Nonperiodic echoes from quantum mushroom-billiard hats

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Nonperiodic tunable quantum echoes have been observed in experiments with an open microwave billiard whose geometry under certain conditions provides Fibonacci-like sequences of classical delay times. These sequences combined with the reflection at the opening induced by the wave character of the experiment and the size of the opening allow to shape quantum pulses. The pulses are obtained by response of an integrable scattering system.

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Wave mechanics in billiards can be implemented by flat microwave cavities [1–4]. They model single particle quantum aspects of mesoscopic structures, e.g., two-dimensional electron gases in quantum dots or more complicated systems [5–8]. Recently billiard systems with large openings have attracted attention [9–14]. In [15] properties of the classical dynamics of open mushroom billiards were investigated. Mushroom billiards, which were proposed by Bunimovich [16] recently, consist of one or more circular or elliptic hats which are attached to stems composed of straight walls. Their phase space has the particular property that the chaotic and the regular areas are sharply separated with no fractal structure in the border region. The mushroom billiards considered in [15] consist of a circular hat and a triangular stem. Trajectories of particles passing the stem belong to the chaotic part of phase space, whereas the hat comprises chaotic and regular trajectories [16–18]. In [15] we were interested in the number of bounces a particle entering the hat of the mushroom experiences before exiting back into the stem. Note that the starting conditions for these trajectories all belong to the chaotic part of the phase space. The result of these studies of the classical dynamics was that for a fixed angular momentum a selective number of bounces, in fact a total of three, is possible. Only trajectories of particles, which are scattered from inside the stem immediately into the hat are considered and followed only until they reenter the stem. Thus, as these particles never touch the boundary of the stem the results presented in [15] are independent of its geometry. Indeed it was shown there, that the selectivity persists when considering the hat as an open scattering system with the opening obtained by removing the stem. Then one observes for a fixed angular momentum only three different delay times, i.e., times a particle sent into the hat spends there before exiting it. These observations lead to the question how far this selectivity influences the pulse structure of the corresponding open quantum billiard. Here we report on measurements of the time resolved response of such a scattering system to incident waves and the detection of aperiodic and selective pulse sequences. In the experiments a pulse is sent into an open mushroom-billiard hat from the outside and the pulses sent back to the exterior are recorded. The sensitive dependence of the corresponding classical response on the size of the opening can be used as a guideline to design desirable pulse sequences in the wave domain. This is in stark contrast to systems whose classical or ray dynamics is mixed with no sharply divided phase space or chaotic. In the former case periodic echo signals were seen [19] and theoretically understood [20,21] for the short time behavior, in the latter a noisy response is expected. The experiment described here was performed for a quantum billiard with the shape of a quarter circle, but the flexibility of design can be enhanced, e.g., by deforming the circle to an ellipse [15]. Keeping in mind the analogy to open nanostructures, e.g., for two-dimensional electron gases [22–24] this paves the way to convert a simple quantum pulse into a complicated nonperiodic pulse sequence and thus to obtain a tunable quantum pulse generator by a simple scattering mechanism.

We recall briefly that billiards, which are paradigmatic dynamical systems [25,26], are two-dimensional domains with free motion except for specular reflections at the walls. The corresponding quantum systems are determined by the time independent Schrödinger equation with Dirichlet boundary conditions at the walls. For the experimental investigation of such quantum billiards we exploit the equivalence of the related Schrödinger equation and of the Helmholtz equation for the electric field strength in a flat, cylindrical microwave resonator of corresponding shape below the frequency, where the first transversal electric mode is excited [27,28]. Up to this frequency the electric field strength is perpendicular to the top and bottom plate of the resonator and the Helmholtz equation is scalar. Such a microwave resonator consists of two parallel plates and a third plate with a hole of the shape of the billiard squeezed in between. Specifically, the hole has the form of a quarter circular boundary with radius $R=240$ mm such that the resonator is open along one straight line of the quarter circle. Note that this geometry is a realization of a desymmetrized open mushroom-billiard with circular hat [29,30]. The size r of the hole can be adjusted by a bar to yield the shape indicated in Fig. 1. The

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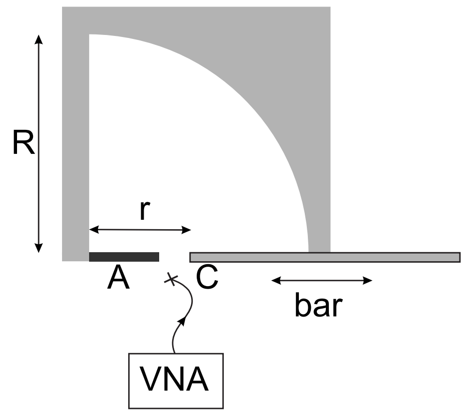

FIG. 1. Sketch of the experimental setup (top view). The inner white part of the light gray area indicates the quarter circle shape of the flat microwave resonator. The radius is R and the size of the adjustable opening is r. An antenna ($\times$) in front of the opening near point C (edge of the bar) couples the microwave signal in and out and is attached to a VNA. The dark gray bar A indicates microwave absorber material inserted into a part of the opening of the cavity.

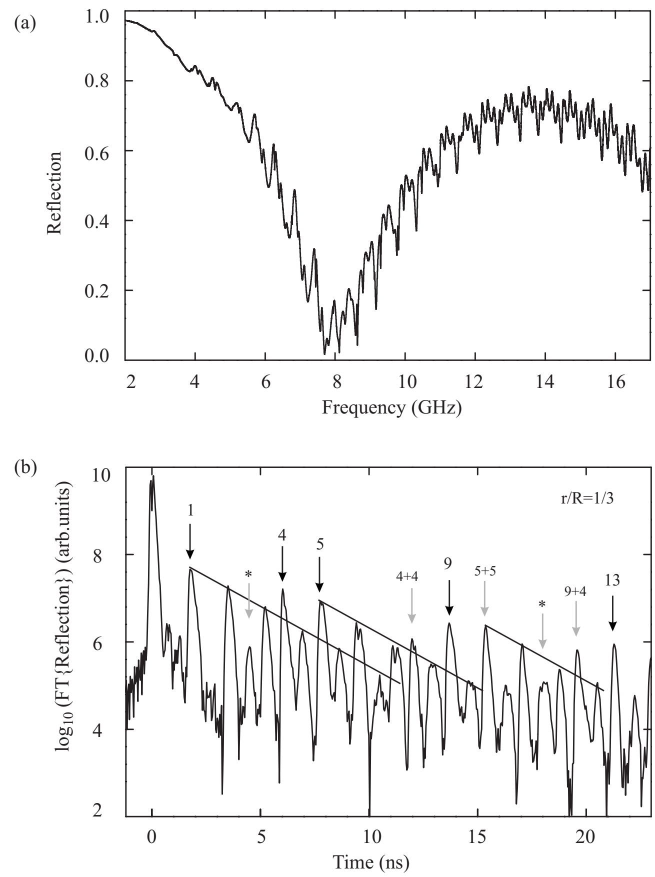

FIG. 2. Upper panel: reflection spectrum at the antenna shown in Fig. 1 (linear scale). Lower panel: Fourier transform of the spectrum in semilog-scale. The black arrows mark times at which classical echoes occur, whereas gray arrows mark times of trajectories facilitated by quantum effects. Bounce numbers corresponding to these times are also shown. Exponentially decaying peak sequences of equal spacings are connected by straight lines. Some peaks correspond to a combination of bounce numbers (e.g., 4+4) or more complex systematics (*).

separation of the parallel plates is 5 mm. Microwave power was emitted into the resonator and received by the same dipole antenna. A vector network analyzer (VNA) provided the rf signal for frequencies between 1–17 GHz well within the limit for the validity of the scalar Helmholtz equation, the excitation frequency being increased with a step size of 50 kHz. The VNA measured the ratio of the received to the emitted microwave power and the relative phase thus yielding the complex scattering matrix elements for the scattering of electromagnetic waves from the antenna into the resonator and back to it. The antenna was placed perpendicular to the billiard plates in 7 mm distance from the opening outside the cavity and 15 mm from corner C of the bar, cf. Fig. 1. We placed microwave absorbing material A along the opening of the billiard up to a distance of 15 mm to the antenna thereby damping multiple wave reflections at the opening. The measurements were performed for different opening parameters r/R of the billiard.

The overall shape of the reflection spectra shows a minimum close to 8 GHz due to the emission characteristics of the antenna, cf. upper panel of Fig. 2. The scattering information we are interested in is contained in the fine structure imprinted on this overall shape. A Fourier transform of the entire spectrum yields the response to a short pulse in the time domain. The modulus square of the signal obtained in this way decays as a power law with decay exponent $\gamma \simeq 1.95$. This value is very close to the predicted one [31] of 2.0 for classical particles which escape from a billiard with integrable dynamics. However, here we are interested in the short time characteristics of the system, which deviates from this behavior. The lower panel of Fig. 2 shows the time response of the open quarter circle with $r/R=1/3$ for short times. An aperiodic sequence of peaks is clearly visible, and their strengths decay on average with increasing time. The peak seen near time 0 is related to the smooth frequency dependence of the emission characteristics of the antenna observed as a broad dip in Fig. 2.

As mentioned above it was shown in [15] that in the corresponding classical scattering system a particle injected from the outside into the quarter circle billiard encounters only a certain number of reflections on the circular boundary (shortly called bounces) from a scarce set of possible numbers before it leaves it. For the opening ratio $r/R=1/3$ the sequence of allowed bounce numbers n is 1,4,5,9,14,23,37,51…. It proceeds up to the number 37 such as a generalized Fibonacci series, i.e., each number is given by the sum of the two previous ones. For larger n the sequence is no longer Fibonacci-like but still each occurring number is the sum of two smaller ones. When analyzed in detail [15] one finds that this sum rule is strictly obeyed within finite intervals of angular momentum values, i.e., of the impact parameter with respect to the center of the circle. Each interval is bordered by two singularities of diverging bounce numbers resulting from the existence of parabolic manifolds [17]. This behavior can be explained in terms of number theoretical properties of the circle map [32].

For an interpretation of the peaks in the Fourier transformed experimental spectra in terms of the bounce numbers n of the classical scattering dynamics these need to be converted into time delays. Using conservation of the angular momentum, respectively the impact parameter b, the time T a particle needs for n reflections at the circular boundary is given as

$$T = 2Rn\sqrt{1-(b/R)^2}/v. \qquad (1)$$

Here v denotes the velocity of the particle. Note that the possible values for b are restricted by the size of the opening, and typically $b \approx 0$ for $n=1$ and $b \approx r$ for large bounce numbers. Applying this formula (with $v=c$ the speed of light and of microwave propagation) to the sequence of bounce numbers given above we obtain classical predictions for the possible delay times, i.e., the positions of the echoes for the opening ratio $r/R=1/3$. We mark these times with black arrows identified by the corresponding bounce numbers n in the lower panel of Fig. 2, and indeed find that they coincide with the dominant peaks of the measured response, i.e., in a microwave experiment mimicking an open quantum billiard of corresponding shape we detected scattering echoes at the predicted times.

However, many additional peaks appear although the predicted peaks protrude above the average decay of the time response. The smaller peaks marked by stars are related to multiple reflections caused by diffraction at the edges of the opening. The peaks corresponding to 1 and 5 bounces at the circular boundary as well as the one denoted by 5+5 are followed by an exponentially decaying sequence of peaks with constant spacing, equal in all three cases. To guide the eye, the exponential decay is indicated by the straight lines in the lower panel of Fig. 2. We see that also the decay rates coincide. The sequences are caused by waves that hit the corners of the opening and there get partially scattered into the 1-bounce orbit. Both escape and reflection of the 1-bounce orbit at the opening may happen repeatedly, leading to the exponentially decaying sequence of peaks. The purpose of the absorbing materials covering part of the opening is to suppress 1-bounce reflections. Experimental setups without the absorber material and with the antenna nearer to the center showed dominance of trivial 1-bounce peaks and many secondary peaks for large openings. Other peaks can be attributed to a wave impinging on the opening and scattered into classical orbits which bounce more than once at the circular boundary before they hit the opening again. These peaks are marked by gray arrows and are labeled by $n+m$ for the scattering of an n bounce orbit into an m bounce orbit. Effects caused by the coupling to regular modes of waves re-entering the cavity due to diffraction at the corners of the opening were investigated in detail in [33], of refracted fields re-entering a dielectric cavity with the shape of a mushroom in [34].

With the argumentation given above we understand the most prominent peaks of the response function, but we may ask why we do not see the exponential decay after the 4 and the 9-bounce peaks. The reason is that the sequence of peaks following the 4 (9) bounce peak coincides with that of the 5 (5+5) bounce peaks within the peak width determined by the spectral range and are thus superimposed and then decay, as mentioned above, exponentially in a sequence of repeated single bounces. A sharp eye might detect the double peaks in

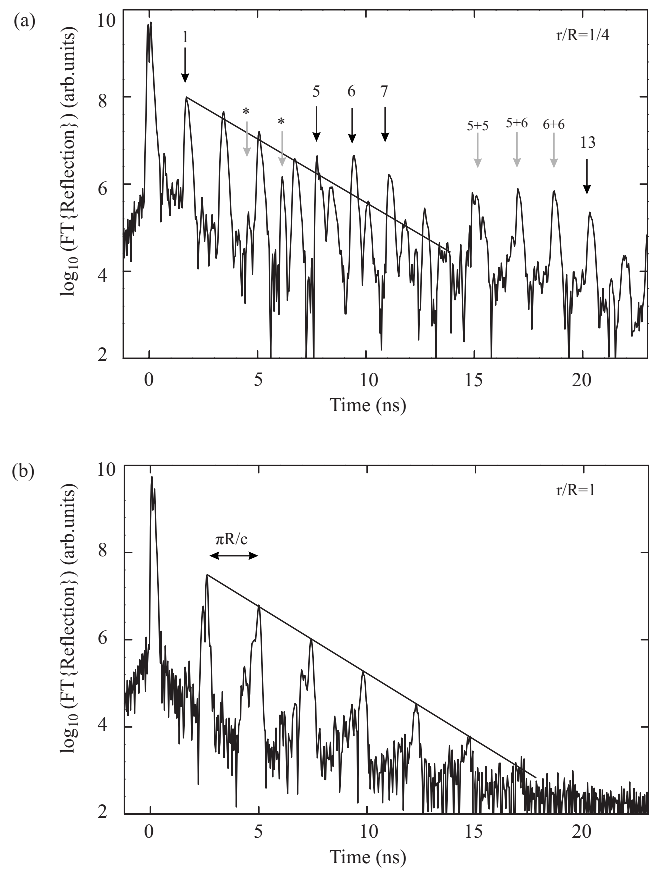

FIG. 3. Same as the lower panel of Fig. 2 for opening sizes $r/R=1/4$ (upper panel) and $r/R=1$ (lower panel), where we find whispering gallery dynamics. The time between two peaks is $\pi R/c$.

these sequences, but this effect is at the limit of our resolution. Other smaller peaks, e.g., those labeled by the stars, admit more complicated assignments but are still understood in terms of diffractive orbits. Summarizing the analysis of Fig. 2 we conclude that the pulse structure is essentially determined by the possible classical escape times (depending on the hole size), by diffractions at the opening (depending on the usage of microwave absorbing material) and by the coupling strength of the waves to the internal states corresponding to classical structures (depending on the antenna position). Modifying the experimental setup and thereby changing any or all of the three determining factors we can drastically change the output pulse sequence.

To support the validity of these conclusions we performed measurements for different opening ratios r/R, that is allowed classical escape times. In all cases the interpretation as given above explains the detected echoes or pulse sequences. In the upper panel of Fig. 3 we show the time response for $r/R=1/4$. Note that the Fibonacci-like behavior is not as prominent as in the former case, as already for $n=7$ the number of bounces does not equal the sum of the previous two possible bounce numbers, but that of the first and the third one. However, the sequence starting with $n=1$ is characterized by a fixed period and the superposition of several of these sequences again leads to a highly aperiodic pulse signal. Finally we see in the lower panel of Fig. 3 that in the geometry of the quarter circle billiard periodic echoes can also be realized, though they might be of less interest. This is feasible for $r/R=1$, i.e., for the fully opened billiard. In this case the wave travels along high-order polygonal orbits which follow the circular boundary closely, known as whispering gallery orbits [35]. The time between two successive echoes is the time needed to travel forth and back along the circular boundary at the speed of light.

Previous work [19] has shown, that the time structure of quantum signals will reflect certain classical properties even if the experiment is carried out far from the classical limit. Yet known examples lead to periodic or near periodic response. Our experiment provides an example for a pulse response in terms of aperiodic echoes by wave scattering off an integrable system. Deviations from the time structure of the classical problem are mainly due to diffraction at the opening. Moreover, in our microwave experiment the quasi-two-dimensional interior of the billiard is coupled to the three-dimensional free space. Such effects are well understood and can be controlled. Though microwave billiards as model systems neglect the many body character of a quantum dot as well as charges and spins of the electrons, the detected aperiodicity is expected to be visible also in ballistic scattering experiments on the nanoscale. As the time resolved treatment of transport through quantum dots was extremely successful in the last decade both theoretically [24,36,37] and experimentally [38–41], the future development of quantum pulse generators, providing a large diversity of pulse responses, seems feasible.

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[1] H.-J. Stöckmann and J. Stein, Phys. Rev. Lett. **64**, 2215 (1990).
[2] E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990).
[3] S. Sridhar, Phys. Rev. Lett. **67**, 785 (1991).
[4] H.-D. Gräf, H. L. Harney, H. Lengeler, C. H. Lewenkopf, C. Rangacharyulu, A. Richter, P. Schardt, and H. A. Weidenmüller, Phys. Rev. Lett. **69**, 1296 (1992).
[5] C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506 (1992).
[6] M. F. Crommie, C. P. Lutz, and D. M. Eigler, Nature (London) **363**, 524 (1993).
[7] S. M. Reimann and M. Manninen, Rev. Mod. Phys. **74**, 1283 (2002).
[8] Y. Kondo and K. Takayanagi, Science **289**, 606 (2000).
[9] S. Oberholzer, E. V. Sukhorukov, and C. Schönenberger, Nature (London) **415**, 765 (2002).
[10] J. S. Hersch, M. R. Haggerty, and E. J. Heller, Phys. Rev. E **62**, 4873 (2000).
[11] S. Gustavsson, R. Leturcq, B. Simovič, R. Schleser, T. Ihn, P. Studerus, K. Ensslin, D. C. Driscoll, and A. C. Gossard, Phys. Rev. Lett. **96**, 076605 (2006).
[12] G. Castaldi, V. Galdi, and I. M. Pinto, IEEE Trans. Antennas Propag. **56**, 2638 (2008).
[13] J. W. Song, N. A. Kabir, Y. Kawano, K. Ishibashi, G. R. Aizin, L. Mourokh, J. L. Reno, A. G. Markelz, and J. P. Bird, Appl. Phys. Lett. **92**, 223115 (2008).
[14] Y.-H. Kim, M. Barth, H.-J. Stöckmann, and J. P. Bird, Phys. Rev. B **65**, 165317 (2002).
[15] B. Dietz, T. Friedrich, M. Miski-Oglu, A. Richter, T. H. Seligman, and K. Zapfe, Phys. Rev. E **74**, 056207 (2006).
[16] L. A. Bunimovich, Chaos **11**, 802 (2001).
[17] E. G. Altmann, A. E. Motter, and H. Kantz, Chaos **15**, 033105 (2005).
[18] A. Bäcker, R. Ketzmerick, S. Löck, M. Robnik, G. Vidmar, R. Höhmann, U. Kuhl, and H.-J. Stöckmann, Phys. Rev. Lett. **100**, 174103 (2008).
[19] C. Dembowski, B. Dietz, T. Friedrich, H.-D. Gräf, A. Heine, C. Mejía-Monasterio, M. Miski-Oglu, A. Richter, and T. H. Seligman, Phys. Rev. Lett. **93**, 134102 (2004).
[20] C. Jung, C. Lipp, and T. H. Seligman, Ann. Phys. **275**, 151 (1999).
[21] C. Jung, C. Mejía-Monasterio, O. Merlo, and T. H. Seligman, New J. Phys. **6**, 48 (2004).
[22] P. Hansen, K. A. Mitchell, and J. B. Delos, Phys. Rev. E **73**, 066226 (2006).
[23] G. Akguc and L. E. Reichl, Phys. Rev. E **64**, 056221 (2001).
[24] M. Prusty and H. Schanz, Phys. Rev. Lett. **98**, 176804 (2007).
[25] M. V. Berry, Eur. J. Phys. **2**, 91 (1981).
[26] L. A. Bunimovich, Chaos **1**, 187 (1991).
[27] H.-J. Stöckmann, *Quantum Chaos-An Introduction* (Cambridge University Press, Cambridge, 1999).
[28] A. Richter, in *Emerging Applications of Number Theory, The IMA Volumes in Mathematics and its Applications*, edited by D. A. Hejhal *et al.* (Springer, New York, 1999), Vol. 109, p. 479.
[29] T. Miyaguchi, Phys. Rev. E **75**, 066215 (2007).
[30] H. Tanaka and A. Shudo, Phys. Rev. E **74**, 036211 (2006).
[31] W. Bauer and G. F. Bertsch, Phys. Rev. Lett. **65**, 2213 (1990).
[32] N. B. Slater, Proc. Camb. Philos. Soc. **63**, 1115 (1967).
[33] I. Březinová, C. Stampfer, L. Wirtz, S. Rotter, and J. Burgdörfer, Phys. Rev. B **77**, 165321 (2008).
[34] J. Andreasen, H. Cao, J. Wiersig, and A. E. Motter, e-print arXiv:0905.4040.
[35] O. Bohigas, D. Boosé, R. Egydio de Carvalho, and V. Marvulle, Nucl. Phys. A. **560**, 197 (1993).
[36] S. Rotter, J.-Z. Tang, L. Wirtz, J. Trost, and J. Burgdörfer, Phys. Rev. B **62**, 1950 (2000).
[37] I. V. Zozoulenko and T. Blomquist, Phys. Rev. B **67**, 085320 (2003).
[38] E. A. Shaner and S. A. Lyon, Phys. Rev. Lett. **93**, 037402 (2004).
[39] Z. Z. Zhong, N. M. Gabor, J. E. Sharping, A. L. Gaeta, and P. L. McEuen, Nat. Nanotechnol. **3**, 201 (2008).
[40] M. J. Biercuk, D. J. Reilly, T. M. Buehler, V. C. Chan, J. M. Chow, R. G. Clark, and C. M. Marcus, Phys. Rev. B **73**, 201402(R) (2006).
[41] W. Lu, J. Zhonqing, L. Pfeiffer, K. W. West, and A. J. Rimberg, Nature (London) **423**, 422 (2003).