Model of binary opinion dynamics: Coarsening and effect of disorder

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We propose a model of binary opinion in which the opinion of the individuals changes according to the state of their neighboring domains. If the neighboring domains have opposite opinions then the opinion of the domain with the larger size is followed. Starting from a random configuration, the system evolves to a homogeneous state. The dynamical evolution shows a scaling behavior with the persistence exponent $\theta \simeq 0.235$ and dynamic exponent $z \simeq 1.02 \pm 0.02$. Introducing disorder through a parameter called rigidity coefficient ρ (probability that people are completely rigid and never change their opinion), the transition to a heterogeneous society at $\rho=0^+$ is obtained. Close to $\rho=0$, the equilibrium values of the dynamic variables show power-law scaling behavior with ρ. We also discuss the effect of having both quenched and annealed disorder in the system.

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A number of models which simulate the formulation of opinion in a social system have been proposed in the physics literature recently [1]. Many of these show a close connection to familiar models of statistical physics, e.g., the Ising and the Potts models. Different kinds of phase transitions have also been observed in these models by introducing suitable parameters. One such phase transition can be from a homogeneous society, where everyone has the same opinion to a heterogeneous one with mixed opinions [2].

In a model of opinion dynamics, the key feature is the interaction of the individuals. Usually, in all the models, it is assumed that an individual is influenced by its nearest neighbors. In this Brief Report, we propose a one-dimensional model of binary opinion in which the dynamics is dependent on the *size* of the neighboring domains as well. Here an individual changes his/her opinion in two situations: first when the two neighboring domains have opposite polarity and, in this case, the individual simply follows the opinion of the neighboring domain with the larger size. This case may arise only when the individual is at the boundary of the two domains. An individual also changes his/her opinion when both the neighboring domains have an opinion which opposes his/her original opinion, i.e., the individual is sandwiched between two domains of same polarity. It may be noted that for the second case, the size of the neighboring domains is irrelevant.

This model, henceforth referred to as model I, can be represented by a system of Ising spins, where the up and down states correspond to the two possible opinions. The two rules followed in the dynamical evolution in the equivalent spin model are shown schematically in Fig. 1 as cases I and II. In the first case, the spins representing individuals at the boundary between two domains will choose the opinion of the left side domain (as it is larger in size). For the second case, the down spin flanked by two neighboring up spins will flip.

The main idea in model I is that the size of a domain represents a quantity analogous to "social pressure," which is expected to be proportional to the number of people supporting a cause. An individual, sitting at the domain boundary, is most exposed to the competition between opposing pressures and gives in to the larger one. This is what happens in case I shown in Fig. 1. The interaction in case II, on the other hand, implies that it is difficult to stick to one's opinion if the entire neighborhood opposes it.

Defining the dynamics in this way, one immediately notices that case II corresponds to what would happen for spins in a nearest-neighbor ferromagnetic Ising model (FIM) in which the dynamics at zero temperature is simply an energy minimization scheme. However, the boundary spin in the FIM behaves differently in case I; it may or may not flip as the energy remains same. In the present model, the dynamics is deterministic even for the boundary spins (barring the rare instance when the two neighborhoods have the same size in which case the individual changes state with 50% probability).

In this model, the important condition of changing one's opinion is the size of the neighboring domains, which is not fixed either in time or space. This is the unique feature of this model and, to the best of our knowledge, such a condition has not been considered earlier. In the most familiar models of opinion dynamics such as the Sznajd model [3] or the voter model [4], one takes the effect of nearest neighbors within a given radius and, even in the case of models defined on networks [5], the influencing neighbors may be nonlocal but always fixed in identity.

We have done Monte Carlo simulations to study the dynamical evolution of the proposed model from a given initial state. With a system of N spins representing individuals, at

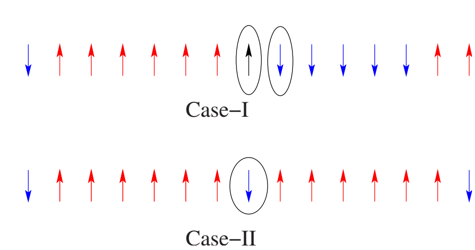

FIG. 1. (Color online) Dynamical rules for model I: in both cases, the encircled spins may change state; in case I, the boundary spins will follow the opinion of the left domain of up spins which will grow. For case II, the down spin between the two up spins will flip irrespective of the size of the neighboring domains.

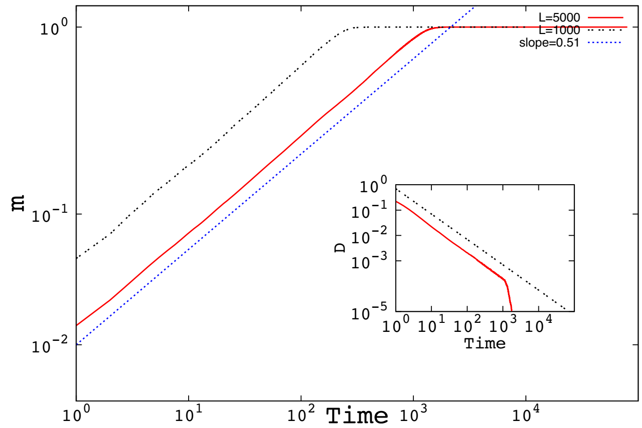

FIG. 2. (Color online) Growth of order parameter m with time for two different system sizes along with a straight line (slope 0.51) shown in a log-log plot. Inset shows the decay of fraction of domain wall D with time.

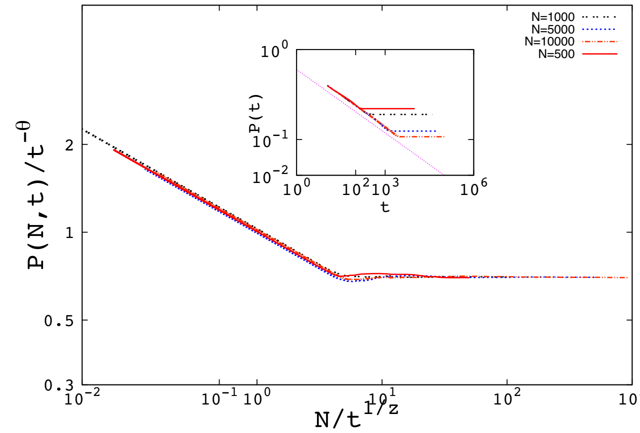

FIG. 3. (Color online) The collapse of scaled persistence probability versus scaled time using θ=0.235 and z=1.04 is shown for different system sizes. Inset shows the unscaled data.

each step, one spin is selected at random and its state updated. After N such steps, one Monte Carlo time step is said to be completed.

If N_+ is the number of people of a particular opinion (up spin) and N_- is the number of people of opposite opinion (down spin), the order parameter is defined as $m=|N_+ - N_-|/N$. This is identical to the (absolute value of) magnetization in the Ising model.

Starting from a random initial configuration, the dynamics in model I leads to a final state with $m=1$, i.e., a homogeneous state where all individuals have the same opinion. It is not difficult to understand this result; in the absence of any fluctuation, the dominating neighborhood (domain) simply grows in size ultimately spanning the entire system.

We have studied the dynamical behavior of the fraction of domain walls D and the order parameter m as the system evolves to the homogeneous state. We observe that the behaviors of $D(t)$ and $m(t)$ are consistent with the usual scaling behavior found in coarsening phenomena; $D(t) \propto t^{-1/z}$ with $z=1.00 \pm 0.01$ and $m(t) \propto t^{1/2z}$ with $z=0.99 \pm 0.01$. These variations are shown in Fig. 2.

We have also calculated the persistence probability that a person has not change his/her opinion up to time t. Persistence, which in general is the probability that a fluctuating nonequilibrium field does not change sign up to time t, shows a power-law decay behavior in many physical phenomena, i.e., $P(t) \propto t^{-\theta}$, where θ is the persistence exponent. In that case, one can use the finite-size scaling relation [6,7]

$$P(t,L) \propto t^{-\theta} f(L/t^{1/z}). \quad (1)$$

For finite systems, the persistence probability saturates at a value $\propto L^{-\alpha}$ at large times. Therefore, for $x \ll 1$, $f(x) \propto x^{-\alpha}$ with $\alpha = z\theta$. For large x, $f(x)$ is a constant. Thus, one can obtain estimates for both z and θ using the above scaling form.

In the present model, the persistence probability does show a power-law decay with $\theta = 0.235 \pm 0.003$, while the finite-size scaling analysis made according to Eq. (1) suggests a z value 1.04 ± 0.01 (Fig. 3). Thus, we find that the values of z from the three different calculations are consistent and conclude that the dynamic exponent $z = 1.02 \pm 0.02$.

It is important to note that both the exponents z and θ are quite different from those of the one-dimensional Ising model [8] and other opinion/voter dynamics models [9–11]. Specifically, in the Ising model, $z=2$ and $\theta=0.375$ and, for the Sznajd model, the persistence exponent is equal to that of the Ising model. This shows that the present model belongs to an entirely different dynamical class.

The model I described so far has no fluctuation. Fluctuations or disorder can be introduced in several ways. We adopt a realistic outlook: since every individual is not expected to succumb to social pressure, we modify model I by introducing a parameter ρ called rigidity coefficient, which denotes the probability that people are completely rigid and never change their opinions. The modified model will be called model II, in which there are ρN rigid individuals (chosen randomly at time $t=0$), who retain their initial state throughout the time evolution. Thus, the disorder is quenched in nature. The limit $\rho=1$ corresponds to the unrealistic noninteracting case when no time evolution takes place; $\rho=1$ is in fact a trivial fixed point. For other values of ρ, the system evolves to a equilibrium state.

The time evolution changes drastically in nature with the introduction of ρ. All the dynamical variables such as order parameter, fraction of domain wall, and persistence attain a saturation value at a rate which increases with ρ. Power-law variation with time can only be observed for $\rho < 0.01$ with the exponent values same as those for $\rho=0$. The saturation or equilibrium values, on the other hand, show the following behavior:

$$m_s \propto N^{-\alpha_1} \rho^{-\beta_1},$$

$$D_s \propto \rho^{\beta_2},$$

$$P_s = a + b\rho^{\beta_3}, \quad (2)$$

where in the last equation $a \simeq 10^{-2}$ and $\beta_3 (\simeq 0.36)$ are weakly dependent on N. The values of the exponents are $\alpha_1 = 0.500 \pm 0.002$, $\beta_1 = 0.513 \pm 0.010$, and $\beta_2 = 0.96 \pm 0.01$

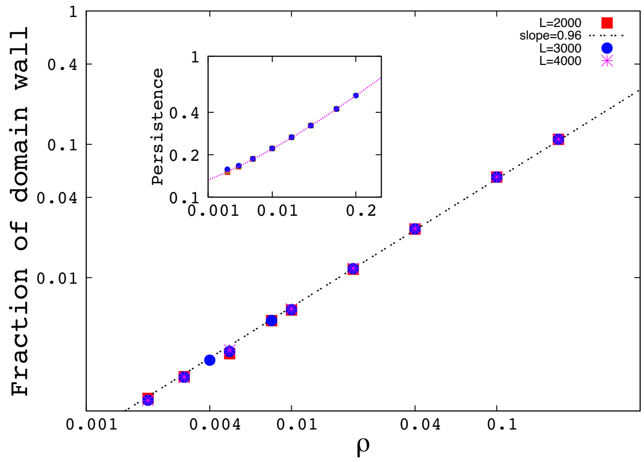

FIG. 4. (Color online) Saturation values of fraction of domain walls D_s and persistence probability P_s (shown in inset) increase with rigidity coefficient ρ in a power-law manner. There is no system size dependence for both quantities.

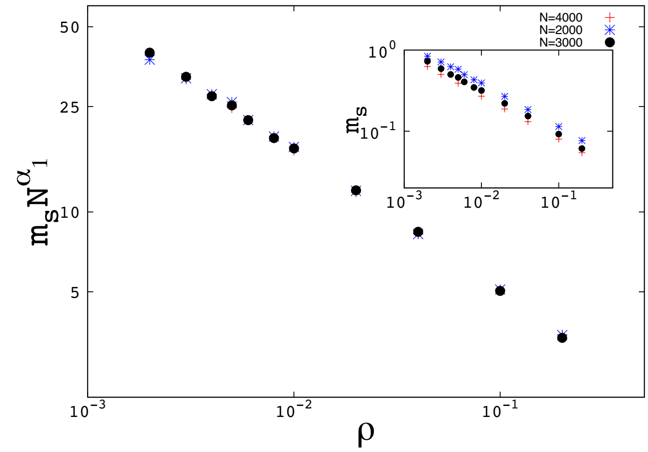

FIG. 5. (Color online) Scaled saturation value of m_s decays with the rigidity coefficient ρ. Inset shows the unscaled data.

(Figs. 4 and 5). The variation of m_s with ρ is strictly speaking not valid for extremely small values of ρ. However, at such small values of ρ, it is difficult to obtain the exact form of the variation numerically.

It can be naively assumed that the $N\rho$ rigid individuals will dominantly appear at the domain boundaries such that in the first-order approximation (for a fixed population), $D \propto 1/\rho$. This would give $m \propto 1/\sqrt{\rho}$, indicating $\beta_1 = 0.5$ and $\beta_2 = 1$. The numerically obtained values are in fact quite close to these estimates.

The results obtained for model II can be explained in the following way. With $\rho \neq 0$, the domains cannot grow freely and domains with both kinds of opinions survive making the equilibrium m_s less than unity. Thus, the society becomes heterogeneous for any $\rho > 0$ when people do not follow the same opinion any longer. The variation of m_s with N shows that $m_s \to 0$ in the thermodynamic limit for $\rho > 0$. Thus, not only does the society become heterogeneous at the onset of ρ, it goes to a completely disordered state analogous to the paramagnetic state in magnetic systems. Thus, one may conclude that a phase transition from a ordered state with $m=1$ to a disordered state ($m=0$) takes place for $\rho = 0^+$. It may be recalled here that $m=0$ at the trivial fixed point $\rho=1$ and, therefore, the system flows to the $\rho=1$ fixed point for any nonzero value of ρ, indicating that $\rho=1$ is a stable fixed point. The saturation values of the fraction of domain walls do not show system size dependence for $\rho=0^+$ further supporting the fact that the phase transition occurs at $\rho=0$.

The effect of the parameter ρ is therefore very similar to thermal fluctuations in the Ising chain, which drives the latter to a disordered state for any nonzero temperature, $\rho=1$ being comparable to infinite temperature. However, the role of the rigid individuals is more similar to domain walls which are pinned, rather than thermal fluctuations. In fact, the Ising model will have dynamical evolution even at very high temperatures; while in model II, the dynamical evolution becomes slower with ρ, ultimately stopping altogether at $\rho=1$. This is reflected in the scaling of the various thermodynamical quantities with ρ, e.g., the order parameter shows a power-law scaling above the transition point.

Since the role of ρ is similar to domain-wall pinning, one can introduce a depinning probability factor μ, which in this system represents the probability for rigid individuals to become nonrigid during each Monte Carlo step. μ relaxes the rigidity criterion in an annealed manner, in the sense that the identity of the individuals who become nonrigid is not fixed (in time). If $\mu=1$, one gets back model I (identical to model II with $\rho=0$) whatever be the value of ρ and, therefore, $\mu=1$ signifies a line of (model I) fixed points, where the dynamics leads the system to a homogeneous state.

With the introduction of μ, one has effectively a lesser fraction ρ' of rigid people in the society, where

$$\rho' = \rho(1-\mu). \quad (3)$$

The difference from model II is, of course, that this effective fraction of rigid individuals is not fixed in identity (over time). Thus, when $\rho \neq 0$, $\mu \neq 0$, we have a system in which there are both quenched and annealed disorder. It is observed that for any nonzero value of μ, the system once again evolves to a homogeneous state ($m=1$) for all values of ρ. Moreover, the dynamic behavior is also the same as model I

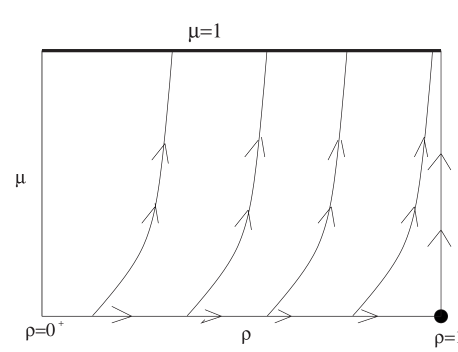

FIG. 6. The flow lines in the ρ-μ plane: any nonzero value of ρ with $\mu=0$ drives the system to the disordered fixed point $\rho=1$. Any nonzero value of μ drives it to the ordered state ($\mu=1$, which is a line of fixed points) for all values of ρ.

with the exponents z and θ having identical values. This shows that the nature of randomness is crucial as one cannot simply replace a system with parameters $\{\rho \neq 0, \mu \neq 0\}$ by one with only quenched randomness $\{\rho' \neq 0, \mu' = 0\}$, as in the latter case, one would end up with a heterogeneous society. We therefore conclude that the annealed disorder wins over the quenched disorder; μ effectively drives the system to the $\mu = 1$ fixed point for any value of ρ. This is shown schematically in a flow diagram (Fig. 6). It is worth remarking that it looks very similar to the flow diagram of the one-dimensional Ising model with nearest-neighbor interactions in a longitudinal field and finite temperature.

In summary, we have proposed a model of opinion dynamics in which the social pressure is quantified in terms of the size of domains having the same opinion. In the simplest form, the model has no disorder and self-organizes to a homogeneous state, in which the entire population has the same opinion. This simple model exhibits coarsening with exponents which are drastically different from those of other known one-dimensional models. With disorder, the model undergoes a phase transition from a homogeneous society (with an order parameter equal to one) to a heterogeneous one, which is fully disordered in the sense that no consensus can be reached as the order parameter goes to zero in the thermodynamic limit. With both quenched and annealed randomness present in the system, the annealed randomness is observed to drive the system to a homogeneous state for any amount of the quenched randomness.

Many open questions still remain regarding models I and II, the behavior in higher dimensions being one of them. In fact, a full understanding of the phase transition occurring in model II reported here is an important issue. Although the phase transition has similarities with the one-dimensional Ising model, there are some distinctive features which should be studied in more detail.

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