Linearly polarized superluminal electromagnetic solitons in cold relativistic plasmas

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We investigate a special class of coupled nonlinear superluminal solitons arising from the interaction of an intense linearly polarized electromagnetic pulse with a cold plasma. These modulated envelope structures are obtained as numerical solutions of the classic Akhiezer-Polovin model equations [Sov. Phys. JETP **3**, 696 (1956)]. We also present a multiple time scale perturbation analysis in the small amplitude limit that provides a close analytic description of these nonlinear solutions.

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I. INTRODUCTION

The interaction of intense electromagnetic radiation with plasmas has ever remained an area of immense and active interest for plasma physicists due to the variety of interesting nonlinear phenomena that can arise during the process. Some well known examples of such nonlinear phenomena are self-focusing, harmonic generation, soliton formation, magnetic field generation, and a host of parametric instabilities [1]. One of the earliest and a classic paper in this field was written by Akhiezer and Polovin [2] who formulated a basic model for this problem and obtained some elegant nonlinear solutions for the electromagnetic and plasma waves in the relativistic regime. This pioneering work, carried out in the late fifties when laser powers of such high intensity were only a theoretical speculation, assumed a great significance in later years as high power lasers began to develop very rapidly and inspired a large number of theoretical investigations that exploited and further explored the model [3–14]. The model has also been extended to include finite ion response effects in several works [15–18]. Apart from the fundamental interest in the basic nonlinear phenomena and the novel solutions of the model equations, many of the investigations were also strongly motivated by possible applications of such solutions to experimental situations arising in radio-frequency heating of plasmas [3,19], laser fusion [20–24] and plasma based particle acceleration schemes [25,26]. Despite some of its limitations (e.g., neglect of ion dynamics, one-dimensional approximation etc.) the Akhiezer-Polovin model is a powerful paradigm for understanding many of the basic nonlinear phenomena associated with the interaction of an intense electromagnetic wave with a plasma [7–11] and the complete range of its nonlinear solutions has not been fully explored yet. Our present work is motivated by a desire to examine in greater details a class of nonlinear solutions of this model that has not received a great deal of attention in the past. To provide a brief historical perspective to these solutions we note that in their original paper Akhiezer and Polovin [2] obtained exact analytic nonlinear solutions in two special cases, namely, pure longitudinal waves and pure transverse waves. In general however these two waves are coupled in the nonlinear regime due to strong $\vec{v}\times\vec{B}$ forces where $\vec{B}$ is the magnetic field associated with the electromagnetic wave and $\vec{v}$ is the transverse electron fluid velocity. For such a case they obtained superluminal solutions in two limiting cases, e.g., $\beta=U/c\gg 1$ and $\beta-1\ll 1$, where U is the phase velocity of the wave and c is the speed of light. Further, their analysis for the $\beta\gg 1$ solutions was carried out for almost transverse modes where the magnitude of the longitudinal component of electron momentum was considered to be small. In a later work, Kaw and Dawson [3] investigated superluminal coupled longitudinal-transverse modes in the same two regimes for arbitrary amplitudes of the electromagnetic fields. They obtained such coupled solutions numerically. These coupled longitudinal-transverse linearly polarized solutions were analytically investigated in the $\beta\gg 1$ regime by Max and Perkins [4] in the limit of large transverse momentum of electrons and later by Chian and Clemmow [6] for arbitrary values of transverse electron momentum. In a review article Decoster [7] has summarized several analytical results in the superluminal ($\beta>1$) as well as subluminal ($\beta<1$) regime. A simplifying assumption in many of the above works was to treat one of the two coupled waves in the problem as a driven wave. This causes one degree of freedom to be suppressed and the resultant solutions are then a specialized subclass of solutions. Kaw, Sen and Valeo [8] pointed this out and studied the full system without suppressing any degree of freedom and showed the possibility of investigating a much wider variety of solutions. Essentially, they used the traveling wave ansatz where all dependent variables are assumed to be functions of a single variable $\zeta=x-Ut$, where U is the wave phase velocity and x and t are the independent space and time variables, respectively. In the wave frame the set of coupled nonlinear partial differential equations of the model reduces to a set of coupled nonlinear ordinary differential equations. For the coupled plasma wave-light wave problem these equations can be derived from a Hamiltonian which is a constant of motion. The problem is thereby converted to the standard classical mechanics problem of a "particle" in a two-dimensional nonlinear potential well and the various "orbits" of this pseudoparticle then correspond to various nonlinear solutions of the original wave propagation problem. Using this analogy and with the help of Poincare plots a wide variety of such nonlinear solutions were displayed and discussed in [8]. Broadly, these solutions can be classified as being periodic, quasiperiodic, amplitude modulated, chaotic, or solitonic in character. The solitonic solutions form a special class that arise at the separatrices of the Poincare plot and have not received much attention in the literature. They have been discussed briefly in the context of a relativistic beam driven model [10] and for a weakly relativistic model [27] in the framework of a nonlinear Schrödinger equation. In the present paper we present a detailed investigation of this class of solitonic solutions through numerical solutions of the Akhiezer-Polovin model in the regions of their existence. Our numerical solutions are further complemented by a multiscale perturbation analysis in the small amplitude limit that provides a close approximate analytic solution of these nonlinear envelope structures.

The paper has been organized as follows. In the next section we discuss the model equations and their reduction to a Hamiltonian system with two degrees of freedom. Also the numerical results corresponding to the soliton solutions are shown. In Sec. III we present a multiple time scale perturbative analysis to obtain envelope and phase equations which are then solved analytically to obtain the envelope soliton solutions. Finally in the last section our main results are summarized and briefly discussed.

II. MODEL EQUATIONS AND SOLITON SOLUTIONS

The interaction of a relativistic laser pulse with a cold plasma with ions forming a neutralizing positive background is well described by the one-dimensional fluid-Maxwell model where one-dimensional variations along the direction of propagation are considered,

$$\frac{\partial n}{\partial t}+\frac{\partial(nu)}{\partial x}=0, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)(\gamma u)=\frac{\partial\phi}{\partial x}-\frac{1}{2\gamma}\frac{\partial A_\perp^2}{\partial x}, \tag{2}$$

$$\frac{\partial^2\phi}{\partial x^2}=n-1, \tag{3}$$

$$\frac{\partial^2 A_\perp}{\partial x^2}-\frac{\partial^2 A_\perp}{\partial t^2}=\frac{nA_\perp}{\gamma}. \tag{4}$$

Equations (1)–(4) are, respectively, the electron continuity equation, electron (parallel) momentum equation, Poisson equation, and the electromagnetic wave equation. Here n, u, ϕ, and A stand for electron density, electron (parallel) fluid velocity, scalar potential, and electromagnetic vector potential, respectively. γ is the relativistic factor defined as

$$\gamma=\sqrt{\frac{1+A_\perp^2}{1-u^2}}$$

and other notations are standard. In writing the above equation we have chosen to normalize the density by the background plasma density n_0. The length is normalized by the corresponding electron skin depth c/ω_{pe0} (where $\omega_{pe0}=\sqrt{4\pi n_0 e^2/m_e}$ is the plasma frequency) and time by the inverse of the plasma frequency ω_{pe0}^{-1}; all velocities are normalized by the speed of light in vacuum, c. The scalar and vector potentials are normalized by $m_e c^2/e$, where m_e and e are the mass and electric charge of the electron, respectively.

Now making the traveling wave ansatz we do a coordinate transformation from the laboratory frame to a frame moving with the phase velocity of the electromagnetic wave viz. $\beta=U/c$. The transform in terms of the normalized quantities is defined as $\zeta=x-\beta t$. It is easy to show that for a linear polarization of the electromagnetic wave, so that $A_\perp=\hat{e}_1 a$, the set of fundamental equations [Eqs. (1)–(4)] can be simplified to obtain

$$(\beta^2-1)a_{\zeta\zeta}+\Lambda a=0, \tag{5}$$

$$\phi_{\zeta\zeta}=\delta n. \tag{6}$$

Here subscript "ζ" stands for derivative with respect to $\zeta=x-\beta t$ and Λ is defined as

$$\Lambda=\frac{\beta}{\sqrt{(a^2+1)(\beta^2-1)+(1+\phi)^2}}. \tag{7}$$

The above set of equations was first derived by Akhiezer and Polovin in [2] where they also obtained solutions corresponding to pure nonlinear longitudinal modes, pure transverse modes and coupled longitudinal-transverse modes. Equations (5) and (6) can be rewritten in terms of new variables defined by

$$(\beta^2-1)^{1/2}a=X,$$

$$1+\phi=-Z,$$

$$\frac{\zeta}{(\beta^2-1)^{1/2}}=\xi, \tag{8}$$

in the form,

$$\ddot{X}+\frac{\beta X}{\sqrt{\beta^2-1+X^2+Z^2}}=0, \tag{9}$$

$$\ddot{Z}+\frac{\beta Z}{\sqrt{\beta^2-1+X^2+Z^2}}+1=0, \tag{10}$$

where the overdots denote derivates w.r.t ξ. The system of these two coupled equations is found to admit the following constant of motion

$$H=\frac{1}{2}(\dot{X}^2+\dot{Z}^2)+\beta\sqrt{\beta^2-1+X^2+Z^2}+Z. \tag{11}$$

By numerically solving the above Eqs. (9) and (10) with Hamiltonian (11), Kaw *et al.* [8] investigated a wide range of possible solutions in different parametric regimes using Poincare surface of section plots for $\beta-1<1$ and for a range of H values. In the parameter regime studied by them, they could not find any chaotic solutions which led to a speculation that perhaps the system was integrable. The work of Grammaticos *et al.* [28] formally established nonintegrability by a mathematical analysis based on Ziglin's theorem. Subsequent numerical explorations succeeded in detecting chaotic orbits [29,30] in the region close to the separatrix orbits in the phase plane. Surprisingly, not much attention has been paid to the class of solitary wave solutions that correspond to the separatrix curves themselves. The only work that we are aware of is that of Hadzievski *et al.* [27] who investigated envelope solitary solutions analytically, for small amplitudes, in the framework of the nonlinear Schrödinger equation and who also discussed numerically observed large amplitude standing/slowly moving solitary solutions. Thus their work was restricted to the subluminal regime. There is no report to the best of our knowledge on the solitary solutions in the superluminal ($\beta \gg 1$) regime.

Our aim in the present work is to investigate in detail the class of solutions representing the solitary wave solutions in the superluminal ($\beta \gg 1$) regime. To do this we solve Eqs. (9)–(11) numerically using a fourth-order Runge-Kutta scheme for various parameter values. We zero in on the soliton solutions by examining the Poincare plots and choosing initial conditions that correspond to the separatrix orbits. Depending upon the values of β and H that we choose we can get soliton solutions over a range of amplitude values. Two typical solutions are shown in Figs. 1 and 2. for ($\beta=30, H=910$) and ($\beta=3, H=20$), respectively. The profiles of X and Z display a fast variation in the amplitude as well as a slow envelope modulation. The length scales of these variations are typically of the order of ξ and $\xi/(\beta^2-1)$, respectively. Note that the basic form of the spatial structure is quite similar for both the solutions although there is a substantial difference in their amplitudes. For the solution of Fig. 1, the amplitude of the electromagnetic wave $a \sim X/\beta \sim 0.15$ (weakly relativistic) while for Fig. 2 $a \sim X/\beta \sim 2$ (strongly relativistic). To get an analytic understanding of the basic form of these localized structures we carry out a multiple scale analysis of the solution corresponding to Fig. 1 where we exploit the wide separation in the variation scales of the "carrier" and "envelope" as well as the smallness of the amplitude. We present such an analysis in the next section.

III. MULTIPLE SCALE PERTURBATIVE ANALYSIS: ENVELOPE SOLUTION IN THE $\beta \gg 1$ REGIME

In the high phase velocity limit of $\beta \gg 1$ and for $(X^2+Z^2)/(\beta^2-1) \ll 1$, we can do a Taylor expansion of the nonlinear terms in Eqs. (9)–(11) and reduce them in the lowest nonlinear approximation to

$$\frac{d^2X}{d\xi^2} + \omega_0^2 X - \frac{\epsilon\omega_0^2}{2} X(X^2+Z^2) = 0, \tag{12}$$

$$\frac{d^2Z}{d\xi^2} + \omega_0^2 Z - \frac{\epsilon\omega_0^2}{2} Z(X^2+Z^2) + 1 = 0. \tag{13}$$

The Hamiltonian for these approximated equations is given by

$$H = \frac{1}{2}(\dot{X}^2+\dot{Z}^2) + \frac{\omega_0^2}{2}(X^2+Z^2) - \frac{\epsilon\omega_0^2}{8}[X^2+Z^2]^2 + Z, \tag{14}$$

where

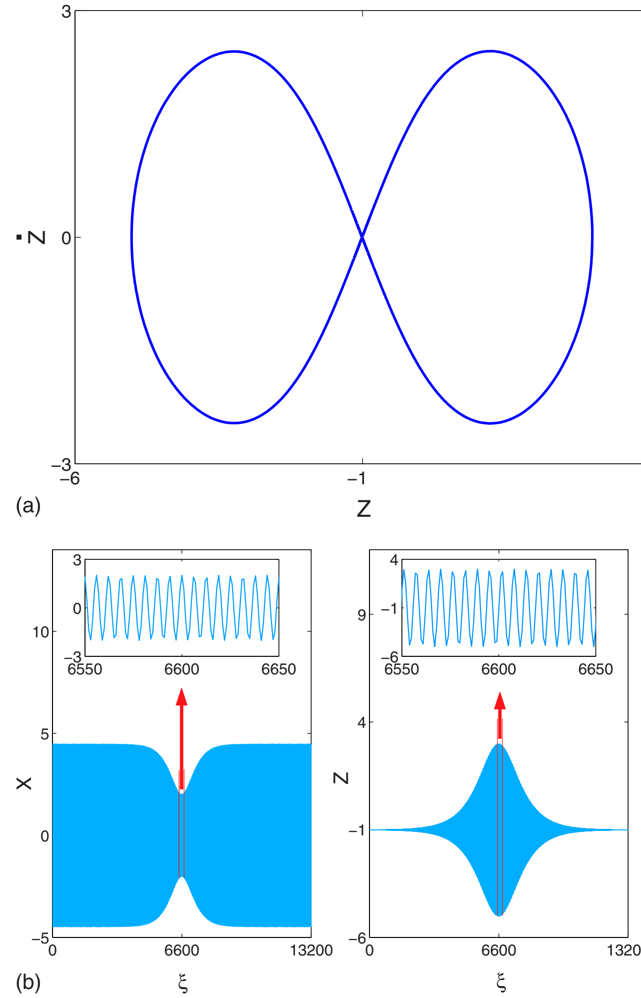

FIG. 1. (Color online) (a) Separatrix curve in Poincare surface of section plot, $\dot{Z}$ vs Z ($X=0, \dot{X}>0$), for $\beta=30$ and $H=910$. (b) (Color online) Soliton solution corresponding to separatrix curve shown in Fig. 1(a). Left plot corresponds to transverse field X and right plot shows the profile of electrostatic field Z. In inset of each of the two plots, expanded view around the central part of the corresponding profile is shown.

$$\omega_0 = \sqrt{\frac{\beta}{\sqrt{\beta^2-1}}}$$

is the frequency of the linearized equations, and

$$\epsilon = \frac{1}{\beta^2-1}$$

is the smallness parameter.

Now following the standard procedure of multiple time scale perturbation analysis [31], we define new time variables $\xi_0=\xi$ and $\xi_1=\epsilon\xi$ so that we obtain

$$\frac{d}{d\xi} = \frac{\partial}{\partial\xi_0} + \epsilon\frac{\partial}{\partial\xi_1},$$

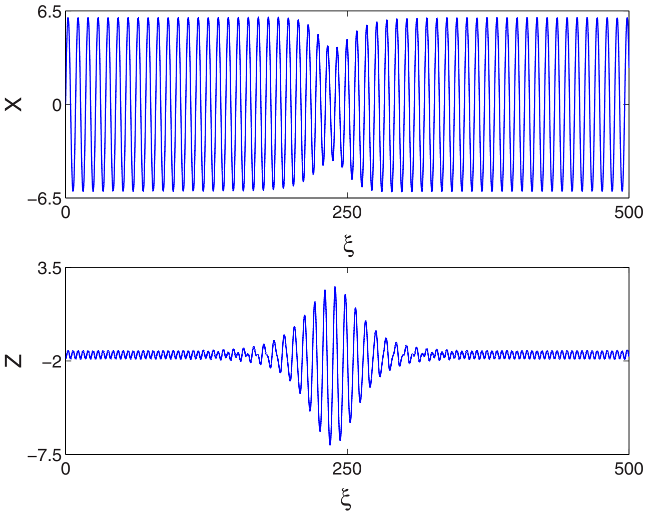

FIG. 2. (Color online) A larger amplitude soliton solution for parameter set $\beta=3$ and $H=20$. Upper plot corresponds to transverse field X and lower plot shows the profile of electrostatic field Z.

$$\frac{d^2}{d\xi^2}=\frac{\partial^2}{\partial\xi_0^2}+2\epsilon\frac{\partial^2}{\partial\xi_1\,\partial\xi_0}+O(\epsilon^2), \tag{15}$$

Also the fields can be expanded as

$$X=X^{(0)}+\epsilon X^{(1)},$$

$$Z=Z^{(0)}+\epsilon Z^{(1)}, \tag{16}$$

Substituting Eqs. (15) and (16) into Eq. (12) and (13) and separating terms of ϵ^0 and ϵ^1 we obtain for the X oscillator, ϵ^0 terms

$$\left(\frac{\partial^2}{\partial\xi_0^2}+\omega_0^2\right)X^{(0)}=0, \tag{17}$$

ϵ^1 terms

$$\left(\frac{\partial^2}{\partial\xi_0^2}+\omega_0^2\right)X^{(1)}=-2\frac{\partial^2}{\partial\xi_1\,\partial\xi_0}X^{(0)}+\frac{\omega_0^2}{2}X^{(0)}[X^{(0)^2}+Z^{(0)^2}]. \tag{18}$$

Similarly for the Z oscillator we obtain
ϵ^0 terms

$$\left(\frac{\partial^2}{\partial\xi_0^2}+\omega_0^2\right)Z^{(0)}=-1, \tag{19}$$

ϵ^1 terms

$$\left(\frac{\partial^2}{\partial\xi_0^2}+\omega_0^2\right)Z^{(1)}=-2\frac{\partial^2}{\partial\xi_1\,\partial\xi_0}Z^{(0)}+\frac{\omega_0^2}{2}Z^{(0)}[X^{(0)^2}+Z^{(0)^2}]. \tag{20}$$

Equations (17) and (19) can be easily solved to obtain the zeroth order solution

$$X^{(0)}=A(\xi_1)\cos[\omega_0\xi_0+\phi_1(\xi_1)], \tag{21}$$

$$Z^{(0)}=-\frac{1}{\omega_0^2}+B(\xi_1)\cos[\omega_0\xi_0+\phi_2(\xi_1)]. \tag{22}$$

While writing the above zeroth order solution we take into account the fact that the amplitudes A, B and the phases ϕ_1, ϕ_2 which are constants on the faster time scale (ξ_0) vary on the slower time scale (ξ_1). Substituting for $X^{(0)}$ and $Z^{(0)}$ into Eqs. (18) and (20) and solving for $X^{(1)}$ and $Z^{(1)}$ we note that the solution both for X as well as Z consists of secular and nonsecular terms. On equating the secular part of the solution to zero and averaging over fast oscillations of frequency ω_0 one obtains the evolution equation for the slowly varying envelopes and the respective phases. The equations thus obtained read as

$$\frac{\partial A}{\partial\xi_1}=\frac{\omega_0AB^2}{16}\sin 2[(\phi_2-\phi_1)], \tag{23}$$

$$\frac{\partial\phi_1}{\partial\xi_1}=-\frac{1}{4\omega_0^3}-\frac{\omega_0}{16}[3A^2+B^2\{2+\cos[2(\phi_2-\phi_1)]\}], \tag{24}$$

$$\frac{\partial B}{\partial\xi_1}=-\frac{\omega_0A^2B}{16}\sin[2(\phi_2-\phi_1)], \tag{25}$$

$$\frac{\partial\phi_2}{\partial\xi_1}=-\frac{3}{4\omega_0^3}-\frac{\omega_0}{16}[3B^2+A^2\{2+\cos[2(\phi_2-\phi_1)]\}]. \tag{26}$$

It follows from Eqs. (23) and (25) that

$$A^2+B^2=\text{constant}=C^2 \tag{27}$$

Now using Eqs. (24), (26), and (27) one obtains

$$\frac{\partial(\phi_2-\phi_1)}{\partial\xi_1}=-\frac{1}{2\omega_0^3}+\frac{\omega_0(2A^2-C^2)}{8}\sin^2(\phi_2-\phi_1). \tag{28}$$

We also rewrite Eq. (23) using Eq. (27) as

$$\frac{\partial A}{\partial\xi_1}=\frac{\omega_0A(C^2-A^2)}{16}\sin[2(\phi_2-\phi_1)]. \tag{29}$$

We now eliminate $(\phi_2-\phi_1)$ from Eqs. (28) and (29) to obtain

$$\left(\frac{dA}{d\xi_1}\right)^2=\frac{1}{64\omega_0^6A^2}[-\omega_0^8C_1^2+(\omega_0^8C_1C^2+8\omega_0^4C_1)A^2 -(\omega_0^8C_1+4\omega_0^4C^2+16)A^4+4\omega_0^4A^6], \tag{30}$$

where $C_1=A^2(C^2-A^2)\sin^2(\phi_2-\phi_1)+4A^2/\omega_0^4$ is a constant of integration and is obtained from the initial conditions. Initial values for A, B, ϕ_1, and ϕ_2 are obtained from the numerical solution itself. In obtaining the numerical solution of Eqs. (12) and (13) the initial conditions are chosen as $X=0.0$, $\dot{X}=0.0$, $Z=-1/\omega_0^2$ and the initial value of $\dot{Z}$ is obtained, for the given parameters (β and H values), from Hamiltonian expression [Eq. (14)].

Similarly for B the equation comes out to be

$$\left(\frac{dB}{d\xi_1}\right)^2 = \frac{1}{64\omega_0^6 B^2}[-\omega_0^8 C_2^2 + (\omega_0^8 C_2 C^2 - 8\omega_0^4 C_2)B^2 + (-\omega_0^8 C_2 + 4\omega_0^4 C^2 - 16)B^4 - 4\omega_0^4 B^6], \quad (31)$$

where $C_2 = B^2(C^2 - B^2)\sin^2(\phi_2 - \phi_1) - 4B^2/\omega_0^4$ is another constant of integration and is obtained from the initial conditions in a similar manner as C_1.

Equations (30) and (31) can now be solved exactly to obtain the expressions for A and B as

$$A(\xi) = \left[\frac{K_3 + \sqrt{K_3^2 - 3K_2K_4}}{3K_4} - \frac{\sqrt{K_3^2 - 3K_2K_4}}{K_4} \times \text{sech}^2\left(\frac{(K_3^2 - 3K_2K_4)^{1/4}}{2}(\epsilon\xi + d_1)\right)\right]^{1/2}, \quad (32)$$

$$B(\xi) = \left[-\frac{L_3 + \sqrt{L_3^2 + 3L_2L_4}}{3L_4} + \frac{\sqrt{L_3^2 + 3L_2L_4}}{L_4} \times \text{sech}^2\left(\frac{(L_3^2 + 3L_2L_4)^{1/4}}{2}(\epsilon\xi + d_2)\right)\right]^{1/2}, \quad (33)$$

where we have used $\xi_1 = \epsilon\xi$; d_1 and d_2 are constants of integration and can be calculated using the initial values of A and B and the values of constants C, C_1, and C_2 which are obtained as described earlier in this section. Other symbols viz. K_1, K_2, K_3, K_4, L_1, L_2, L_3, and L_4 have the following definitions:

$$K_1 = \frac{\omega_0^2 C_1^2}{16},$$

$$K_2 = \frac{1}{16\omega_0^6}(\omega_0^8 C_1 C^2 + 8\omega_0^4 C_1),$$

$$K_3 = \frac{1}{16\omega_0^6}(\omega_0^8 C_1 + 4\omega_0^4 C^2 + 16),$$

$$K_4 = \frac{1}{4\omega_0^2},$$

$$L_1 = \frac{\omega_0^2 C_2^2}{16},$$

$$L_2 = \frac{1}{16\omega_0^6}(\omega_0^8 C_2 C^2 - 8\omega_0^4 C_2),$$

$$L_3 = \frac{1}{16\omega_0^6}(\omega_0^8 C_2 - 4\omega_0^4 C^2 + 16),$$

$$L_4 = \frac{1}{4\omega_0^2}.$$

In Fig. 3 we compare the profile of the analytically obtained envelope solutions A and B [Eqs. (32) and (33)] with the exact numerical solutions for X and Z, shown in Fig. 1(b). As can be seen, the analytical solutions envelope the numerical solutions quite well.

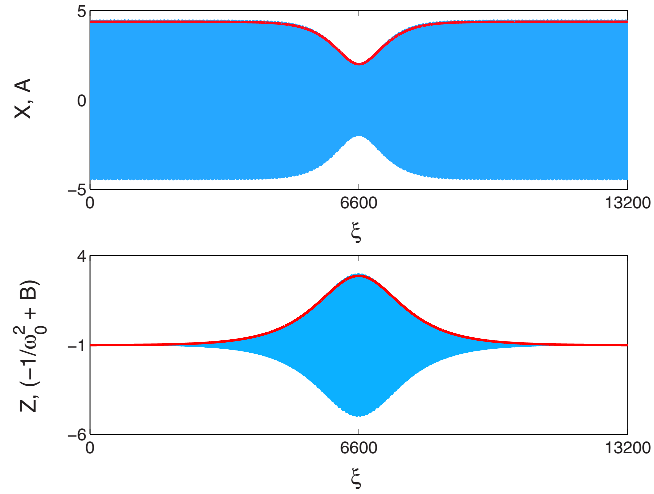

FIG. 3. (Color online) Comparison of the analytically obtained envelopes with the numerically obtained exact solution for $\beta=30$ and $H=910$. The analytical envelope curves drawn in solid lines are seen to fit the envelope of the numerical profiles quite well.

IV. SUMMARY AND DISCUSSION

We have investigated a special class of coupled nonlinear stationary solutions of the Akhiezer-Polovin model that consists of envelope solitary wave structures. These localized structures travel with superluminal phase velocities and physically constitute modulated structures of light waves and plasma waves that can arise from the interaction of a linearly polarized intense laser wave with a relativistic cold plasma. Our numerical investigations show that these solutions can be found for a variety of amplitudes ranging from low, weakly relativistic ones to the large strongly relativistic regime. The basic spatial structure of these solitons is quite similar over this range of amplitudes and we can get an analytic understanding of this form by subjecting the low amplitude solitons to a multiple scale analysis. The minimal set of the coupled equations then take the form of Eqs. (12) and (13) which we are able to solve analytically to obtain the explicit solutions (21) and (22) with Eqs. (32) and (33). These solutions agree quite well with our numerical findings and constitute the principal result of our present work. In a sense our work is complementary to that of Hadzievski *et al.* [27] who had carried out a similar analysis for linearly polarized solitons in the weak relativistic limit. The principal difference between our works is that their solitons travel at subluminal velocities and obey a generalized form of the nonlinear Schrödinger equation in contrast to the set of envelope equations that our superluminal solitons satisfy [Eqs. (30) and (31)]. The difference in the nature of these equations would also introduce subtle differences in the shapes and sizes of the localized structures which can be obtained by a direct comparison of their "implicit" solution to our explicit analytic solutions.

We next discuss the question of the stability of these localized structures. Past analysis of subluminal electromagnetic solitons indicate that in general solitons created from circularly polarized light waves tend to be more stable than those from linearly polarized ones. There is however no general principle or fundamental reason for this to happen and the stability question needs to be addressed independently and specifically for each class of solutions. As reported in [27,32] there are parameter regimes where stable subluminal linearly polarized solitons can exist. Even when they do not exist the relevant question is the time scale for which they persist before disintegrating. For superluminal localized structures Rozanov [33] has put forward heuristic arguments stating that they would always be unstable and have a finite life time of the order of $L/(U-c)$ where L is the typical size of the structure, U is its phase speed and c is the speed of light. For our typical solutions this life time turns out to be of the order of several plasma periods. Over this life time they can give rise to interesting physical phenomena [33] such as the excitation of wake fields (e.g., due to the Vavilov-Cerenkov effect) whose signatures may be detected in intense laser plasma interactions. For a more conclusive view on their existence and the time scale for which they persist their stability analysis has to be worked out. Our approximate analytic solutions may prove useful in such a calculation and we hope to report on such a work in the future.

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