Experimental determination of heat transfer coefficient in the slip regime and its anomalously low value

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In this paper, the measurement of the heat transfer coefficient in rarefied gases is presented; these are among the first heat transfer measurements in the slip flow regime. The experimental setup is validated by comparing friction factor in the slip regime and heat transfer coefficient in the continuum regime. Experimental results suggest that the Nusselt number is a function of Reynolds and Knudsen numbers in the slip flow regime. The measured values for Nusselt numbers are smaller than that predicted by theoretical or simulation results, and can become a few orders of magnitude smaller than the theoretical values in the continuum regime. The results are repeatable and expected to be useful for further experimentation and modeling of flow in the slip and transition regimes.

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I. INTRODUCTION

The subject of this paper is measurement of the amount of heat that can be transferred from a heated circular tube to colder rarefied gas flowing through the tube. Although results with gas at atmospheric or higher pressures are readily available, we consider the case of such low gas pressures that the flow is in the slip regime ($10^{-3} < \mathrm{Kn} < 10^{-1}$, where Kn is the Knudsen number defined as the ratio of mean free path of the gas (λ) to the tube diameter, D). The slip regime is characterized by velocity slip and temperature jump at the wall, implying that both the velocity and temperature of the gas at the wall is different from that of the wall [1]. The amounts of velocity slip and temperature jump at the wall depend on the gas-solid pair, amount of rarefaction, among other parameters [2,3]. Owing to this jump, there is difficulty in prescribing the boundary conditions. The heat transfer coefficient is dependent on the velocity slip and temperature jump at the wall and their precise values need to be obtained empirically; this motivated us to experimentally determine the value of the heat transfer coefficient. It is noted that such a problem has hitherto been considered theoretically [4–14] and experimental data is totally missing. The aim of this work is therefore to provide the experimental data set which can be used, for example, for testing the theoretical results.

The results will also be useful for understanding gas flow in microchannel, where Knudsen number again becomes a relevant parameter due to the small length scale. Study of gas flow through microchannels is important because of potential applications of microdevices in engineering, medical, and other scientific areas, and has therefore resulted in a large number of publications in this area [15–20].

The validity of the continuum hypothesis in the slip and transition ($10^{-1} < \mathrm{Kn} < 10$) regimes has been debated in the literature [19,21,22]. The applicability of the classical Navier-Stokes and Fourier equations therefore becomes questionable. The Navier-Stokes-Fourier equations have been replaced by the more general equations such as Burnett, super-Burnett, and regularized 13-moment equations [4,23,24]. These more general equations are accurate to second order or higher in Knudsen number and obtained from the Chapman-Enskog expansion of the Boltzmann equation. The numerical solution of the Boltzmann equation itself is expensive and only limited solutions to these more general equations are available [25,26]. In the following we review the available analytical solutions which are all based on the Navier-Stokes-Fourier equations. It should be noted that these equations are still difficult to solve analytically; appropriate assumptions are therefore employed to remove certain terms so as to reduce the level of the complexity.

Ameel *et al.* [6] and Kavehpour *et al.* [7] were among the first to obtain an analytical solution of gas flow in microtubes and microchannels with heat transfer. Both of them employed a velocity and temperature jump conditions at the walls, and found a substantial reduction in Nusselt number (Nu) with an increase in Knudsen number (Kn). Larrode *et al.* [8] considered a circular tube with constant wall temperature at the wall, hydrodynamically fully developed flow, constant properties fluid, high-Peclet number, and negligible energy dissipation. The Nusselt number was obtained as a series solution. The authors concluded that with increasing rarefaction, the Nusselt number may increase, decrease, or remain unchanged depending on the ratio of velocity to temperature jump coefficients. The reason offered for higher heat transfer coefficient with rarefaction as compared to continuum regime, was a larger velocity at the wall (because of velocity slip) leading to increased convective heat transfer. Hadjiconstantinou and Simek [9] included the effect of axial conduction for flow in microchannels with constant wall temperature in the slip and transition regimes. They found a decrease in Nusselt number with an increase in Knudsen number. Tunc and Bayazitoglu [10] solved the two-dimensional energy equation with viscous dissipation using the integral transform technique, for both uniform temperature and uniform heat flux boundary conditions. The viscous term was nondimensionalized in terms of the Brinkman

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number and the effect of Brinkman number on Nusselt number was studied. Note that the Brinkmann number is a measure of the ratio of viscous energy dissipation to heat conduction

$$\mathrm{Br} = \frac{\mu u^2}{k_f \Delta T}. \tag{1}$$

where, μ is the dynamic viscosity of gas; u is the mean stream wise velocity; k_f is the thermal conductivity of the gas; and T is the absolute temperature. Brinkmann number is analogous to Eckart number (Br=Ec.Pr where Ec is Eckart number and Pr is Prandtl number). Under fully developed condition, the Nusselt number was found to increase with Brinkman number for the constant wall temperature case, while the opposite was reported for the constant heat flux case. Further, a decrease in Prandtl number was found to increase the Nusselt number because of a corresponding decrease in the temperature jump. However, no explanation for the difference in behavior of Brinkman number with boundary condition was offered. Vasudevaiah and Balamurugan [11] solved the energy equation for an ideal Newtonian gas for plane and corrugated channels. Zhu and Liao [12] solved the more general problem of the heat transfer in microchannels of arbitrary cross-sections. Lockerby *et al.* [27] suggested that a Knudsen layer exists near the walls under rarefied condition and the slip boundary condition is somewhat artificial, employed to accommodate the Knudsen layer.

Choi *et al.* [28] measured the friction factor and heat transfer coefficient for microtubes with inside diameter of 3 and 81 μm, with nitrogen as the working fluid. However, due to the large inlet pressures (5.7–10 MPa), the Knudsen number is relatively small in their measurements and the results were not presented in terms of Kn. Interestingly, the Nusselt number was found to depend on Reynolds number (Re), with the exponent of Re being 1.17 and 1.96 in the laminar and turbulent regimes, respectively. Yan and Farouk [29] used direct simulation Monte Carlo to compute fluid flow and heat transfer for a mixture of noble gases. These authors proposed a correlation connecting the Nusselt number to Knudsen and Peclet numbers.

The literature survey shows that there is near consensus about a decrease in Nusselt number with an increase in Knudsen number; however, there are differences in the magnitude of reduction. In particular, whereas Refs. [6,8,10,30] suggest a Nusselt number value of around 3.6 (which is close to the continuum value), Hooman [31] suggest a value of 8.58 at Knudsen number of 0.01 for constant wall temperature boundary condition case. Experimental determination therefore becomes all the more relevant, especially in the absence of any data on Nusselt number in the slip regime. This paper provides such set of experimental data, and substantial difference with respect to theoretical analysis is suggested by the results. The scope of the present work is experimental and we provide only a few hypotheses that may partially explain the large reduction in the value of the heat transfer coefficient.

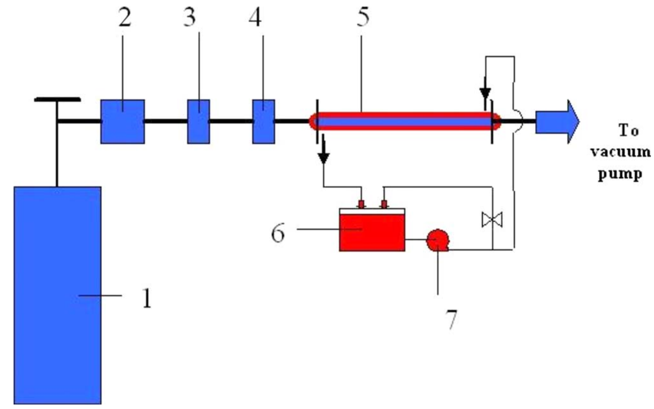

FIG. 1. (Color online) Layout of the experimental setup: 1-nitrogen cylinder; 2-pressure regulator; 3-particle filter; 4-mass flow controller; 5-counter flow tube-in-tube heat exchanger; 6-hot water tank; and 7-water pump.

II. EXPERIMENTAL SETUP AND PROCEDURE

Experimental setup built for the purpose of this work and the procedure followed while performing the experiments are discussed in this section. It is noted that our interest is in working with relatively large Knudsen numbers, which can be achieved by either reducing the dimensions of the tube or increasing the mean free path of the gas (i.e., by reducing the pressure in the tube). We have chosen the latter approach because of difficulty in making measurements at small scales required in the former approach. However, the results are applicable to both micro and macrodomains under similar values of the nondimensional parameters.

Figure 1 shows the schematic of experimental setup used for the measurement of heat transfer coefficient in a circular pipe with nitrogen flowing in the slip regime. The experimental facility consists of an oil diffusion pump (Hindhivac VS150D) connected to a tube-in-tube heat exchanger. The flow of nitrogen in the inner tube of the heat exchanger is driven by the vacuum pump. The ultimate vacuum achievable by this vacuum pump is about 10^{-6} mbar in an empty, clean, and outgassed chamber. A mass flow controller (MKS 1179A) is used to control and meter the mass of nitrogen flowing in the tube. Particles of size greater than 20 μm are trapped by a particle filter device at the upstream of the controller. The pressures at the inlet and outlet of the inner tube are measured by an absolute pressure transducer (MKS 626A, range of 0.01–100 mbar), so that the corresponding Knudsen numbers can be calculated. The tap at upstream is kept at a distance greater than the development length for laminar flow in a circular tube [32].

Figure 2 shows the tube-in-tube heat exchanger made of stainless steel. The inner tube is a circular tube of length 0.96 m, with an inner diameter of 25 mm and an outer diameter of 30 mm. The surface roughness of the commercial stainless steel pipe chosen in the present study is around 0.05 mm. The outer tube is also circular with an inner diameter of 50 mm. This heat exchanger is used for the measurement of heat transfer coefficient with nitrogen at low pressure flowing in the inner tube and hot water in the annulus. The hot water heats up the nitrogen. The mass flow rate for water is main-

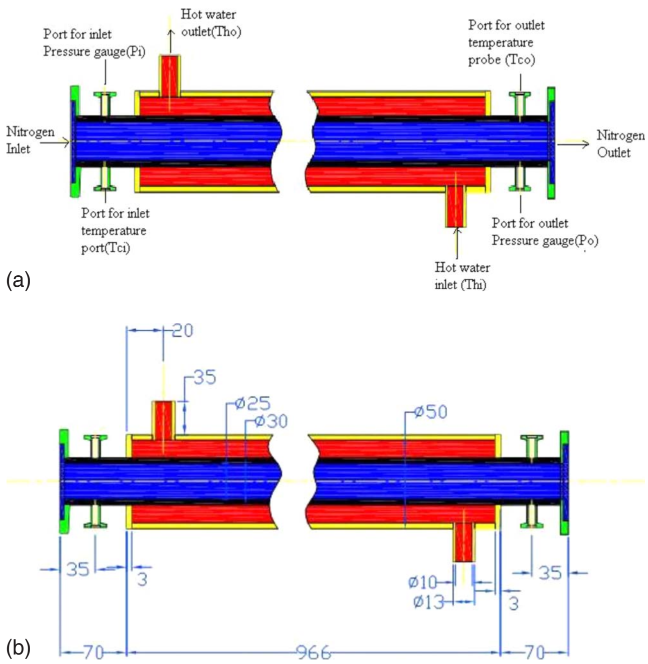

FIG. 2. (Color online) (a) Schematic of the tube-in-tube heat exchanger and (b) schematic with dimensions (in mm) of the heat exchanger.

tained constant at about 44 g/s. The inlet and outlet temperatures of both nitrogen and water are measured using thermocouples situated at the inlet and outlet of each tube. The thermocouples on the nitrogen side are secured with flanges and care is taken to ensure that there is no leakage of atmospheric air into the tube. The thermocouples on the water side are held in position by sealant. The entire heat exchanger unit is insulated with thermowool in order to minimize the losses from the hot water to the atmosphere.

Nitrogen is drawn from a cylinder at 1.5 bar which flows through the tube-in-tube heat exchanger. A pressure regulator, at downstream of the cylinder, is used to control the pressure upstream (1.5–2 bar) of the mass flow controller. The mass flow controller is used to adjust the mass flow rate of gas flowing in the system.

The setup is tested for leakage and the inner tube of the tube-in-tube heat exchanger is evacuated and kept under high vacuum (i.e., $<10^{-3}$ mbar) for six hours before any measurement is conducted. This is done to avoid outgassing effect. The leakage into the setup is estimated to be less than 0.1% of the total flow rate in the tube. In order to ensure steady state condition, measurements were taken after every fixed time interval (of 10 min). Steady state is assumed to have reached only if three consecutive readings show the same value. In order to evaluate the time required for reaching steady state precisely, few cases are allowed to run for very long time, but negligible difference in the results were obtained. The water is heated in a tank (of capacity 12 L) to a temperature of around 60 °C to 80 °C. The temperature in the tank is maintained constant by an adjustable thermostat. The hot water is drawn from the tank by a centrifugal pump (Grunfos) and supplied to tube-in-tube heat exchanger, and routed back to the hot water tank after passing through the heat exchanger. A by-pass line from the exit of the pump is used to control the mass flow rate of the water flowing through the tube-in-tube heat exchanger. The mass flow rate of gas and water are maintained constant with the help of mass flow controller and pump, respectively. Temperature of hot water and nitrogen at the inlet and outlet of tube-in-tube heat exchanger, pressure at the inlet and outlet of the tube-in-tube heat exchanger in the tube side (i.e., nitrogen side) for a given mass flow rate of nitrogen and hot water are recorded after the system reaches the steady state. Experiments are repeated for different Knudsen numbers and Reynolds numbers. The ranges of Knudsen and Reynolds numbers covered in the present study are 0.00011–0.01516 and 0.5–174. The pressure ratio (outlet pressure inlet pressure) is the range of 0.877–0.996. The Reynolds number on the water side is 1270.

III. DATA REDUCTION

The data reduction techniques employed is discussed in this section. The Reynolds number for a given mass flow rate ($\dot{m}$) is given by

$$\mathrm{Re} = \frac{4\dot{m}}{\pi D \mu}. \tag{2}$$

where D is hydraulic diameter of the tube. The hydraulic diameter for inner tube is 25 mm and annulus is 20 mm.

The mean free path of the gas is calculated as [32]

$$\lambda = \frac{\mu}{p}\sqrt{\frac{\pi R T}{2}}, \tag{3}$$

where R is the specific gas constant and p is the pressure. The viscosity of nitrogen at mean temperature is used while calculating the Knudsen number. The Knudsen number (λ/D) at the inlet and outlet of the nitrogen side of the tube-in-tube heat exchanger is calculated using the inlet and outlet pressures and temperatures, and all results are presented in terms of the arithmetic average of the inlet and outlet Knudsen numbers.

The Grashof number is estimated as

$$\mathrm{Gr} = \frac{\rho^2 g \beta (T_w - T_m) D^3}{\mu^2}, \tag{4}$$

where, ρ is density of the fluid, g is the acceleration due to gravity, β is coefficient of volume expansion, T_w and T_m represent temperature of the wall and mean gas temperature, respectively. In continuum flows friction factor (f) for incompressible flow can be expressed in terms of pressure drop (Δp). However, in slip flow regime nonlinearity in pressure distribution is present due to compressibility and rarefaction effects [16,32]. The total pressure drop in such a case for flow in a straight uniform cross-section channel is expressed in terms of both frictional and acceleration pressure drop components as

$$\frac{p_i - p_o}{p_i} = \frac{1}{2} G^2 \frac{RT_m}{p_i^2} \left[f \frac{L}{D} \frac{p_i}{p_m} + 2 \left(\frac{p_i}{p_o} - 1 \right) \right], \tag{5}$$

where p_i and p_o are the pressures at inlet and outlet of the nitrogen side of tube-in-tube heat exchanger, p_m is the mean pressure, T_m is the mean temperature, L is the total length of the tube, G is the mass velocity ($G = \frac{\dot{m}}{A}$), and A is the inner cross-sectional area of the tube. Note that in Eq. (5) the first term on the right-hand corresponds to frictional pressure drop and second term gives the acceleration pressure drop [33].

The procedure involved in the calculation of Nusselt number on the nitrogen side (i.e., tube side) is as follows: knowing the inlet and outlet temperatures of both hot and cold fluids, the log mean temperature difference (ΔT_{LMTD}) across the heat exchange is calculated as

$$\Delta T_{LMTD} = \frac{(T_{ci} - T_{ho}) - (T_{co} - T_{hi})}{\ln \frac{(T_{ci} - T_{ho})}{(T_{co} - T_{hi})}}, \tag{6}$$

where, T_{hi} and T_{ho} are the temperatures of the hot fluid at inlet and outlet, respectively [Fig. 2(a)]. Similarly, T_{ci} and T_{co} are the temperatures of the cold fluid at inlet and outlet, respectively. The overall heat transfer coefficient for the heat exchanger (UA) can be obtained from the energy balance, using

$$(UA)\Delta T_{LMTD} = \dot{m} C_p (T_{co} - T_{ci}), \tag{7}$$

where C_p is the specific heat of nitrogen. UA is also given by

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(r_o/r)_i}{2\pi k L} + \frac{1}{h_o A_o}. \tag{8}$$

where r_i is inner radius of the inner tube, r_o is outer radius of the inner tube, k is the thermal conductivity of the tube material, A is surface area, h is the heat transfer coefficient, and subscripts i and o refer to the inner (nitrogen) and outer (water) side of the tube. The flow rate of water in the annulus of the tube-in-tube heat exchanger being kept always laminar (Re=1270). For the lowest value of gas heat transfer coefficient the relative importance of the three terms of Eq. (8) has been analyzed. Heat transfer coefficient at the water side is calculated by using relation $\mathrm{Nu} = 3.66 = h_o D_o / k$. The ratio of conductive resistance of the wall to the convective resistance of the gas is 7.33×10^{-8} and ratio of convective resistances of water and gas is 3.01×10^{-6}. Similar calculations for highest value of gas heat transfer coefficient have also been done. The ratio of resistance, of the wall and the gas is 1.48×10^{-4} and water to the gas is 7.62×10^{-3}. Therefore, second and third terms on the right-hand side of the Eq. (8) are very small as compared to $(h_i A_i)^{-1}$ and can be neglected. Equation (8) reduces to

$$UA = h_i A_i, \tag{9}$$

and h_i can now be readily calculated. The validity of Eq. (9) was verified for all measurements. Finally, the Nusselt number is given by

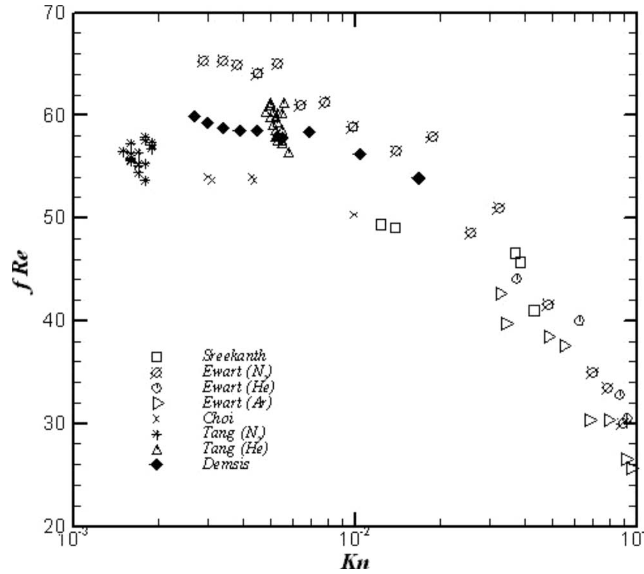

FIG. 3. Friction factor times Reynolds number as a function for Knudsen number—present data and comparison with literature. The friction factor for Sreekanth was calculated based on data in his paper.

$$\mathrm{Nu} = \frac{h_i D}{k_f}. \tag{10}$$

The maximum uncertainties in Nusselt and Knudsen numbers are 10 and 0.5%, respectively. The uncertainties reduce with an increase in the mass flow rate—at the highest mass flow rate covered all the nondimensional numbers mentioned have uncertainties of less than 5%.

IV. VALIDATION OF THE SETUP

The friction factor under rarefied condition and measurement of the heat transfer coefficient under normal (continuum) conditions are considered as validation parameters for the setup.

The friction factor (f) times Reynolds number as a function of Knudsen number is shown in Fig. 3. Under normal (continuum) conditions, for a tube, f Re is a constant with a value of 64. This is however not the case for rarefied gas—the value of f Re is less than 64 and decreases monotonically with an increase in Knudsen number (see also Verma *et al.* [34]). This finding matches with the earlier results [17,18,28,32].

The setup is also validated for heat transfer coefficient in the continuum regime. The Nusselt number from the present experiment is found to compare with the Dittus-Bolter correlation within 12% for two values of Reynolds number investigated. The above exercises validate our setup in the rarefied regime and tools employed in heat transfer measurements. Note that further validation in the slip regime is not possible due to nonavailability of heat transfer results in the literature. The overall agreement with available results establishes confidence in the experimental setup and measurement techniques.

TABLE I. Experimental data used for the calculation of the Nusselt number for different Knudsen numbers and Reynolds numbers.

$\dot{m}$ (N_2) (g/s)	$\dot{m}$ (H_2O) (g/s)	T_{co} (°C)	T_{ci} (°C)	T_{hi} (°C)	T_{ho} (°C)	p_m (mbar)	h (W/m^2 K)	Nu	Kn_m	Re	Gr/Re^2
1.87×10^{-4}	44.0	45.7	42.4	66.9	66.4	0.164	3.68×10^{-4}	3.47×10^{-4}	1.69×10^{-2}	0.54	4.39×10^{-3}
3.74×10^{-4}	44.0	45.2	41.0	64.4	64.9	0.244	1.01×10^{-3}	9.51×10^{-4}	1.13×10^{-2}	1.07	2.16×10^{-3}
5.62×10^{-4}	44.0	46.1	41.5	65.1	65.1	0.31	1.71×10^{-3}	1.61×10^{-3}	8.87×10^{-3}	1.61	1.49×10^{-3}
7.49×10^{-4}	44.0	46.6	41.5	63.7	64.4	0.3645	2.70×10^{-3}	2.55×10^{-3}	7.55×10^{-3}	2.14	1.08×10^{-3}
9.36×10^{-4}	44.0	47.6	40.7	66.1	66.4	0.4135	4.08×10^{-3}	3.85×10^{-3}	6.65×10^{-3}	2.68	9.71×10^{-4}
1.12×10^{-3}	44.0	46.9	41.5	66.4	66.6	0.461	3.79×10^{-3}	3.58×10^{-3}	5.96×10^{-3}	3.21	8.47×10^{-4}
1.31×10^{-3}	44.0	48.6	40.5	63.4	63.4	0.51	7.93×10^{-3}	7.48×10^{-3}	5.39×10^{-3}	3.75	6.33×10^{-4}
1.50×10^{-3}	44.0	50.1	41.0	64.4	64.4	0.559	1.02×10^{-2}	9.63×10^{-3}	4.92×10^{-3}	4.29	5.75×10^{-4}
1.68×10^{-3}	44.0	49.1	39.7	62.9	64.2	0.6005	1.17×10^{-2}	1.11×10^{-2}	4.57×10^{-3}	4.82	5.25×10^{-4}
1.87×10^{-3}	44.0	49.6	40.5	63.4	63.9	0.6495	1.30×10^{-2}	1.22×10^{-2}	4.23×10^{-3}	5.36	4.82×10^{-4}
2.06×10^{-3}	44.0	47.9	40.7	65.1	65.1	0.6875	9.86×10^{-3}	9.30×10^{-3}	3.99×10^{-3}	5.89	5.07×10^{-4}
2.25×10^{-3}	44.0	48.1	41.0	64.9	65.1	0.727	1.10×10^{-2}	1.03×10^{-2}	3.78×10^{-3}	6.43	4.66×10^{-4}
2.43×10^{-3}	44.0	48.1	40.2	66.9	66.1	0.7725	1.20×10^{-2}	1.13×10^{-2}	3.55×10^{-3}	6.96	4.88×10^{-4}
2.62×10^{-3}	44.0	48.6	40.5	67.1	66.9	0.8155	1.33×10^{-2}	1.25×10^{-2}	3.37×10^{-3}	7.50	4.69×10^{-4}
2.81×10^{-3}	44.0	46.6	38.7	63.7	63.7	0.8515	1.48×10^{-2}	1.39×10^{-2}	3.21×10^{-3}	8.03	4.14×10^{-4}
3.00×10^{-3}	44.0	46.6	39.7	62.4	62.4	0.8965	1.50×10^{-2}	1.42×10^{-2}	3.05×10^{-3}	8.57	3.70×10^{-4}
3.18×10^{-3}	44.0	48.4	40.7	65.6	65.6	0.942	1.61×10^{-2}	1.52×10^{-2}	2.91×10^{-3}	9.11	3.95×10^{-4}
3.37×10^{-3}	44.0	48.4	40.0	64.9	64.6	0.982	1.92×10^{-2}	1.81×10^{-2}	2.79×10^{-3}	9.64	3.73×10^{-4}
3.56×10^{-3}	44.0	47.1	38.7	62.2	62.2	1.024	2.18×10^{-2}	2.05×10^{-2}	2.67×10^{-3}	10.2	3.37×10^{-4}
3.73×10^{-3}	44.0	48.4	39.7	64.4	64.2	1.069	2.23×10^{-2}	2.10×10^{-2}	2.56×10^{-3}	10.7	3.50×10^{-4}

V. RESULTS FOR HEAT TRANSFER COEFFICIENT IN THE SLIP REGIME

Experimental data measured for the calculation of Nusselt number for different Reynolds and Knudsen number is given in Table I. The extreme values of (Gr/Re^2) indicate that the flow is typically in the forced convection regime. Note that the gas conductivity is not a function of pressure and the standard value (of 0.0265 W m^{-1} K^{-1}) has been employed for data reduction. Similarly, specific heat and viscosity are independent of pressure.

Figure 4 shows that the Nusselt number reduces with an increase in Knudsen number. A reduction in Nu with increasing Kn is expected from physical considerations—as the gas becomes more rarefied, its ability to transfer heat from a hot surface reduces, and is inline with the theoretical and simulation results in the literature reviewed above. The result is analogous to the behavior of friction factor with Knudsen number [34]. However, what is unexpected is that the Nusselt number can become extremely small (minimum value is about 0.00062) in the slip regime. It is however reassuring that the Nusselt number approaches the continuum value on a reduction in Knudsen number. To check for the repeatability of the measurements in the slip regime, the experiments were repeated several times (Fig. 4 inset shows few of these runs), and pointed out that the numbers obtained were quite stable. It is observed that the scatter is within ±3%.

To further test the correctness of the results in the slip regime, a bent thermocouple is employed (Fig. 5) for the measurements—with the uninsulated part of the thermocouple lying along an isocline. The difference in temperature rise of nitrogen with straight and bend thermocouple is within ±10% (maximum of 0.27 °C), and a similar observation applies for the heat transfer coefficient obtained

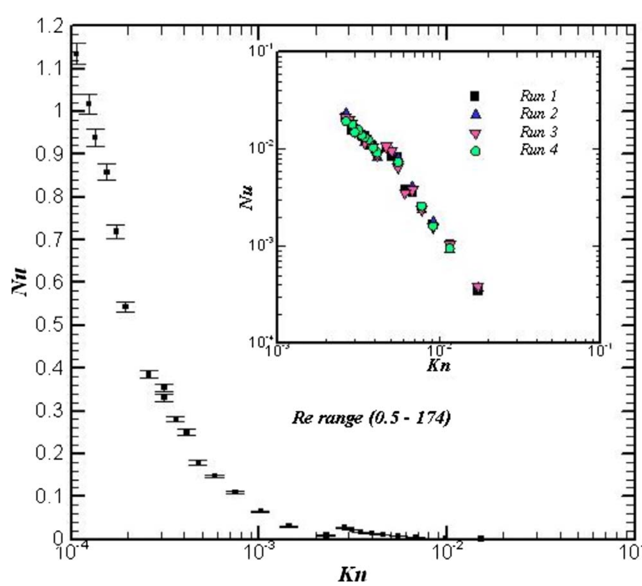

FIG. 4. (Color online) Variation of Nusselt number as a function of Knudsen number; inset shows variation of Nusselt number as a function of Knudsen number for four different runs (in the slip regime), showing repeatability in the measurements. Note that the results have been plotted on a log-log scale to highlight the low values of Nusselt number and depict the trend clearly in the slip regime.

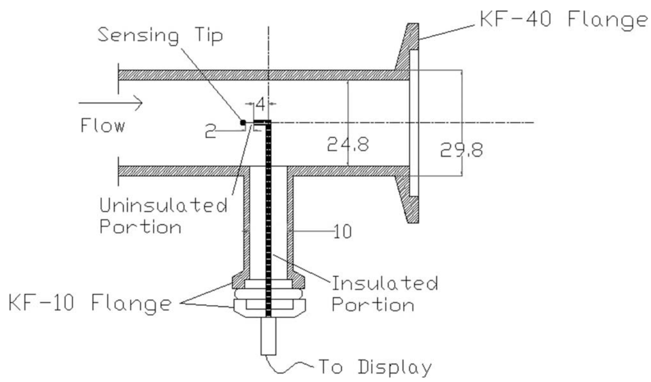

FIG. 5. Schematic of the bended thermocouple.

(Table II). Independent measurements of temperature were carried out with resistance temperature detector (RTD) to reconfirm the measurement of fluid temperatures. Results obtained using RTD are within $\pm 10\%$ from that obtained using thermocouples. Further, tests were done by changing the heat load (hot water temperature); again the value of heat transfer coefficient is within the experimental uncertainty implying that the values reported herein are independent of the water temperature.

We are not aware of any experimental results in the open literature against which these measurements could be compared. The theoretical and simulation results available in the literature [6–14] are unable to predict such low values of Nusselt number. The smallest value of Nusselt number for nitrogen that has been reported in the literature is Nu $=0.112$ at Re$=0.067$ and Kn$=1.619$ (Yan and Farouk [29]). This result has been obtained in a microchannel through the direct simulation Monte Carlo technique. A *rough* estimate of the value of Nusselt number from the data of Choi *et al.* [28] at Kn$=2\times10^{-4}$ and Re$=100$ is 0.30, which is 10% smaller than the present measurements at comparable values of Knudsen and Reynolds numbers. Owing to limited information in Choi *et al.* [28], it is difficult to make precise comparison. The plot shows that the values of Nusselt number corresponding to a certain Knudsen number in the slip regime are considerably very small as compared to theoretically suggested values in the literature.

The value of Nusselt number was also found to be responding to changes in Reynolds number while the Knudsen number is held constant (Fig. 6). An increase in Reynolds number resulted in close to linear increase in Nusselt number; this result is in qualitative agreement with the measurements of Choi *et al.* [28] and simulation results of Yan and Farouk [29]. An otherwise attempt, i.e., the dependence of Nusselt number on Knudsen number at constant Reynolds number is also made, and the results are summarized in Fig. 7. The resulting plot shows that Nusselt number depends on Knudsen number in the laminar slip regime.

Therefore, heat transfer depends on Reynolds and Knudsen numbers in the laminar slip regime. The value of the heat transfer coefficient can be 3–4 orders of magnitude smaller than the continuum regime value; such low values of heat transfer coefficient have never been reported, to the best of our knowledge.

TABLE II. Percentage difference in temperature rise and heat transfer coefficient as obtained from straight and bended thermocouples.

$\dot{m}(N_2)$ (kg/s)	Percentage difference in ΔT_{gas}	Percentage difference in h
5.18×10^{-7}	10.59	9.65
2.09×10^{-6}	−7.68	−13.76
3.11×10^{-6}	−6.31	−6.09
4.1×10^{-6}	−5.46	−8.77

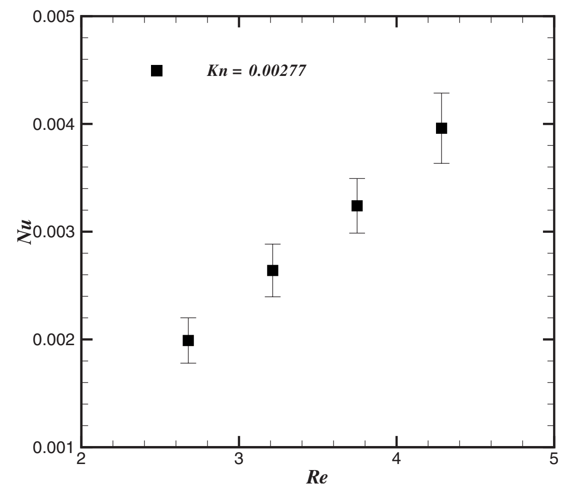

FIG. 6. Nusselt number as a function of Reynolds number at constant Knudsen number as obtained from experiment.

VI. DISCUSSION

The experimental values of Nusselt number being much smaller than the theoretical and numerical values warrants some discussion. A careful look at the analysis and simulations was undertaken to identify the reason for large difference between theoretical and experimental findings. The per-

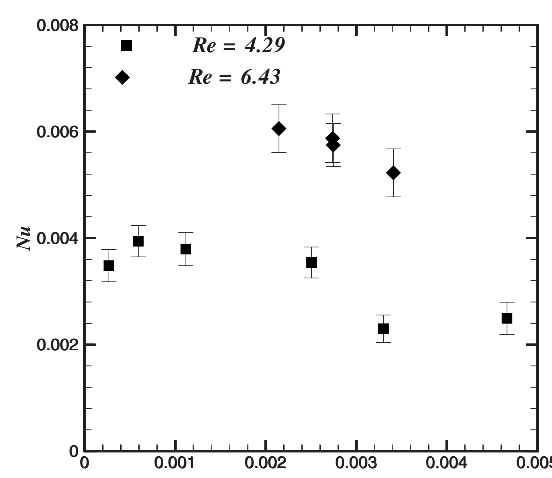

FIG. 7. Nusselt number as a function of Knudsen number at constant Reynolds number as obtained from experiment

tinent details from each of these studies are presented below.

Sparrow and Lin [35], Ameel *et al.* [6], and Larrode *et al.* [8] solved the simplified energy equation

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{11}$$

with velocity and temperature jump at the walls of the tube where r is the radial coordinate and α is the thermal diffusivity. Tunc and Bayazitoglu [10], Aydin and Avci [30], and Hooman [31] included the viscous dissipation term in the simplified energy equation given above. They solved the problem for a microtube, with velocity slip and temperature jump as the boundary conditions. Hadjiconstantinou and Simek [9] and Zhu and Liao [12] included the axial conduction term in the simplified energy equation given above. They solved the problem for a microchannel with velocity and temperature jump at the walls of the channel.

Note that all of the above analysis neglect convective heat transfer due to the $v\,\partial T/\partial r$ term. In continuum, under hydrodynamically fully developed condition, v (radial velocity) is identically zero; therefore, this term does not appear in the governing equation. However, as is well known that v is not zero in microchannels [15,16,19,36], because of a slight movement of gas toward the wall due to expansion of the gas. Although the magnitude of v is small as compared to u, the temperature gradient is larger along r than along x. An order of magnitude analysis suggests that $u\,\partial T/\partial x$ and $v\,\partial T/\partial r$ terms are of the same order, and therefore $v\,\partial T/\partial r$ should be retained while simplifying the governing equation. The importance of the $v\,\partial T/\partial r$ term in microchannels is also discussed by Mahulikar and Herwig [37,38], albeit in slightly different context (liquid flow in microchannels with variation in properties due to temperature).

The directions of conduction and convection fluxes need to be examined in order to decide on enhancement or suppression of heat transfer [37,38]. In the present case, due to heated wall and colder gas, conduction is from the wall toward the tube centerline; whereas v is toward the wall. Therefore, the radial convection flux opposes the conduction flux by moving the fluid from the centerline of the tube toward the wall. Due to the opposing effects of conduction and convection, the heat transfer coefficient should reduce. However, the radial convection term is neglected in the analysis, as already noted.

Most of the theoretical analyses cited above indeed show a reduction in Nusselt number. This is however due to the presence of a temperature jump at the walls of the tube. In addition to the temperature jump, the presence of $v\,\partial T/\partial r$ term is expected to lead to a further reduction in Nu, which has not been considered in the analysis. It is also worthwhile to note that there are inconsistencies in the theoretical analysis themselves. For example, the analysis of Larrode *et al.* [8] suggests that the Nusselt number may decrease, remain unchanged, or even increase, which is inconsistent with analysis of most other researchers.

If one examines the simulations carefully (e.g., Kavehpour *et al.* [7]; Hong and Asako [39]), there seems to be significant influence of the viscous dissipation and compressibility effect terms. In particular, Hong and Asako [39] have shown that the Nusselt number at Knudsen number approximately 0.0014 reduces drastically when the compressibility effects (pressure work term in the energy equation) are included. Note that the compressibility effect terms have not been included in any of the aforementioned analysis.

VII. CONCLUSIONS

This study is motivated by gas flow and heat transfer in microchannels and flow of rarefied gases in tubes, where the phenomenon of slip plays an important role. There is a paucity of experimental data in slip regime in the literature. A tube-in-tube heat exchanger was constructed with the means for mounting of measuring instruments. Hot water is the fluid passing in the outer side of the counterflow heat exchanger and nitrogen is allowed to pass in the inner tube at very low pressure. The ranges of Knudsen number and Reynolds numbers covered under this study are 0.00011–0.01516 and 0.5–174, respectively. The pressure ratio (outlet pressure to the inlet pressure) ranges between 0.877–0.996. Measurements of mass flow rates, temperatures, and pressures are taken to obtain values of the overall heat transfer coefficient.

The values obtained for Nusselt number are between 0.00062–1.13 which are substantially smaller than the corresponding values in the continuum regime. The results suggest that the Nusselt number is dependent on several parameters including Knudsen and Reynolds numbers. The results also suggest that while there is a modest decrease in friction factor, the heat transfer coefficient drastically reduces by rarefaction of the gas.

The present measurements are significant because: first, these are the first heat transfer measurements in the slip regime. Second, they show substantial difference from the theoretical models, implying that the theoretical analysis for such flows is incomplete. In particular it is noted that while the friction factor decreases by 50%, the Nusselt number decreases by up to four orders of magnitude in the slip regime. We expect our results to trigger development of physics—including further proof of the importance of radial convection hypothesized in the manuscript.

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