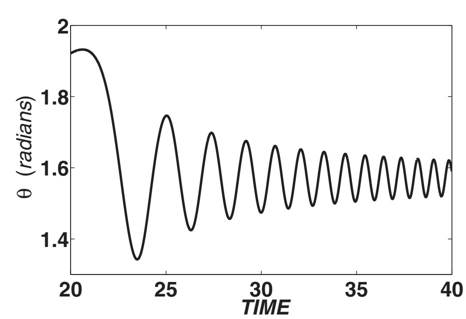

FIG. 5. Variation of the angle between acoustic and vorticity vectors with time (same initial conditions as Fig. 1).

$$C(\tau) = [0 \quad -2Rv_y(\tau)]^T.$$

It is easy to see that Eq. (18) is in the form of Lighthill's famous acoustic analogy [27], with the source terms corresponding to nonlinear velocity coupling, viscous dissipation, and entropy being neglected. However, it is important to realize that the operator ∂_τ^2 in the convected coordinate system is equivalent to the operator $(\partial_\tau + Ay\partial_x)^2$ in the laboratory frame and hence Eq. (18) is actually a convective form of Lighthill's equation. Note that the second term on the left-hand side of Eq. (18) represents the Laplacian operator multiplied by $-1/k_x^2$. The source term is readily identified as arising due to interaction of the velocity fluctuations with the mean shear. For $\tau < \tau^*$, the right-hand side becomes $-2Rv_{yv}(\tau)$ for which analytical expression is known.

Equation (18) can be solved numerically to verify the abrupt nature of acoustic wave emergence. The characteristic frequencies of $B(\tau)$ are $i[1+\beta(\tau)^2]^{1/2}$ and $-i[1+\beta(\tau)^2]^{1/2}$ which represent the "slow"' and "fast" acoustic modes, respectively, in the transformed coordinate system. As τ approaches τ^*, the characteristic frequencies of $B(\tau)$ become closer and closer to the forcing frequency which is zero (vorticity driving is nonperiodic). From Fig. 2, it can be inferred that as τ approaches τ^*, the source term becomes stronger and stronger, hitting a peak at $\tau = \tau^*$. Also it is well known that when an operator is non-normal, there can be a large response even when the forcing frequency is far away from the characteristic frequencies. Thus, the non-normal nature of $B(\tau)$ also plays a crucial role. All these factors are collectively responsible for the emergence of density fluctuations at around $\tau = 16$. Equation (18) also explains why the phenomenon is not "strongly noticeable" at very low values of the shear parameter R [1,28]. However, it is important to note that for $\tau > \tau^*$, the right-hand side becomes $-2R[v_{yv}(\tau) + v_{ya}(\tau)]$ and the acoustic velocity fluctuations modulate its own density fluctuations by interacting with the mean shear. This is the crucial mechanism which makes the density fluctuations grow even after the vorticity "driving" dies down.

V. PHYSICS OF THE MECHANISMS

First, we consider the physics of exchange of energy between the mean flow and "pure" acoustic and vorticity modes. A pure vorticity mode being nondivergent should have its velocity fluctuation vector (V_v) perpendicular to the wave number vector (K). We consider the case $k_y(\tau)/k_x > 0$. A virtual fluid particle at a distance y from the x axis has a total velocity of $U = Ay\boldsymbol{e_x} + V_v(\tau)$ at time τ. After a small interval of time $\delta\tau$, the particle drifts up to a height of $y + \delta y$ because of $V_v(\tau)$, making the total velocity of the particle $U(\tau+\delta\tau) = A(y+\delta y)\boldsymbol{e_x} + V_v(\tau+\delta\tau)$. From Eq. (8), it is clear that the vorticity mode being nondivergent cannot "directly" contribute to the potential energy which manifests itself in the form of density fluctuations, though it does contribute to the potential energy indirectly by driving the acoustic disturbance. Since the fluid particle is not "compressed," its total velocity remains constant (conservation of momentum) and hence $V_v(\tau+\delta\tau) = V_v(\tau) - A\,\delta y\boldsymbol{e_x}$. It is easy to visualize that if $k_y(\tau)/k_x > 0$, the angle between $V_v(\tau)$ and $-A\,\delta y\boldsymbol{e_x}$ is less than $\Pi/2$ and hence $|V_v(\tau+\delta\tau)| > |V_v(\tau)|$; the vorticity mode gains energy from the mean flow. By similar arguments, it can be shown that the vorticity mode gives energy to the mean flow if $k_y(\tau)/k_x < 0$. This can be confirmed analytically. From Eqs. (16) and (17), it can be shown that $\partial_\tau E_v = (\Omega_v(0)/k_x)^2\{R\beta(\tau)/[1+\beta(\tau)^2]^2\}$, where R is a positive constant. Thus, E_v grows for $\beta(\tau) > 0$ and decays for $\beta(\tau) < 0$. This explains the transient growth followed by decay of the energy of the vorticity mode as seen in Fig. 3. Careful examination reveals that the above explanation is closely related to the well-known "lift-up mechanism" [29,30].

A pure acoustic mode being irrotational should have its velocity fluctuation vector (V_a) parallel to the wave-number vector (K). Consider a fluid particle which drifts in the positive y direction under the influence of V_a. The particle undergoes two processes, namely, the kinematic process in which the particle gains or loses kinetic energy from the mean flow and the interaction process in which the kinetic energy is converted into potential energy. To gain a better understanding of the physics of exchange of energy between the mean flow and the acoustic mode, we dismember the above two processes in time [9]. In other words, we assume that the processes happen one after the other. In reality, the two processes occur simultaneously and the temporal dismembering is done only to gain a better understanding of the phenomena of exchange of energy. Proceeding in exactly the same way as mentioned above, we can seen that if $k_y(\tau)/k_x > 0$, the angle between $V_a(\tau)$ and $-A\,\delta y\boldsymbol{e_x}$ is greater than $\Pi/2$ and hence $|V_a(\tau+\delta\tau)| < |V_a(\tau)|$; the acoustic mode gives energy to the mean flow. It can also be shown that the acoustic mode gains energy from the mean flow if $k_y(\tau)/k_x < 0$.

The above conclusions can be arrived at mathematically as follows. From Eqs. (8)–(10), it is easy to show that $\partial_\tau E = -Rv_x v_y$. Now a "divergence-free" constraint will cause v_x and v_y to be negatively correlated in the regime $\beta(\tau) > 0$ and positively correlated in the regime $\beta(\tau) < 0$, which would mean that the disturbance energy grows if $\beta(\tau) > 0$ and decays if $\beta(\tau) < 0$. On the other hand, an "irrotational" constraint will cause v_x and v_y to be positively correlated in the regime $\beta(\tau) > 0$ and negatively correlated in the regime $\beta(\tau) < 0$.

From Fig. 3, we see that the vorticity mode behaves exactly as predicted using physical arguments. The acoustic energy shows a transient growth, followed by a momentary dip, and then continues to grow. The emergent acoustic mode has a rotational component and a divergent component. The velocity fluctuation vector V_a' will be at an angle to K in this case. The rotational component behaves like the vorticity mode and hence gains energy from the mean flow if $\beta(\tau) > 0$ and gives back energy to the mean flow if $\beta(\tau) < 0$. From Eq. (18), it is clear that the vorticity mode "drives" the acoustic mode. Thus, when the vorticity mode gains energy from the mean flow $[\beta(\tau) > 0]$, the driving becomes stronger and it becomes weaker when the vorticity mode starts dying down $[\beta(\tau) < 0]$. These two factors are responsible for the transient growth followed by the momentary dip of the acoustic energy near $\tau = \tau^*$. In the regime $\beta(\tau) < 0$, the divergent component gains energy from the mean flow and this eventually causes the total acoustic energy to grow. At large times, the rotational component dies down and V'_a becomes parallel to K or orthogonal to V_v as is seen from Fig. 5. This causes $E_{interaction}$ to go to zero at large times as seen in Fig. 3.

The fact that the acoustic disturbance assumes rotational character is evident from Eq. (13). For understanding how an acoustic disturbance becomes rotational, we consider an irrotational initial disturbance. This would mean that V' is parallel to K. Now from Eq. (6), we understand that the wave-number vector K tilts in the direction of the shear as τ increases. Also, the additional velocity gained by the perturbation by virtue of momentum conservation $(-A\,\delta y \mathbf{e_x})$ has a component perpendicular to K at all times except at $\tau=\tau^*$ when K is aligned along the x axis. Both these factors can cause V' and K to be misaligned to each other. In other words, they cause an originally irrotational perturbation to assume rotational character. Equation (12) can be rewritten as

$$\partial_\tau \Omega = iRk_x[v_x + \beta(\tau) v_y]. \tag{19}$$

The first term on the right-hand side of Eq. (19) arises because of the rotation of K and the second term arises due to the additional velocity gained by the disturbance $(-A\,\delta y \mathbf{e_x})$ not being parallel to K.

VI. IMPACT OF VISCOUS DAMPING

Proceeding in the same way as in Sec. II and including the effects of viscous damping, the governing equations can be written as

$$\partial_\tau d = v_x + \beta(\tau) v_y, \tag{20}$$

$$\partial_\tau v_x = - R v_y - d - 1/(3Re)\{[4 + 3\beta(\tau)^2] v_x + \beta(\tau) v_y\}, \tag{21}$$

$$\partial_\tau v_y = - \beta(\tau) d - 1/(3Re)\{\beta(\tau) v_x + [3 + 4\beta(\tau)^2] v_y\}, \tag{22}$$

where $Re = c_s / k_x \nu$ is a characteristic Reynolds number for the problem. From Eqs. (21) and (22), it can be shown that

$$\partial_\tau \Omega_v = - 1/Re[1 + \beta(\tau)^2]\Omega_v, \tag{23}$$

$$\Omega_v(\tau) = \Omega_v(0)\exp(- 1/Re\{\tau + R^2/3[(\tau^*)^3 - (\tau^* - \tau)^3]\}). \tag{24}$$

Using Eqs. (15) and (24), v_{xv} and v_{yv} can be analytically obtained as

$$v_{xv}(\tau) = [\beta(0) v_{xv}(0) - v_{yv}(0)]\{\beta(\tau)/[1 + \beta(\tau)^2]\} \times \exp(- 1/Re\{\tau + R^2/3[(\tau^*)^3 - (\tau^* - \tau)^3]\}), \tag{25}$$

$$v_{yv}(\tau) = - [1/\beta(\tau)] v_{xv}(\tau). \tag{26}$$

The total wave number of the disturbances increases with time, which would imply a higher rate of viscous damping. Hence, the inviscid growth cannot be sustained as $\tau \to \infty$ [21]. The effect of viscous damping on the transient and long-term behaviors is investigated.

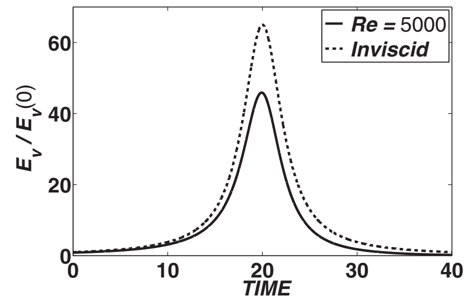

FIG. 6. Evolution of energy of vorticity disturbance with time for the inviscid and viscous cases (same initial conditions as Fig. 1).

The time evolution of the energy of the vorticity mode, which reflects the transient behavior of the system is shown in Fig. 6 for inviscid ($Re \to \infty$) and viscous ($Re = 5000$) cases for the same initial conditions as Fig. 1. As expected, we see a fall in the peak value of energy amplification and a faster decay to zero at large times in the viscous case. From Eqs. (20)–(22), it can be shown that $\partial_\tau E = -Rv_x v_y - 1/(3Re)\{[4 + 3\beta(\tau)^2] v_x^2 + [3 + 4\beta(\tau)^2] v_y^2 + 2\beta(\tau) v_x v_y\}$ from which it is evident that for a finite value of Re, the rate of energy dissipation due to viscous effects would overcome the rate at which the disturbance gains energy from the mean flow at large times.

To get a better understanding of the asymptotic behavior of the system, we make the following simplifying assumptions. From Sec. V, we know that at large times, the rotational components of the disturbance will have died down and hence $v_y \sim \beta(\tau)\, v_x$. Based on this assumption, it can be easily shown that the total energy of the disturbance decays for all $\tau > \tau_c$, where τ_c is the solution of $R\beta(\tau) + 4/(3Re)[1 + \beta(\tau)^2]^2 = 0$. Further, using the fact that $\beta(\tau)^2 \gg 1$ (at large times), τ_c can be obtained as $\tau_c = \tau^* + (3Re/4R^2)^{1/3}$. From the expression for τ_c, we can conclude that it remains finite for all finite values of Re. Thus the disturbance energy eventually decays to zero for all values of Re, except in the inviscid case when $Re \to \infty$. Also for the inviscid case, $\tau_c \to \infty$ indicating asymptotic instability.

The time evolution of the total energy, normalized with the energy of the initial disturbance, is shown in Fig. 7 for the viscous case ($Re = 5000$). As expected, we see that the energy of the disturbance does not increase indefinitely, but eventually decays to zero under the influence of the accelerated viscous dissipation rate caused by the increasing total

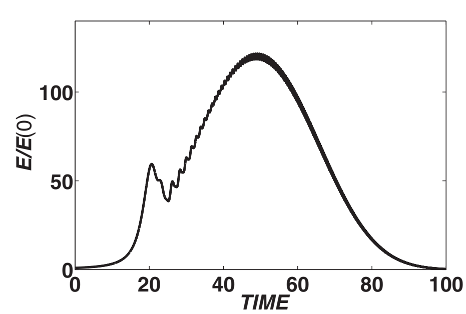

FIG. 7. Evolution of total energy with time for $Re = 5000$ (same initial conditions as Fig. 1).

wave number. The momentary dip in the total energy occurs when the vorticity mode starts giving back energy to the mean flow. Now, the acoustic mode starts gaining energy from the mean flow. However, the rate of loss of energy due to viscous effects becomes greater than the rate at which the acoustic mode gains energy from the mean flow eventually. Thus, under the influence of viscous damping, the system shows linear stability. However, the initial energy of the perturbation is amplified by large factors, at which point nonlinearities could play a significant role in causing further instability. For the problem of interest, the optimal growth of energy density has been calculated [21]. There have been some recent attempts to study the nonlinear effects [31,32]. The impact of nonlinearities in the light of this paper will be presented in a forthcoming paper.

VII. CONCLUSIONS

The impact of non-normality on the "nonresonant" phenomenon of emergence of acoustic waves from aperiodic vorticity disturbances has been studied. It is found that the phenomenon occurs when the non-self-adjoint nature of the underlying operator hits a peak. The acoustic waves develop rotational character under the influence of shear and hence they interact with the vorticity perturbation. The relevant acoustic source has been identified and the abrupt nature of the phenomenon is quantified. The time evolution of the energies corresponding to the individual modes of disturbances and an additional term arising due to their interaction have been plotted and their behavior explained through physical arguments. The inviscid growth is not sustained under the influence of viscous dissipation and the disturbance energy decays to zero at large times. Transient growth by large factors is observed due to non-normality of the governing operator which could "trigger" nonlinear instabilities.

ACKNOWLEDGMENTS

The authors wish to thank Professor P. J. Schmid (École Polytechnique) and Professor Mathew Juniper (Cambridge University) for their valuable comments and suggestions.

[1] G. D. Chagelishvili, A. G. Tevzadze, G. Bodo, and S. S. Moiseev, Phys. Rev. Lett. **79**, 3178 (1997).
[2] S. C. Reddy, P. J. Schmid, and D. S. Henningson, SIAM J. Appl. Math. **53**, 15 (1993).
[3] T. Kato, *Perturbation Theory for Linear Operators* (Springer-Verlag, New York, 1976).
[4] P. J. Schmid and E. de Langre, ASME J. Appl. Mech. **70**, 894 (2003).
[5] K. Balasubramanian and R. I. Sujith, Phys. Fluids **20**, 044103 (2008).
[6] K. Balasubramanian and R. I. Sujith, J. Fluid Mech. **594**, 29 (2008).
[7] B. F. Farrell and P. J. Ioannou, J. Atmos. Sci. **53**, 2025 (1996).
[8] B. F. Farrell and P. J. Ioannou, J. Atmos. Sci. **53**, 2041 (1996).
[9] G. D. Chagelishvili, A. D. Rogava, G. R. Khujadze, and J. G. Lominadze, Phys. Fluids **9**, 1955 (1997).
[10] S. C. Reddy and D. S. Henningson, J. Fluid Mech. **252**, 209 (1993).
[11] A. D. D. Craik and W. O. Criminale, Proc. R. Soc. London, Ser. A **406**, 13 (1986).
[12] W. O. Criminale and P. G. Drazin, Stud. Appl. Math. **83**, 123 (1990).
[13] P. Marcus and W. H. Press, J. Fluid Mech. **79**, 525 (1977).
[14] L. H. Gustavsson, J. Fluid Mech. **224**, 241 (1991).
[15] K. M. Butler and B. F. Farrell, Phys. Fluids A **4**, 1637 (1992).
[16] G. D. Chagelishvili, R. G. Chanishvili, T. S. Khristov, and J. G. Lominadze, Phys. Rev. E **47**, 366 (1993).
[17] G. D. Chagelishvili, R. G. Chanishvili, J. G. Lominadze, and A. G. Tevzadze, Phys. Plasmas **4**, 259 (1997).
[18] G. D. Chagelishvili, A. D. Rogava, and D. G. Tsiklauri, Phys. Rev. E **53**, 6028 (1996).
[19] A. D. Rogava and S. M. Mahajan, Phys. Rev. E **55**, 1185 (1997).
[20] G. D. Chagelishvili, A. D. Rogava, and I. N. Segal, Phys. Rev. E **50**, R4283 (1994).
[21] B. F. Farrell and P. J. Ioannou, Phys. Fluids **12**, 3021 (2000).
[22] T. Gebhardt and S. Grossmann, Phys. Rev. E **50**, 3705 (1994).
[23] J. S. Baggett, T. A. Driscoll, and L. N. Trefethen, Phys. Fluids **7**, 833 (1995).
[24] F. Volponi and Z. Yoshida, J. Phys. Soc. Jpn. **71**, 1870 (2002).
[25] L. M. B. C. Campos, J. M. G. S. Oliveira, and M. H. Kobayashi, J. Sound Vib. **219**, 739 (1999).
[26] V. V. Golubev and H. M. Atassi, J. Sound Vib. **198**, 601 (1996).
[27] M. J. Lighthill, Proc. R. Soc. London, Ser. A **211**, 564 (1952).
[28] Y. Auregan, A. Maurel, V. Pagneux, and J. F. Pinton, *Sound-Flow Interactions* (Springer-Verlag, New York, 2002).
[29] M. T. Landahl, SIAM J. Appl. Math. **28**, 735 (1975).
[30] P. J. Schmid and D. S. Henningson, *Stability and Transition in Shear Flows* (Springer-Verlag, New York, 2001).
[31] J. Vanneste, P. J. Morrison, and T. Warn, Phys. Fluids **10**, 1398 (1998).
[32] F. Rincon, Phys. Fluids **19**, 074105 (2007).