Group delay of evanescent signals in a waveguide with barrier

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The time history of subsequent tunneled wave packets can represent a more meaningful way to describe signal evolutions and to determine the group delay than the detection of a demodulated monopulse envelope. In the present experimental research, the delay of transmitted tunneled packets was found to be shorter than that measurable in vacuum, in good agreement with the *phase time* and Esposito's equation and independent of the barrier length. The wave packets did not show narrowing and "reshaping," so these phenomena did not appear to be the reason of the short delay experienced. The latter was not found to be proportional to the amplitude of the incident signal.

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I. INTRODUCTION

More than 70 years have passed since MacColl [1] stated that "there is no appreciable delay in the transmission of packet through a barrier." Thirty years later, Hartman [2] rectified this assumption stating that a delay occurs, but it becomes independent of the barrier length for a sufficiently thick section. Despite the innumerable studies on this matter carried out over the last few decades, both theoretical and experimental, the interpretation of this phenomenon remains delicate and disputed.

Tunneling was investigated both in microwave [3–12] and optical regime [13–17]. The time delays, experienced or theorized, were found to be up to few times shorter than that expected for traversing the same distance in a vacuum. Concomitantly, a great number of studies have raised serious doubts on the invoked superluminal behavior [18–29]. The reasons for these criticisms are of different natures: technical (e.g., determination of the signal front), theoretical (e.g., compatibility with causality), and related to the actual mechanism of the phenomenon (e.g., assumption of a group delay as traversal time implying a velocity). Some technical problems regard the evolution of the tunneled wave packet and, particularly, attenuation, distortion, "reshaping," narrowing, and consequent determination of the group velocity. The measurement of the time duration at half pulse peak (the so-called "half width") was suggested as a method for the determination of the group velocity in the case of attenuated Gaussian pulses [30]. A better parameter associated with the signal speed is the front of the wave, but unfortunately, it does not seem possible to obtain a sharp edge by means of which the beginning of a pulse can be easily identified. Moreover, the signal noise can compromise the correct detection of the beginning and peak of a tunneled largely attenuated wave. Sauter [23] stressed that a Gaussian pulse does not remain constant nor Gaussian shaped along the barrier; the peak can travel superluminally but it cannot be used to calculate the speed of information, which remains subluminal. Similarly, Büttiker and Washburn [24] inferred that the tunneled pulse is "front loaded" so that the peak of its envelope is advanced with respect to the peak of the incident wave packet. Pulse "reshaping" was also considered a cause of the observed superluminality [13,19,24]. Pulse narrowing, which was put in relationship with reshaping [28], was discussed by Japha and Kurizki [21]. The authors attributed the narrowing to destructive interferences between successive wave packets along accessible retarded paths; the phenomenon would depend on the width of the pulse with respect to the shortest causal arrival time. Finally, the presence of significant pulse jitter and the detection of the envelope by means of demodulators were also considered sources of instrumental uncertainty [18].

From a theoretical point of view, time delays of different meaning were proposed to describe the time spent during the interaction of a massive particle or an electromagnetic wave with a barrier in the tunneling process. Without presumption of completeness, we can remember the following: the *phase time*, introduced by Wigner [31]; the *dwell time*, introduced by Smith [32] and Büttiker [33]; the Larmor-Baz'-Rybachenko times [34,35]; the Büttiker-Landauer *traversal time* [36]; the Sokolovski and Baskin *complex* time [37] (see also, for instance, the articles or reviews in [28,33,38,39]); and the *universal tunneling time*, suggested by Haibel and Nimtz [40], Nimtz and Stahlhofen [41], and revisited by Esposito [42]. Sometimes phase time and dwell time were found to be equal, as inferred for tunneling through photonic band gaps [28]. The different definitions of time associated with the tunneling have contributed to generate additional confusion in this already complicated matter.

With regard to the interpretation of the tunneling mechanism, in a series of theoretical studies Winful [25,27,28] strenuously stressed that the anomalous short delay observed for the wave packet is the effect of energy storage in the barrier and its subsequent release. In other words, he inferred that this delay is not a propagation delay since the evanescent waves are standing waves that do not propagate in the barrier. The measured group delay appears to be proportional to the stored energy and its saturation in the barrier explains the "Harman effect." In this sense, the undersized section should be considered a lumped element inserted along the normal-sized line and the fact that the envelope length is much greater than that of the barrier should involve the quasistationary condition of the observed phenomenon. As it is technically difficult (at least at the moment) to prolong the

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barrier without causing signal attenuation and degradation that make the envelope useless for any analysis, this criticism means that the experimenter does not know which way to turn. Before resigning ourselves to abandoning this approach, we should be aware that an element is considered "lumped" if its physical length is smaller than 1/10 or, better, 1/20 [43] of the operational wavelength, where the latter refers to the carrier and not to the modulating signal (or rather the length of the packet envelope). Only if the element is really lumped we can assume that there is no appreciable phase shift between input and output. This is not the case of most tunneling experiments. Moreover, if the group delay is proportional to the energy stored in the cavity-lumped element (as the barrier is considered), it should not too difficult to experimentally verify this dependence.

Waveguides with one or more undersized sections playing the role of "opaque" barriers have represented a test bench for research on tunneling [3–5,7,11,38,44]. Because of the use of frequencies close to around 10 GHz, the literature is lacking in contributions where both carrier and modulating signals are shown and used for a time-domain analysis. Today it is commonly accepted that the group delay of a tunneled pulse (measured at the peak) can be shorter than that for a reference traveling in a normal-sized section, but the determination of a starting or ending point for a single pulse remains critical. We can therefore now proceed further, exploiting the information contained in the time history of the signal describing subsequent wave packets. The present study is aimed at providing a contribution on the following points: (i) description of true not demodulated tunneled wave packets in the time domain (this makes it possible to better verify distortion, reshaping, and narrowing); (ii) determination of the group delay by geometrical analysis and by using a setup for measuring the time spent in the barrier by the transmitted packets; (iii) assessment of the agreement between the measured group delay and the current theory; and (iv) evaluation of the influence of the incident signal amplitude on the group delay.

II. MATERIALS AND METHODS

A. Theoretical aspects

From the theory of waveguides [45], we know that for k_z there should be two roots (plus and minus): $k_z = \pm\sqrt{(\omega^2/c^2)-(\pi^2/a^2)}$, where ω is the angular frequency of the wave and a is the length of the greater side of the rectangular section. This means that waves can travel along the two opposite directions of the guide (with a positive or negative phase velocity). If ω is less than $\omega_c=\pi c/a$ the wave number k_z as well as the wavelength in the guide becomes imaginary. In any case, we can write $k_z=\pm ik'$ and, then, $k'=\sqrt{(\pi^2/a^2)-(\omega^2/c^2)}$, where k' is a positive real number, but unfortunately the analysis of the field **E** along the longitudinal axis (z) shows that the wave does not propagate and the field penetrates only for a distance of about $1/k'$. If $\omega \ll \omega_c$ the field undergoes an exponential attenuation along the guide: $1/e$ in the a/π distance.

How long does the interaction of a wave packet with a barrier last? Let us consider few essential equations from literature and, then, operate the necessary substitution to pass from the quantomechanical approach to the electromagnetic evanescent waves. First of all, the group delay, in a common meaning, is the time that the envelope of a wave packet spends to travel from two spatial positions along a medium and is considered the time necessary for the information transport. From a practical point of view, the group delay can be measured as the time elapsing between two homologous points of wave packets (e.g., the peaks of Gaussian or sine-shaped packet envelopes).

The phase time, which is the energy derivative of the phase shift, was found to be [33]

$$\tau_\phi = \frac{m}{\hbar k\kappa}\frac{2\kappa d k^2(\kappa^2-k^2)+k_0^4\sinh(2\kappa d)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa d)}, \tag{1}$$

where d is the barrier length.

For the electromagnetic case, τ_ϕ can be calculated operating the substitution $\hbar/m \to c^2/2\pi\nu$. Büttiker and Landauer [46] criticized the use of τ_ϕ to describe the tunneling time because of the wave packet distortion and the lacking of causal relation between incident and transmitted packets. Conversely, Esposito [42] underlined that τ_ϕ can be assumed as a physically meaningful quantity to measure the tunneling transit time. The phase time is also called *group delay* but to avoid confusion with the commonly meant group delay we prefer to use the first term.

The *universal tunneling time* is simply given by [40,41]

$$\tau_u \approx \frac{1}{\nu}. \tag{2}$$

It applies for all wave packets, independently of the kind of field and barrier, and was found to be in good or quite good agreement with the results of several tunneling experiments [40–42]

Esposito [42] refined the calculation of τ_u. For the case of an undersized waveguide the following equation applies:

$$\tau_{\rm th} = \frac{1}{\nu}\frac{1}{\pi}\sqrt{\frac{\nu^4}{(\nu^2-\nu_1^2)(\nu_2^2-\nu^2)}}, \tag{3}$$

where ν is the radiation frequency and ν_1 and ν_2 are the cutoff frequencies of the normal-sized and under-sized sections, respectively.

Esposito equation [42] origins from the formal analogy between the Schrödinger and Helmholtz equations and the case of a nonrelativistic electron with mass interacting with a potential barrier.

Equations (1) and (3) give the same result for a wide range of radiation and cutoff frequencies in time-saturation condition (for enough long barriers).

At the moment, only the study by Enders and Nimtz [3] can be correctly used to compare the theory with the results of tunneling experiments by using waveguides with an opaque barrier. The experiment by Ranfagni *et al.* [47,48] was carried out by using frequencies that are too close to the cutoff of the barrier for ensuring us that it was really a test of

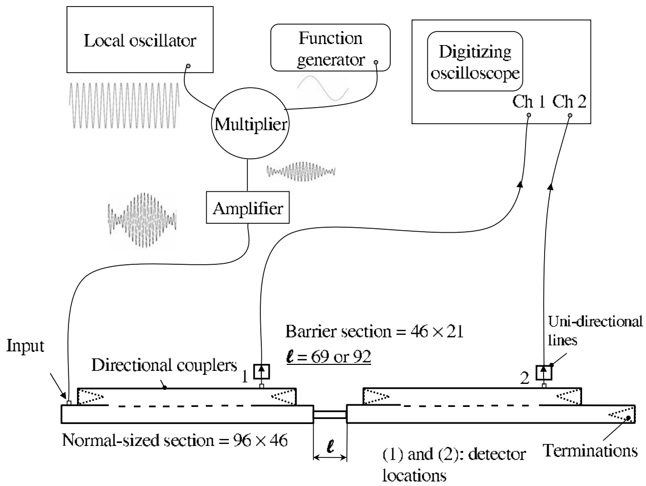

FIG. 1. Experimental setup. (Note: all dimensions are in mm. The figure is not to scale.)

tunneling [49]. In the study by Enders and Nimtz [3], for a radiation frequency of 8.7 GHz, we have $\tau_{\text{exp}}=130$ ps, $\tau_{\phi}=\tau_{\text{th}}=128$ ps, and $\tau_u=115$ ps. The agreement among these times is excellent, but we will see how some results can diverge in other circumstances.

B. Experimental setup

The experimental setup used in the present study is sketched in Fig. 1. It is based on a rectangular waveguide made of aluminum (96×46 mm^2) with an undersized (46×21 mm^2) segment ($\omega<\omega_c$) located in the middle. Two barrier lengths were considered in the experiment: 69 and 92 mm (+1/3 with respect to the first length). Longer barriers were not used because the attenuation involved a critical deterioration of the signal on the other side of the undersized section. The normal-sized and the undersized sections have cutoff frequencies of 1.561 and 3.259 GHz, respectively. Both normal-sized waveguide portions, separated by the barrier, are equipped with a multihole directional coupler. The detectors, located in the directional couplers, are connected to an oscilloscope by means of unidirectional matched lines. This setup aimed to measure only the transmitted signal along the waveguide. The time delay and attenuation of the wave packets due to the barrier are measured by comparing the time delay and attenuation between signals at points (1) and (2) (Fig. 1) in presence and absence of the interposed barrier (in this last case the barrier is removed and the two normal-sized guides are connected to each other). Preliminarily, two normal-sized segments having the same length of the barriers (69 and 92 mm) were also used to test the measurement reliability of the instrumental chain.

An amplitude modulated signal was generated by combining a sine wave carrier at 2 GHz with a sine wave modulating signal at 70 MHz. Modulation was obtained by using a signal multiplier. Because of the attenuation induced by the multiplier, a microwave amplifier was also used. The modulated amplified signal illuminates the waveguide by means of a transmitting antenna normally inserted with respect to the longitudinal and transversal axes of the guide, as is provided

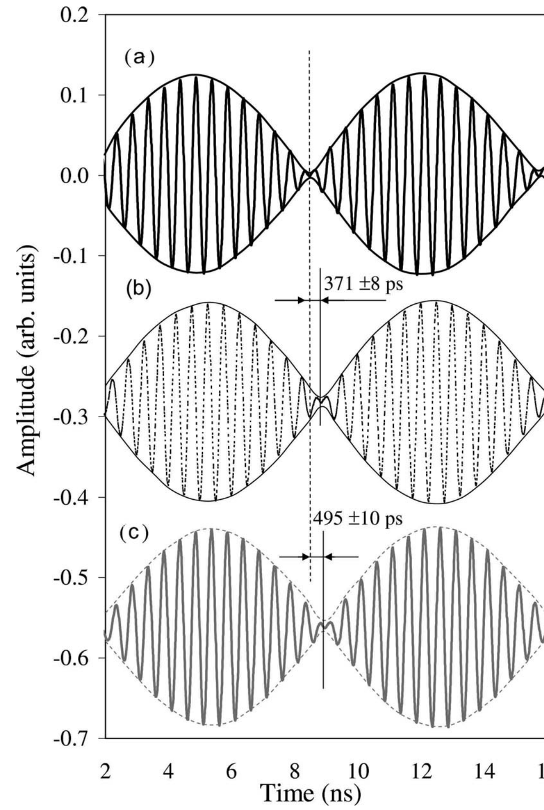

FIG. 2. Wave packet envelopes and time history of the modulated signals detected at point (2) (Fig. 1) that have traveled in normal-sized waveguide lengths. (a) Without interposed normal-sized segment; with (b) 69 mm and (c) 92 mm normal-sized segments (the cross section is the same for the waveguides supporting the directional couplers). The (b) and (c) signals were shifted down along the amplitude axis to facilitate reading. The displayed time delays are obtained by averaging the measurements related to 12 subsequent wave packets (10 ps resolution). The measured group velocity is $1.86\pm0.04\times10^8$ m/s.

for the configuration in the TE_{10} mode. The signals captured from points (1) and (2) were sent to a digitizing oscilloscope with 20 GHz bandwidth interfaced to a personal computer. Received signals were acquired after the average of 512 repetitions with a resolution of 10 ps. The group delay was calculated by measuring the time between the geometrical centers of the envelopes (at the narrowing) of the wave packets by averaging the data of 12 subsequent packets.

The influence of the incident signal amplitude on the group delay was also determined (for the short barrier) by reducing the initial amplitude value by 1/3.

III. RESULTS AND DISCUSSION

A random period of the digitized time history of the signals detected at point (2) (setup in Fig. 1) by using normal-sized lengths instead of the barriers is shown in Fig. 2. The three plots show the signals (A) without normal-sized interposed segment and (B) with 69 mm and (C) 92 mm normal-sized segments. Group delay and group velocity can be easily geometrically determined by means of the envelope of the

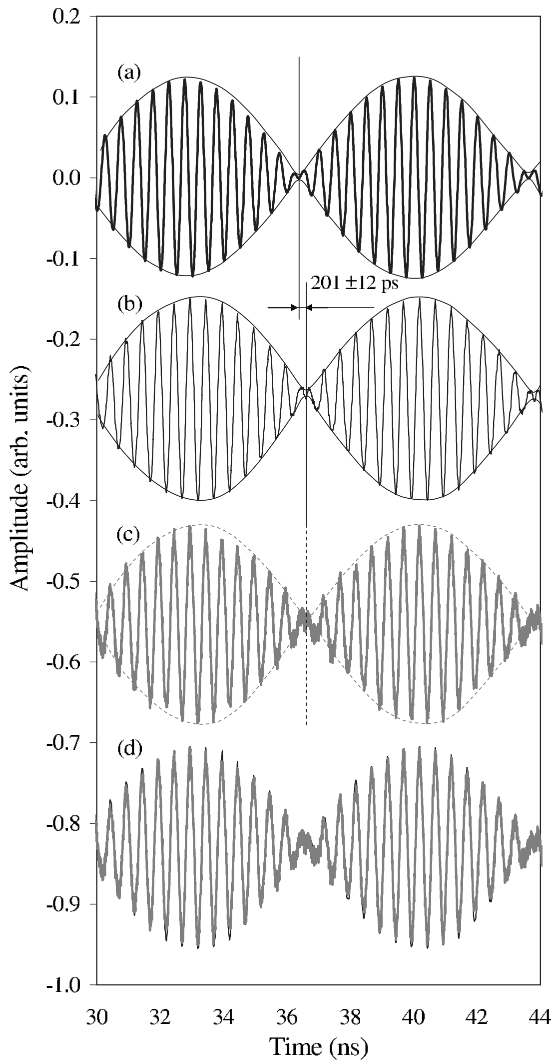

FIG. 3. Wave packet envelopes and time history of the modulated signals detected at point (2) by using the set up in Fig. 1. (a) Without barrier; (b) with 69 mm barrier; (c) with 92 mm barrier; and (d) superposition of (b) and (c) signals. The normalized (b)–(d) signals were shifted down along the amplitude axis to facilitate reading. The displayed time delay is obtained by averaging the measurements related to 12 subsequent wave packets (10 ps resolution).

wave packets. The group delay was found to be 371 ± 8 ps for the short normal-sized segment and 495 ± 10 ps for the long one. The measured group velocity was found to be $1.86 \pm 0.04 \times 10^8$ m/s. The latter appears in good accordance with the theoretical one (1.87×10^8 m/s) and then, the instrumental chain can be considered properly calibrated.

Figure 3 shows the time history of the signals detected at point (2) by using the setup in Fig. 1 without or with barriers. The several plots show the normalized signals for the waveguide (a) without barrier, (b) with 69 mm, (c) 92 mm barriers, and (d) the superposition of (b) and (c) signals.

Although largely attenuated, the (b) tunneled signal appears well defined, undistorted in shape, not narrowed with respect to the (a) signal that has traveled along the normal-sized guide without barrier, and with low noise. The geometrical method based on the envelope of the wave packets and supported by the comparison of the signal time histories appears suitable for the determination of the group delay with an acceptable tolerance. The (b) tunneled wave packets are delayed by 201 ± 12 ps with respect to the (a) not tunneled packets. From (c) plot, the signal noise on the other side of the long barrier does not appear so negligible as for the short one. On the other hand and despite the noise, from the (d) signal superposition, one can see that the beginning (and end) of the packets tunneled through the short and long barriers is time or quasitime coincident.

The tunneled signal was found to be attenuated by 33.3 and 44.4 dB for the 69 and 92 mm barriers, respectively. Theoretical attenuations for the two barriers are 32.3 and 43.1 dB, showing a good agreement with the experimental results.

The group delay for the test with reduced signal amplitude $(-1/3)$ was found to be 200 ± 14 ps: it can be considered the same for the full amplitude. As for distributed circuits the signal amplitude did not influence the group delay.

Now we can compare the measured delays with the times obtained from Eqs. (1)–(3): $\tau_{\text{exp}} = 201 \pm 12$ ps, $\tau_\phi = \tau_{\text{th}} = 198$ ps, and $\tau_u = 500$ ps. For both barrier lengths the values of τ_{exp} and τ_ϕ are coincident or quasicoincident, revealing that the time is already saturated for the short barrier. The time delay between wave packet envelopes, measured in geometrical way, is in good agreement with the phase time and the Esposito tunneling transit time. We remember that the used setup measures only the transmitted signal, so we should be authorized to speak of *velocity* that is $1.15c$ for the short barrier and $1.53c$ for the long one.

Shorter time delays were experienced in other studies. In a frequency modulation experiment by Aichmann *et al.* [50], with carrier at 8.7 GHz and bandwidth of 2 kHz, a signal speed of $4.7c$ was inferred. Barbero *et al.* [7] obtained a group velocity of $5.71c$ by means of numerical simulation. The related very short time delay could be attributed not only to the different transparency of the used barriers, cutoff frequency of the normal-sized sections, and carrier frequencies but also to a different setup or methodology that does not ignore reflective components. Studies on tunneling carried out with undersized waveguides are still few to confirm that the measured delay always agrees with a given equation. Further research should be addressed to analyze the influence of the experimental setup characteristics on the measured time delay.

IV. CONCLUSIONS

The time-domain analysis of wave packet trains, together with a geometrical determination of the starting and ending points of the envelopes of the packets, is a useful method for the calculation of the group delay of tunneled signals. With respect to the analysis based only on the demodulation of a single Gaussian pulse, it makes it possible to avoid or minimize the uncertainty due to the determination of the incoming or the peak of the wave packet and some technical problems inferred if demodulators are used.

The results of the present study confirm that the group delay for the signal subjected to tunneling is shorter than that traveling in a normal-sized waveguide and in vacuum. The group delay of the transmitted signal was found to be in agreement with the theoretical phase time and the Esposito transit time.

Moreover, the group delay does not change by modifying the barrier length, for long enough barriers. Wave packets do not suffer from narrowing or distortion caused by the barrier. Finally, the group delay does not appear to be influenced by the incident signal amplitude, at least in the examined conditions. The barriers used in the experiments and simulations reported in the literature should not be considered lumped elements: this is not the reason for considering the tunneling a quasistationary phenomenon.

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