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Effects of strategy-migration direction and noise in the evolutionary spatial prisoner's dilemma

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Spatial games are crucial for understanding patterns of cooperation in nature (and to some extent society). They are known to be more sensitive to local symmetries than, e.g., spin models. This paper concerns the evolution of the prisoner's dilemma game on regular lattices with three different types of neighborhoods—the von Neumann, Moore, and *kagomé* types. We investigate two kinds of dynamics for the players to update their strategies (that can be unconditional cooperator or defector). Depending on the payoff difference, an individual can adopt the strategy of a random neighbor [a voter-model-like dynamics (VMLD)] or impose its strategy on a random neighbor, i.e., invasion-process-like dynamics (IPLD). In particular, we focus on the effects of noise, in combination with the strategy dynamics, on the evolution of cooperation. We find that VMLD, compared to IPLD, better supports the spreading and sustaining of cooperation. We see that noise has nontrivial effects on the evolution of cooperation: maximum cooperation density can be realized either at a medium noise level, in the limit of zero noise or in both these regions. The temptation to defect and the local interaction structure determine the outcome. Especially, in the low noise limit, the local interaction plays a crucial role in determining the fate of cooperators. We elucidate these both by numerical simulations and mean-field cluster approximation methods.

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I. INTRODUCTION

In the last decades, a great deal of interest has been devoted to understanding the evolution of cooperation [1], i.e., how cooperation emerges and persists in a population composed of selfish individuals. Evolutionary game theory [2,3]—a research field cultivated traditionally by both evolutionary biologists, political scientists, economists, and mathematician [4,5] that recently has attracted a growing interest from the physics community [6,7]—provides a general framework to address such issues. One famous metaphor for the problem of cooperation is the prisoner's dilemma (PD) game [8,9], in which defection could benefit the individual in the short term, whereas cooperation has only potential benefits in a longer time perspective. In the common mathematical formulation of PD with pairwise interaction, each of the two encountering cooperators (defectors) get a payoff R (P), a defector confronting a cooperator acquires payoff T, while the cooperator gains S. The four parameters is required to satisfy the conditions $T>R>P>S$ and $2R>T+S$. In a well-mixed population, defectors are unbeatable and cooperators are doomed to extinction [4].

The seminal work of Nowak and May [10], where a PD game is played on two-dimensional grids with agents lacking memory and ability to form complex strategies, showed that spatial structure and nearest-neighbor interactions can enable cooperators to survive by forming clusters within which they benefit from mutual cooperation and protecting them from exploitation by defectors. This work has inspired numerous subsequent investigations of evolutionary games on spatial grids [11,12]. Now the spatial (or network) reciprocity is regarded as one of the five main mechanisms supporting cooperation [5,13]. Furthermore, coevolution of strategy distribution and underlying interaction graphs can also be useful in characterizing the PD game [14–17]. The crucial observation is that through switching partners, cooperators are capable of frequently meeting other cooperators, which substantially increase their reproduction rate. In addition, a lots of other mechanisms favoring cooperation have also been proposed: volunteering participation [18], separation of interaction and learning graphs [19], dilution and random diffusion of agents on the grid [20], success-driven migration [21], memory effects [22], reputation-based interaction [23], moderate aspiration level [24], etc. For a comprehensive review of this research field, we refer to Refs. [5,6].

It is worth pointing out that the scale-free topology, thanks to the capability of mutual protection of the hubs if occupied by cooperators, can enhance the spreading of cooperative behavior and resist the invasion of defection, hence becoming a promoter of cooperation [25]. However, it was also argued that the adoption of averaged payoffs instead of accumulated ones in the score function, as well as an introduction of participation costs, might weaken this mechanism [26]. These results suggest that the distinct ability of strategy breeding of the players [27], rather than the interaction graph, matters much for the emergence of cooperation. The heterogeneous or asymmetric strategy migration ability can be implemented in different ways, such as dynamic preferential selection [28], introduction of two types of players with different teaching activity [29], social diversity [30,31], nonlinear attractive effect [32,33], etc. In principle, as long as some distinguished players have higher influence to spread their strategy [31] and also the connections among these influential players are coupled in some appropriate way [27,29,30], the dilemma can be relaxed and the promotion of cooperation is warranted.

These results suggest that the heterogeneity in the migration of strategies might be an important factor for the stability of cooperation. To our knowledge, in most previous stud-

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ies of spatial games, a common simplifying assumption is that, whenever updating strategy, a player selects a neighbor as reference and attempts to adopt the neighbor's strategy according to some prescribed criterion. In other words, the focal player is always a recipient of the strategy. Only few works have studied the reverse situation, with strategy donors and their influence on the evolution of cooperation. Ohtsuki *et al.* [34] implemented both death-birth and birth-death updating for games on graphs and found that for birth-death updating, selection does not favor cooperation. Antal *et al.* studied evolutionary dynamics on degree-heterogeneous graphs evolved either by one individual dying and being replaced by the offspring of a random neighbor (voter model dynamics) or by an individual giving birth to an offspring taking over a random neighbor (invasion process dynamics). They found that the fixation probability of a single fitter mutant under the voter model dynamics is k^2 times of corresponding value for the invasion process dynamics (k is the node degree) [35]. With this paper, we continue investigating the effects of the direction of strategy migration on the evolution of cooperation in the spatially explicit PD game. Another theme of recent research is the effects of noise on dynamic processes [36]. Indeed, noise (or mutation) plays an important role in the outcome of evolutionary games [37,38]. In recent years, the effects of noise on spatial games have also been studied [39–43]. The spatial PD model proposed in [10] is a deterministic cellular automaton and can be extended to a stochastic version by introduction of noise in different ways. Perhaps the first spatial PD model with a stochastic strategy adoption process was proposed in Ref. [44]. We take the approach of Szabó and Tőke [39], where the players updates their strategies according to a Fermi function and in which noise have the role of temperature in the kinetic Ising model. For this simple "noise-guided" evolutionary model, the authors found rich dynamic phenomena—two absorbing states consisting of only cooperators and defectors separated by a coexistence region. Particularly, the phase transition involving the extinction of cooperators or defectors is found to belong to the universality class of directed percolation [39]. In a similar vein, they also studied how the noise affects the phase diagram of PD on different two-dimensional lattices and Newman-Watts networks [40]. In somewhat different approach, Perc *et al.* [43] and Tanimoto [45] introduced noise to the payoff matrix. The reported results show a coherence-resonance phenomenon where the fraction of cooperators reaches its maximum at an intermediate noise level [43]. In Ref. [40] the noise effect in PD games was studied, mainly focusing on the threshold of extinction of cooperation as a function of noise intensity. A clear picture for how the average fraction of cooperation evolves by varying the noise level under different intensities of the temptation to defection, to the best of our knowledge, is still lacking. In the present work, we will follow the research line of Refs. [34,35,40] to study how the strategy migration direction and selection noise affect the cooperation. In the rest of the paper, we first define our model and then treat the problems sketched above both by computer simulations and cluster approximation methods. Finally, we make some discussions and draw our conclusion.

II. MODEL

In the present study, we consider the evolutionary PD on three types of regular lattices with periodic boundary conditions, namely, square lattice with von Neumann neighborhood (von Neumann lattice), square lattice with Moore neighborhood (Moore lattice), and *kagomé* lattice, such that the number of encounters of each player are four, eight, and six, respectively. The reason we concentrate on these simple lattices is twofold: first, previous studies have found that local interaction does affect the spreading of cooperation [12,40]; second, their regular structures allow us to implement mean-field cluster approximation analysis [6,7,40].

Following many studies [6,31,39,40], we use the Nowak-May parametrization [10] of the spatial PD, i.e., the temptation to defect $T=b$ (where $1<b<2$), the reward for mutual cooperation $R=1$, the punishment of mutual defection $P=0$, and the sucker's payoff $S=0$. Initially, we let the players be either unconditional cooperators (C) or defectors (D) with equal probability. The evolution of strategies is governed by random sequential updating. For each of the considered lattice topologies, we first let a randomly chosen player i reap its payoff P_i by playing the PD game with its nearest neighbors. Then, we select a random neighbor j of i, and let it acquire its payoff P_j by playing the game with its nearest neighbors.

When updating strategy, the focal player i can be of the role of either strategy donor or strategy recipient. If i is the strategy recipient, we implement a voter-model-like dynamics (VMLD) [35], which is probably the most common approach [6,25,27,29–32,39–43]. In this case, the player i imitates the strategy of j with a probability dependent on their payoff difference [39]

$$W_{ij}=\frac{1}{1+\exp[(P_i-P_j)/\kappa]}, \tag{1}$$

where $\kappa\in[0,\infty)$ denotes the noise level (or, in the language of evolutionary biology, selection intensity [13]). In the $\kappa=0$ limit, the adoption of a *successful* strategy is deterministic, while in the $\kappa\to\infty$ limit, the strategy learning is blind [40]. If, however, we treat the player i as the strategy donor, we implement exactly invasion-process-like dynamics (IPLD) for the game [35]. In this case, the neighbor j will try to take the focal player is strategy with a probability defined as

$$W_{ji}=\frac{1}{1+\exp[(P_j-P_i)/\kappa]}. \tag{2}$$

By tuning the values of b and κ in the framework of the two updating schemes, we can obtain pictures of how the noise intensity and direction of strategy migration impact the final outcome of the evolutionary spatial PD game.

In our simulation, we consider the possible strategy migration for the players one by one according to a random sequence. One pass through all the agents is called a *Monte Carlo (MC) sweep*. Between each MC sweep we reshuffle the sampling sequence. The total population is of size 200×200 for the von Neumann and Moore lattices and $3\times100\times100$ for the *kagomé* lattice. When repeating the

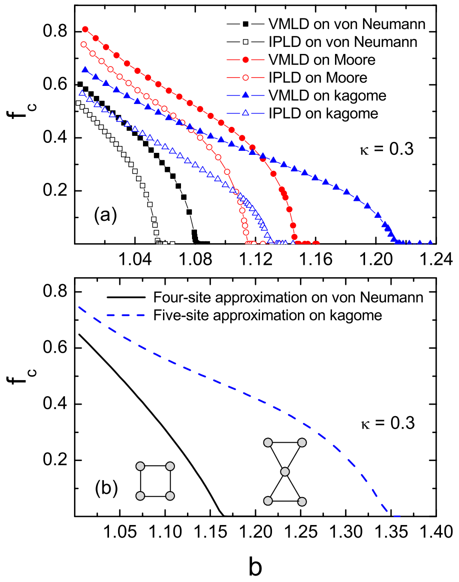

FIG. 1. (Color online) (a) Average fraction f_c of cooperation as a function of the temptation to defect b for a fixed noise intensity κ=0.3 on three types of lattices. Solid and open symbols correspond to the two strategy-migration dynamics VMLD and IPLD, respectively. (b) Theoretical estimations by the four-site cluster approximation on the von Neumann lattice and by the five-site cluster approximation on the *kagomé* lattice, correctly predict the evolving tendency of f_c but do not differentiate between VMLD and IPLD dynamics. Note the different scaling of the *x* axis in the two panels.

above described elementary steps the system develops into a final stationary state characterized by the average fraction of cooperators f_c, which is measured for the last 3000 sweeps of the total 2×10^4. All the simulation results presented below are averages over 20 independent realizations of initial strategy distribution.

III. RESULTS AND DISCUSSION

We start by comparing the results from the game with the VMLD strategy-updating rule with the corresponding IPLD results. In Fig. 1(a), we present the simulation results for f_c as a function of the temptation to defect b on the three lattices, and the noise level κ is fixed as 0.3. We note that f_c decreases monotonously with increasing b up to a threshold b_c, where cooperation vanishes. Although the qualitative properties of the curves are somewhat similar, there exist quantitative differences. By comparison, the stationary cooperation level in IPLD is lower than the corresponding value for VMLD. Furthermore, IPLD results in a lower b_c threshold. Thus, our first finding is that VMLD is more favorable for the spreading of cooperation than IPLD.

At a first glance, it may seem strange that VMLD and IPLD on homogeneous graph give rise to different results, since what is regarded as VMLD from the point of view of the strategy recipient is exactly the IPLD in the viewpoint of the strategy donor. (On heterogeneous graph, this difference is evident since high-degree individuals have greater chance to affect others for VMLD and to be affected by others for IPLD.) In fact, the four- or five-site cluster approximation, developed in the spirit of mean-field theory [6], do not differentiate between the VMLD and IPLD and give out exactly the same results for both strategy updating dynamics [see Fig. 1(b), the two curves for the VMLD and IPLD coincide with one another, and for simplicity we just show one]. A crucial observation is that this method neglects spatial correlations in the population, which cause the update probabilities of VMLD and IPLD to differ. Indeed, if the two probabilities are the same, the final results will be identical on homogeneous graphs. This argument can be deduced from the studies of voter model on graphs. In Ref. [46], Castellano studied both voter and antivoter dynamics on networks (corresponding to the VMLD and IPLD in this work) and found that on homogeneous network, the consensus time in both dynamics is the same. In [35], Antal *et al.* pointed out that the fixation probability of a neutral genotype on homogeneous graph is the same for both voter-model and invasion-process dynamics. The essential difference between the voter model and evolutionary games is that in voter model, when a randomly selected individual is to update its state, only the state of the selected neighbor matters, while in the PD game case, both the state of the selected neighbor and those of this neighbor's neighbors. For our setup, the average payoffs collected by cooperators on the boundary is greater than that by defectors [33], we argue that VMLD favors the diffusion of cooperation on the rough boundaries separating cooperators from defectors stronger than IPLD. (In the Appendix, we have presented a concrete example to analyze the difference between the VMLD and IPLD determining the probability of strategy migration.) A defector at a rough boundary would have, on average, greater chance to be convinced by its cooperating neighbors in the VMLD picture than a cooperating neighbor to convince a defector in the IPLD. To test this point, we have traced the time series of N_C/N_D, the number of events of D switching to C divided by the reverse case, for two special values of b on the von Neumann lattice. The results presented in Fig. 2 give further evidence for the above speculation.

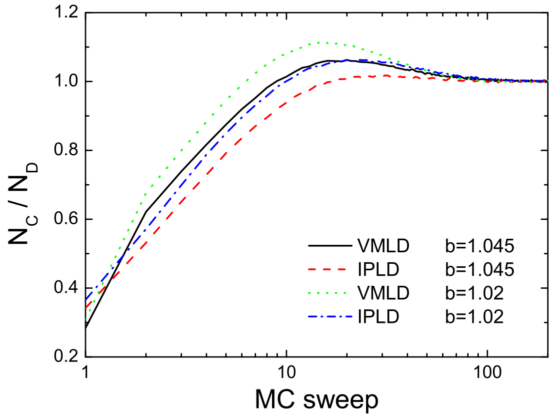

FIG. 2. (Color online) Time series of N_C/N_D, the number of events of D switching to C divided by the reverse case, for b =1.02 and 1.045 on the von Neumann lattice.

Before moving forward, we briefly comment the importance of the network structures in maintaining cooperation.

In previous studies [40] the overlapping triangles (especially, one-site overlapping triangles) in the connectivity structure are found to support the spreading (maintenance) of cooperation. Here, we want to point out that this conclusion is drawn in terms of the magnitude of b_c for the evolutionary PD on different lattice structures [40]. Looking back on our simulations in Fig. 1, we believe that this is not the whole story. If we measure the capability of promoting cooperation by the average fraction of cooperation in the equilibrium state, we observe that when b is small (e.g., $b=1.1$), the one-site overlapping triangles (the *kagomé* lattice) is an inferior structure compared to the case of multiple overlapping sites (the Moore lattice). This indicates that the temptation parameter is an indicative variable when evaluating the role of connectivity structure, giving a more nuanced picture than the previously mentioned effect of overlapping triangles in Ref. [40].

Now let us turn to the effect of noise. As mentioned above, the extinction threshold of cooperation b_c as a function of κ on different types of lattices has been studied in [29,40]. Here we investigate how κ influences f_c in the stationary state for different values of b. In Fig. 3(a), we show simulation results of f_c versus κ for two values of $b=1.02$ and $b=1.045$ on the von Neumann lattice. We observe so-called coherence resonance for both VMLD and IPLD, where f_c reaches its maximum at an intermediate κ. If, in this case, the noise is too weak or strong cooperation may be extinct (this is in accordance with previous results [43]). For the parameters used, the maximum f_c emerges at about $\kappa=0.3$. We note that for the same temptation to defect, comparing with the case of VMLD, IPLD results in a narrower region of κ, where cooperators can maintain a nonzero fraction in the population. That is to say, cooperation is more robust against the fluctuation of noise in the case of VMLD. This result strengthens our previous finding from Fig. 1. Note that $\lim_{\kappa\to\infty} f_c=1/2$. In this paper we focus on low noise levels, i.e., situations where the competitive interactions dominate the dynamics and do not study how cooperation reemerges for very large κ.

The observed resonant behavior can be understood as follows. As mentioned above, the noise parameter κ measures the stochastic uncertainties in the process of strategy learning. In the limit of weak noise, the irrational choices of the individuals become very rare. The strategies of those individuals who get higher payoffs would always migrate successfully. Though the payoffs of the defectors at the interface are on average smaller than those of cooperators [33], the highest payoff is always obtained by those defectors surrounded by three other cooperators. As such, if the individuals imitate with complete rationality ($\kappa\to 0$), the defective strategy would spread more easily in the population, which does harm to the formation of clusters of cooperators. On the other hand, in the limit of large noise, the payoff information becomes less important in determining the success of strategy migration. In such situation, it would be definitely inefficient for the cooperators at the interface to spread their behavior (though they acquire higher payoff on average than those defectors [33]). If, however, the noise κ is appropriately chosen, i.e., not too large or too small, the cooperators would have more chance to diffusion, and the defectors with highest payoff would have less probability to be followed by cooperators, leading to the surviving and enhancement of cooperation. Henceforth the optimal noisy intensity κ emerges.

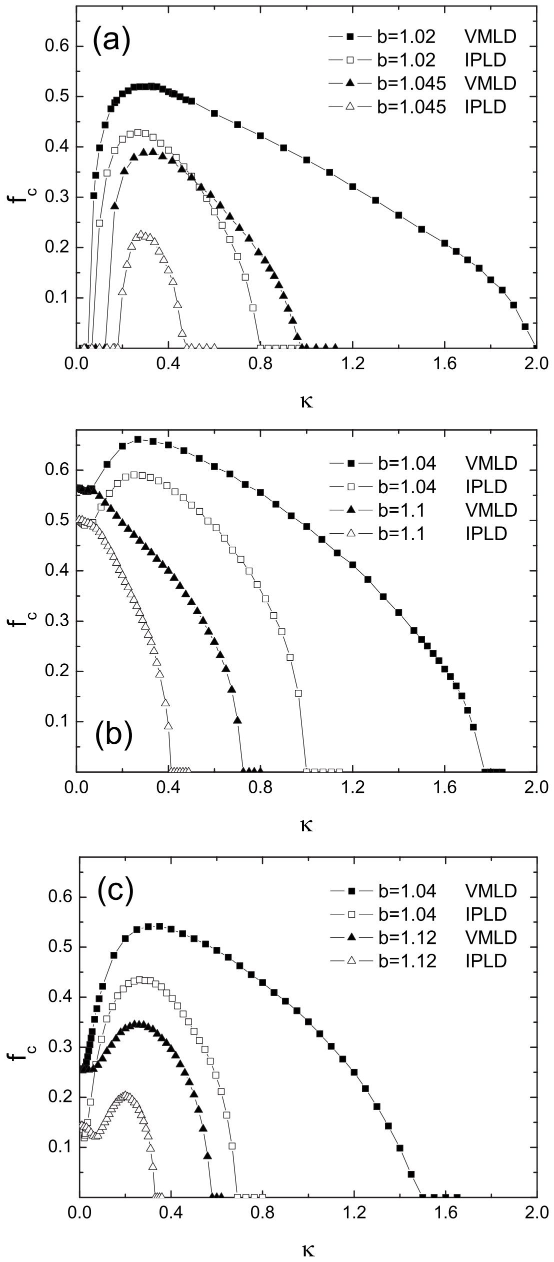

FIG. 3. Average fraction f_c of cooperation as a function of the noise parameter κ for fixed values of b on (a) the von Neumann lattice, (b) the Moore lattice, and (c) the *kagomé* lattice. The solid and open symbols are as shown in Fig. 1. The lines are guides to the eyes.

The experimental tests of the PD game on the Moore and *kagomé* lattices display somewhat similar results [Figs. 3(b) and 3(c)]. For an appropriate temptation to defect, when varying κ, we also notice an emergent of coherence resonance. Moreover, the VMLD is also found to be superior to the IPLD in promoting cooperation. Despite these two aspects, when κ is sufficiently small, the simulation results illustrated in Fig. 3(a) are in stark contrast to those in Figs.

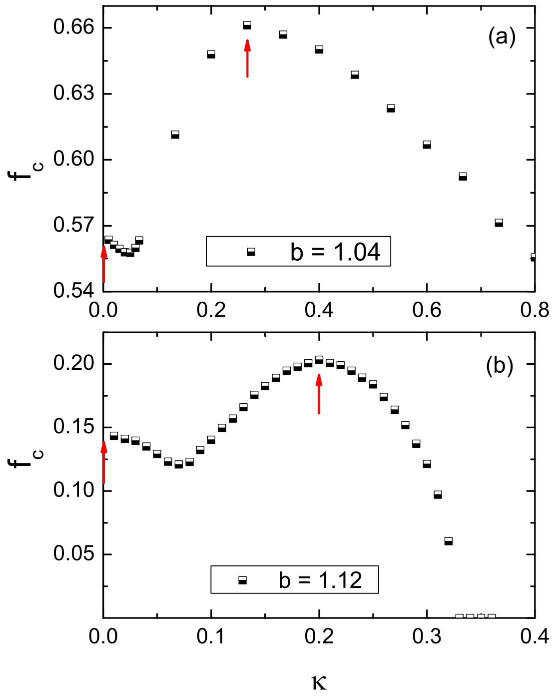

FIG. 4. (Color online) Replotting of the data in Fig. 3 for (a) $b=1.04$ on the Moore lattice with the VMLD and (b) $b=1.12$ on the *kagomé* lattice with the IPLD. The places where f_c reaches its maximum value are indicated by arrows.

3(b) and 3(c). In the former case, the cooperators are doomed to extinction in the limit of zero noise, whereas they can persist in the population with finite f_c in the latter two cases. We argue that the presence of overlapping triangles [40] in the latter two lattices contributes to the difference since triangles are absent in the von Neumann lattice. Note, however, that the importance of overlapping triangles is evaluated with b_c in [40] and with f_c here. Comparing the curves for $b=1.04$ in Figs. 3(b) and 3(c), keeping Fig. 1 in mind, we conclude that for high temptation values, one-site overlapping triangles gives more favorable conditions for cooperation, than the multiple-site case. For low temptation values the situation is reversed.

It is worth noting that the PD game on Moore and *kagomé* lattices there is a nontrivial dependence of f_c on κ (which depends both on the temptation parameter and the direction of strategy migration). In particular, for sufficiently small b, we obtain only one maximum of f_c as a function of κ [the curve in Fig. 3(c) with $b=1.04$ and VMLD]. With increasing b, peculiarly, we observe two peaks of $f_c(\kappa)$, one located at an intermediate value and the other at $\kappa=0$. This phenomenon is yet more prominent in the case of IPLD (cf. Fig. 4). Finally, if the value of b is sufficiently large [the curves of $b=1.1$ in Fig. 3(b) and also in Fig. 5], the peak of f_c at finite κ disappears and only the no-noise maximum remains. For comparison, in Fig. 6 we present analytic results via the four-site cluster approximation on the von Neumann lattice and five-site cluster approximation on the *kagomé* lattice (for details, please see the Appendix in Ref. [6]) and observe that the theoretical estimations correctly predict the tendency of the results obtained by our MC simulations.

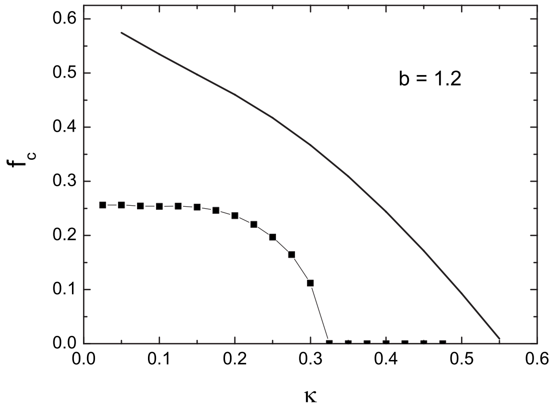

FIG. 5. Average fraction f_c of cooperation versus κ in the case of "high" temptation to defect $b=1.2$ on the *kagomé* lattice. The solid line denotes the analytical results obtained by the five-site cluster approximation.

The simulation results shown in Figs. 3–5 allow us to speculate that in the limit of weak noise, the local interaction topology may play a decisive role in maintaining cooperation. To test this point further, we present in Fig. 7 the simulation results for the evolutionary PD on the Moore lattice and the *kagomé* lattice, where f_c as a function of b is plotted for a fixed value of $\kappa=0.01$. Here we only consider the VMLD. We find that the cooperators vanish at about $b_c=1.5$ for the *kagomé* lattice (which has already obtained in Ref. [40]) and at about $b_c=1.2$ for the Moore lattice. Despite the value of b_c, we note that the average fraction of coopera-

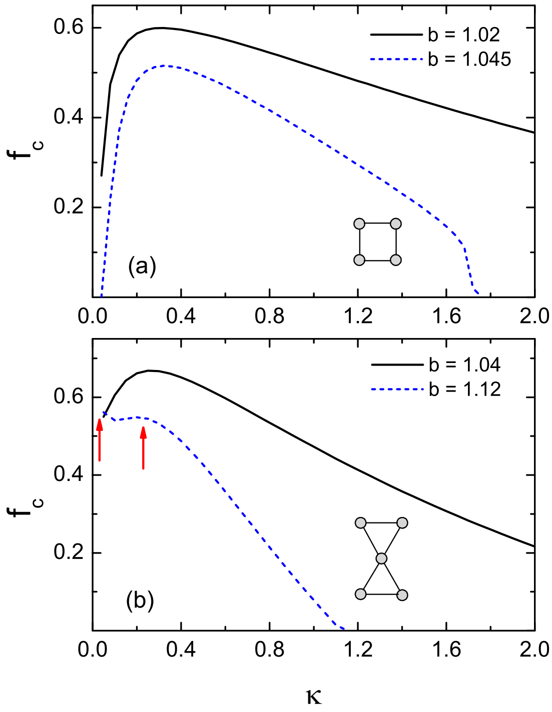

FIG. 6. (Color online) Theoretical estimations for f_c as a function of κ by using (a) the four-site cluster approximation method on the von Neumann lattice and (b) the five-site cluster approximation on the *kagomé* lattice. The arrows mark the places where f_c maximizes when $b=1.12$.

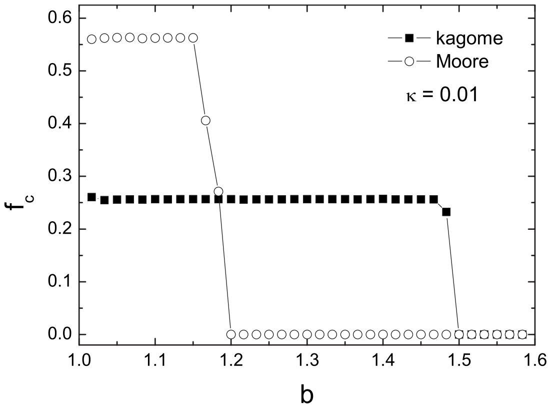

FIG. 7. Average fraction f_c of cooperation as a function of b in the case of very weak noise $\kappa=0.01$ on the *kagomé* lattice (solid squares) and the Moore lattice (open circles).

tion f_c seems to be insensitive to the temptation to defect staying as a plateau as cooperators are capable of surviving (an effect not studied in Ref. [40]). In the following, we provide a local configuration analysis for f_c in the stationary state on *kagomé* lattices.

In the limit of weak noise, the strategy-updating rules Eq. (1) or Eq. (2) is equivalent to the deterministic imitation dynamics [10], i.e., as long as the payoff of the strategy donor is greater than that of the strategy recipient, the strategy will migrate. Since either an isolated cooperator or a connected pair of cooperators cannot prevail surrounded by defectors, consider a five-site cluster of cooperators fully surrounded by defectors. If $b<3/2$, three cooperators forming a triangle will expand its territory to a five-site cluster, see Fig. 8. In that case we may expect that one of the neighboring defectors, say j, would since ($b<2$) imitate is strategy in the next sweep. Then their common neighbor k may acquire the payoff $2b$ in the next sweep. However, since i gets a total payoff 3; then, as long as $b<3/2$, there is no chance that ks strategy can migrate to i. In other words, such a local configuration in a sea of defectors if $b<3/2$ is stable. However, if a neighbor l of k was a cooperator, the total payoff of k would be $3b$ (which is greater than 3), and in that case k's strategy could migrate to i, hence destroying the original cluster. Taken together, the necessary condition for a five-site cluster of cooperators existing stably on the *kagomé* lattice requires that $b<3/2$ and, at the same time, there is no cooperator in their next-nearest neighborhood. Since the second-nearest neighbors may be shared by other five-site cluster of cooperators, we can estimate f_c to $5/(5+8+14/2)=1/4$ (8 and 14 are the number of nearest and next-nearest neighbors of the five-site cluster). This estimation is very close to the one obtained by MC simulations 0.256(2). Thus this local-configuration stability analysis gives a good prediction for f_c of evolutionary PD in the limit of weak noise on the *kagomé* lattice. Indeed, we have visualized the networked population and found many isolated five-site clusters of cooperators on the *kagomé* lattice (results now shown here). The same analysis, with the same conclusions can be done for the Moore lattice.

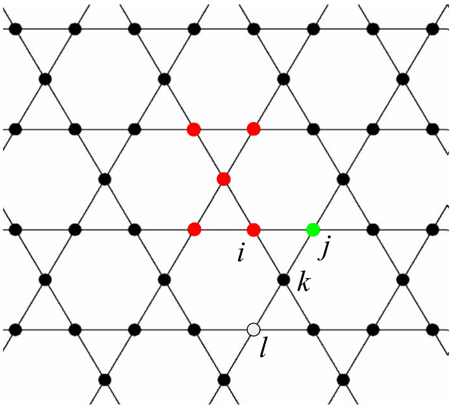

FIG. 8. (Color online) Local configuration stability analysis in the limit of zero noise on the *kagomé* lattice. The central five-site cluster of cooperators cannot be destroyed as long as $b<3/2$ even if no neighbor or next-nearest neighbor is a cooperator. See the text for details.

The above results might shed some light on the stability of cooperation in society. Real social networks often have distinct communities, where individuals in one groups is densely connected, having relatively few connections to other groups [47]. The emergence of community structure may hinder the spread of defection, just as in the case of the five-site cluster on the *kagomé* lattice above. In this sense, our society may not be as sensitive to defection as indicated by the PD on the von Neumann lattice. On the other hand, we also show that VMLD is more efficient in maintaining and promoting cooperation than IPLD. Therefore, in the social situations, if everyone tries learn more from others, it would perhaps be easier for people to work together efficiently.

IV. CONCLUSION

In summary, we have studied the effects of the direction of strategy migration and noise on the evolution of cooperation in the spatial prisoner's dilemma game. To this end, we have considered two types of strategy updating dynamics, namely, voter-model-like dynamics and invasion-process-like dynamics, on three types of lattice structures—square lattice with von Neumann and Moore neighborhoods and *kagomé* lattice. It was found that the VMLD, rather than the IPLD, better sustain and promote cooperation on all the tested lattices. Furthermore, we found noise to have a nontrivial effect on the evolution of cooperation in spatial population. First, for low temptation to defect, coherence resonance is found for all three types of lattice, i.e., there is an optimal noise level for promoting cooperation. Second, for an intermediate temptation to defect, we observe a two-peak behavior (for Moore and *kagomé* lattices)—the first maxima at zero noise, the other at a moderate noise levels (on the von Neumann lattice, however, we observe no such behavior). Third, for even higher temptation levels, only no noise maximizes cooperation. Moreover, we find that in the low-noise limit, the local lattice structure determines the fate of cooperators. We find, in the case of high temptation values, a structure of triangles overlapping at one site to benefit coop-

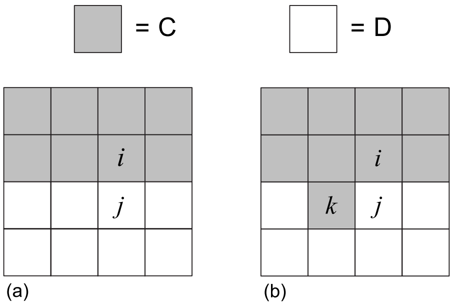

FIG. 9. Illustration of a small system with 4×4 players. (a) Both VMLD and IPLD give out the same transformation probability for j to change from a defector to a cooperator. (b) The VMLD favors better the diffusion of cooperation than the IPLD when the interface is rugged.

eration compared to a structure of triangles overlapping at multiple sites. For low temptation values, the situation is reversed and the multiply overlapping triangles are beneficial. Our MC-simulation results are in good agreement with theoretical predictions obtained from mean-field cluster approximation methods. These results may enrich our knowledge of the evolution of cooperation in spatially structured populations. A possible extension of the present work is to consider evolutionary PD on degree-heterogeneous networks with the incorporation of asymmetric direction of strategy migration. We expect that degree-heterogeneity would induce a stronger difference in the final results, as was found in voter model on such networks [35,46].

ACKNOWLEDGMENTS

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APPENDIX: A CONCRETE EXAMPLE

In this appendix we present a small example illustrating how the voterlike dynamics supports cooperation better than the invasionlike dynamics. For the sake of simplicity we consider a small system—a 4×4 square lattice with von Neumann neighborhood and periodic boundary conditions. Initially, half of the sites of the lattice are occupied by cooperators, and the other half by defectors as shown in Fig. 9(a). We focus our attention on the probability of strategy migration of one focal player, say j, changing from D to C. In the case of VMLD, this may happen when the individual j is selected to update its state and choose the neighbor i as reference, which happens with probability 1/4. According to the payoff matrix of the PD, the individual i will get payoff 3 since there are three cooperators and one defector in its neighborhood. In contrast, the individual j acquires only payoff b, which comes from the interaction with i. As a consequence, the expected probability for the player j changing from D to C is

$$W_{D\to C} = \frac{1}{4}\frac{1}{1+\exp[(b-3)/\kappa]} \tag{A1}$$

if VMLD is adopted. This is also the transformation probability if IPLD is implemented, rather than VMLD. Thus, on a smooth interface, both VMLD and IPLD will result in the same probability for strategy migration. Since the defectors on the interface gain much less payoff than their cooperative neighbors (b is just little larger than 1), we can expect cooperators could invade some of them as time evolves. Now, assume the configuration in Fig. 9(b), where the player j is surrounded by two cooperators, i and k. In such case, if j is selected to update strategy and VMLD is used, the probability for the player to become a cooperator is

$$\begin{aligned} W_{D\to C} &= \frac{1}{4}\left(\frac{1}{1+\exp[(2b-3)/\kappa]} + \frac{1}{1+\exp[(2b-1)/\kappa]}\right) \\ &= \frac{1}{4}(w_{ij}+w_{kj}), \end{aligned} \tag{A2}$$

where $w_{ij}/4$ and $w_{kj}/4$ denote, respectively, the probability of i's strategy, k's strategy convincing j. If, however, IPLD is adopted, the individual j could be imposed on the cooperative strategy either by i or k, whose probability is given by

$$W_{D\to C} = 1-(1-w_{ij}/4)(1-w_{kj}/4) = \frac{1}{4}\left(w_{ij}+w_{kj}-\frac{w_{ij}w_{kj}}{4}\right), \tag{A3}$$

which is smaller than the corresponding value from Eq. (A2). Note that we have assumed the neighbors of i, j, and k have not changed their strategies between the two events (from the updating time of i to that of j). In sum this illustrates how the VMLD favors the migration of cooperative strategy along the interface separating cooperators and defectors as compared to the IPLD. Note that this argument cannot be reversed to hold for cooperators instead of defectors [considering say i in Fig. 9(a) being a defector]. The reason is that, on average, the payoffs of defectors along the interface are smaller than those of the cooperators [18,33], which disables the benefit of the VMLD, and thereby inhibits the diffusion of defection.

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