Thermal entanglement in two-atom cavity QED and the entangled quantum Otto engine

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The simple system of two two-level identical atoms couple to single-mode optical cavity in the resonance case is studied for investigating the thermal entanglement. It is interesting to see that the critical temperature is only dependent on the coefficient of atom-atom dipole-dipole interaction. Based on the mode, we construct and investigate a entangled quantum Otto engine (QOE). Expressions for several important performance parameters such as the heat transferred, the work done in a cycle, and the efficiency of the entangled QOE in zero G are derived in terms of thermal concurrence. Some intriguing features and their qualitative explanations are given. Furthermore, the validity of the second law of thermodynamics is confirmed in the entangled QOE. The results obtained here have general significance and will be helpful to understand deeply the performance of an entangled QOE.

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I. INTRODUCTION

Quantum theory, a field opened by Planck and other pioneers in 20th century has intrigued scientists. Entanglement, first noted by Schrödinger, Einstein, Podolsky, and Rosen, is one of the most striking features of quantum mechanics, and has recently been associated to various phenomena in different areas of physics, for example Hawking radiation in cosmology [1,2], symmetry breaking in high-energy physics [3], and in particular, to many areas of condensed-matter physics such as critical phenomena [3–5]. Entanglement provides the key ingredient for teleportation schemes [6,7], one-way quantum computer [8,9], and many quantum cryptography protocols [10,11]. In addition, the relation between thermodynamics and entanglement is also of fundamental importance, and has been studied by many authors in recent years [12–14]. Some investigations supporting the way of seeing entanglement as a thermodynamical property have recently appeared [15]; in particular the magnetic susceptibility [14,16] of some solids and their heat capacity [17] were identified as entanglement witnesses.

In recent years, quantum thermodynamic cycles (such as quantum engine, quantum refrigerator, quantum heat pump, quantum amplifier, quantum afterburner, and so on) have become one of the interesting research subjects for people working in thermodynamics and statistical physics. Several authors have intensively investigated the performance characteristics of quantum thermodynamic cycles working with harmonic oscillators, uncoupled spins, particles in a potential, and two-level systems (TLSs) like qubits, multilevel systems or quantum optics systems, etc [18–31]. These quantum analyses provide the application foundation of the equilibrium or nonequilibrium statistical mechanics to the practical engineering cycles.

Quantum heat engines are characterized by three attributes: the working medium, the cycle of operation, and the dynamics that govern the cycle. However, in the previous works, the working medium is considered as an ensemble of many noninteracting systems. In general, entanglement between two systems can be generated if they interact with a single-mode cavity in controlled way. Cavity QED, where two two-level atoms resonantly interact with a quantized electromagnetic field inside a cavity, have already proven to be a useful tool for testing fundamental quantum properties [32,33]. Here, we propose an implementation of the QHE via a cavity QED scheme and investigate the influence of both the quantum entanglement and the interacting systems on QHE features. These give rise to the following questions: with the quantum entanglement and the interacting working medium what is the work extraction of quantum heat engine? Can such a quantum heat engine improve the work extraction?

II. MODEL DESCRIPTION

The dipole-dipole interaction of the atoms cannot be neglected when the relative distance of two atoms and the de Broglie wavelength of two atoms can compare in the cavity. To see this, we consider two two-level atoms simultaneously interacting with a single-mode cavity field. The Hamiltonian for the system is given by

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_I, \tag{1}$$

$$\mathbf{H}_0 = \hbar\omega \mathbf{a}^{\dagger}\mathbf{a} + \sum_{j=1,2} \hbar\omega_j S_j^z, \tag{2}$$

$$\mathbf{H}_I = \hbar g \sum_{j=1,2} (e^{-i\delta t}\mathbf{a}^{\dagger}S_j^- + e^{i\delta t}\mathbf{a}S_j^+), \tag{3}$$

where $S_j^z = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$ with $|e_j\rangle$ and $|g_j\rangle$ $(j=1,2)$ being the excited and ground states of the jth atom, $\mathbf{a}^{\dagger}$ and $\mathbf{a}$ are the creation and annihilation operators for the cavity mode, ω_j is the atomic transition

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frequency, ω is the cavity frequency, and g is the atom-cavity coupling strength [34].

In the absence of the dipole-dipole interaction, the space of the two-atom system is spanned by four product states

$$|g_1g_2\rangle,|g_1e_2\rangle,|e_1g_2\rangle,|e_1e_2\rangle \tag{4}$$

with corresponding energies

$$\begin{cases} E_{gg} = -\hbar G \\ E_{eg} = -\hbar\Delta \\ E_{ge} = \hbar\Delta \\ E_{ee} = \hbar G, \end{cases} \tag{5}$$

where $G=\frac{1}{2}(\omega_1+\omega_2)$ and $\Delta=\frac{1}{2}(\omega_2-\omega_1)$. The product states $|e_1g_2\rangle$ and $|g_1e_2\rangle$ form a pair of nearly degenerated states. When we include the dipole-dipole interaction between the atoms, the product states combine into two linear superpositions (entangled states), with their energies shifted from $\pm\hbar\Delta$ by the dipole-dipole interaction energy.

In the case $\delta=\omega_j-\omega \gg g\sqrt{n}$, with $\bar{n}$ being the mean photon number of the cavity field, there is no energy exchange between the atomic system and the cavity. Then the effective Hamiltonian the two-atom system is [35]

$$\mathbf{H} = \sum_{j=1,2} \hbar\omega_j S_j^z + \sum_{i\neq j} \hbar\Omega_{ij} S_i^+ S_j^- . \tag{6}$$

In the basis of the product states [Eq. (4)], effective Hamiltonian (6) can be written in a matrix form as [36]

$$\mathbf{H} = \hbar\begin{pmatrix} -G & 0 & 0 & 0 \\ 0 & -\Delta & \Omega_{21} & 0 \\ 0 & \Omega_{12} & \Delta & 0 \\ 0 & 0 & 0 & G \end{pmatrix}, \tag{7}$$

where $\Omega=\Omega_{12}=\Omega_{21}$ is the coefficient of atom-atom dipole-dipole interaction.

Consider a system constituted by two two-level identical $(\Delta=0)$ atoms and a single-mode optical cavity. The resulting energies and corresponding eigenstates of the system are [37,38]

$$\begin{cases} E_g = -\hbar G \\ E_s = \hbar\Omega \\ E_a = -\hbar\Omega \\ E_e = \hbar G, \end{cases} \tag{8}$$

and

$$\begin{cases} |g\rangle = |g_1\rangle|g_2\rangle \\ |s\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle) \\ |a\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle) \\ |e\rangle = |e_1\rangle|e_2\rangle, \end{cases} \tag{9}$$

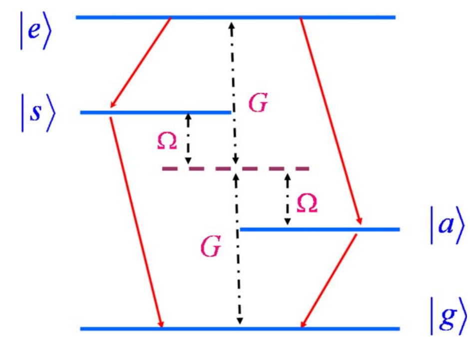

FIG. 1. (Color online) Collective states of two two-level identical atoms. The energies of the symmetric and antisymmetric states are shifted by the dipole-dipole interaction Ω. The arrows indicate possible one-photon transitions.

where $|s\rangle=\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle+|g_1\rangle|e_2\rangle)$ and $|a\rangle=\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle-|g_1\rangle|e_2\rangle)$ are maximally entangled states. Eigenstates (9), first introduced by Dicke [37], are known as the collective states of two interacting atoms.

The collective states of two two-level identical atoms are shown in Fig. 1. It is seen that in the collective states representation, the system behaves as a single four-level system, with the ground state $|g\rangle$, the upper state $|e\rangle$, and two intermediate states: the symmetric state $|s\rangle$ and the antisymmetric state $|a\rangle$. The ground state $|g\rangle$ and the upper state $|e\rangle$ are not affected by the dipole-dipole interaction, whereas the states $|s\rangle$ and $|a\rangle$ are shifted from their unperturbed energies by the amount $\pm\hbar\Omega$, the dipole-dipole energy. The energies of the intermediate states depend on the dipole-dipole interaction and these states suffer a large shift when the interatomic separation is small. The most important property of the collective states $|s\rangle$ and $|a\rangle$ is that they are an example of maximally entangled states of the two-atom system.

III. THERMAL ENTANGLEMENT

The state of the system at thermal equilibrium is $\boldsymbol{\rho}(T)=\frac{1}{Z}\exp(-\frac{\mathbf{H}}{kT})Z$, where $Z=Tr[\exp(-\frac{\mathbf{H}}{kT})]$ is the partition function and k is the Boltzmann's constant. As $\boldsymbol{\rho}(T)$ represents a thermal state, the entanglement in the state is called thermal entanglement. In order to determine the amount of thermal entanglement between two two-level identical atoms and a single-mode optical cavity, we adopt the thermal concurrence C defined by Wootters [39],

$$C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \tag{10}$$

where the λ_i $(i=1,2,3,4)$ are the square roots of the eigenvalues in decreasing order of density-matrix operator $\varrho_{12}=\rho_{12}(\sigma_{1y}\otimes\sigma_{2y})\rho_{12}^*(\sigma_{1y}\otimes\sigma_{2y})$, where the asterisk indicates complex conjugation and σ_y is the Pauli matrix. The thermal concurrence varies from $C=0$ for an unentangled state to $C=1$ for a maximally entangled state.

In the standard basis $\{|g_1g_2\rangle,|g_1e_2\rangle,|e_1g_2\rangle,|e_1e_2\rangle\}$, the density matrix $\boldsymbol{\rho}(T)$ is written as

$$\boldsymbol{\rho}(T)=\frac{1}{Z}\begin{pmatrix} \exp\left(-\frac{\hbar G}{kT}\right) & 0 & 0 & 0 \\ 0 & \exp\left(-\frac{\hbar\Omega}{kT}\right)\left[1+\exp\left(\frac{2\hbar\Omega}{kT}\right)\right] & -\exp\left(-\frac{\hbar\Omega}{kT}\right)\left[\exp\left(\frac{2\hbar\Omega}{kT}\right)-1\right] & 0 \\ 0 & -\exp\left(-\frac{\hbar\Omega}{kT}\right)\left[\exp\left(\frac{2\hbar\Omega}{kT}\right)-1\right] & \exp\left(-\frac{\hbar\Omega}{kT}\right)\left[1+\exp\left(\frac{2\hbar\Omega}{kT}\right)\right] & 0 \\ 0 & 0 & 0 & \exp\left(-\frac{\hbar G}{kT}\right) \end{pmatrix} \quad (11)$$

where $Z=\exp(-\frac{\hbar G}{kT})+\exp(\frac{\hbar G}{kT})+\exp(-\frac{\hbar\Omega}{kT})+\exp(\frac{\hbar\Omega}{kT})$. The square roots of the eigenvalues of the matrix ϱ_{12} are

$$\lambda_1=\lambda_2=\frac{1}{\exp\left(-\frac{\hbar G}{kT}\right)+\exp\left(\frac{\hbar G}{kT}\right)+\exp\left(-\frac{\hbar\Omega}{kT}\right)+\exp\left(\frac{\hbar\Omega}{kT}\right)}, \quad (12a)$$

$$\lambda_3=\frac{\exp\left(\frac{\hbar G}{kT}\right)}{\exp\left(\frac{\hbar G}{kT}\right)+\exp\left(\frac{\hbar\Omega}{kT}\right)+\exp\left[\frac{\hbar(2G+\Omega)}{kT}\right]+\exp\left[\frac{\hbar(G+2\Omega)}{kT}\right]}, \quad (12b)$$

$$\lambda_4=\frac{\exp\left[\frac{\hbar(G+2\Omega)}{kT}\right]}{\exp\left(\frac{\hbar G}{kT}\right)+\exp\left(\frac{\hbar\Omega}{kT}\right)+\exp\left[\frac{\hbar(2G+\Omega)}{kT}\right]+\exp\left[\frac{\hbar(G+2\Omega)}{kT}\right]}. \quad (12c)$$

From Eqs. (10) and (12a)–(12c), the concurrence is given by

$$C=\max\left\{\frac{\sinh\left(\frac{\hbar\Omega}{kT}\right)-1}{\cosh\left(\frac{\hbar\Omega}{kT}\right)+\cosh\left(\frac{\hbar G}{kT}\right)},0\right\}. \quad (13)$$

Then we know $C=0$, if $\sinh(\frac{\hbar\Omega}{kT})<1$, i.e., the critical temperature is given by

$$T_C=\frac{\hbar\Omega}{k\,\mathrm{arcsinh}(1)}. \quad (14)$$

The entanglement vanishes for $T\geq T_C$. It is interesting to see that the critical temperature is only dependent on the coupling constant Ω of atom-atom dipole-dipole interaction.

In Fig. 2 we give the plot of thermal concurrence as a function of G and T for $\Omega=10g$, $g=2\pi\times24\times10^3$ Hz is the coupling constant [40]. For $G=0$, the maximally entangled state $|a\rangle$ is the ground state with eigenvalue $E_a=-\hbar\Omega$. Then the maximum entanglement is at $T=0$, i.e., $C=1$. As T increases, the thermal concurrence decreases as seen from Fig. 2 due to the mixing of other states with the maximally entangled state. For a high value of critical value of G_C when the state $|g\rangle=|g_1\rangle|g_2\rangle$ becomes the ground state, which means there is no entanglement at $T=0$. However, by increasing T, the maximally entangled states $|s\rangle$ and $|a\rangle$ will mix with the state $|g\rangle=|g_1\rangle|g_2\rangle$, which makes the entanglement increase (see Fig. 2). Moreover, we can see that for a higher value of Ω, the system has a stronger entanglement, which is consistent with Fig. 3.

IV. THERMAL ENTANGLED QOE MODEL

The entangled quantum Otto engine (QOE) considered here is the quantum analog of the classical Otto engines. The

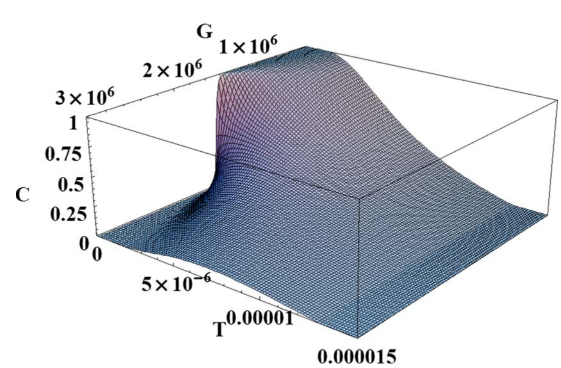

FIG. 2. (Color online) Thermal concurrence vs G and T, for the parameters $\Omega=10g$.

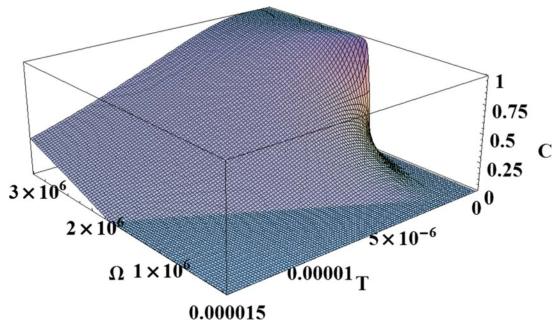

FIG. 3. (Color online) Thermal concurrence vs Ω and T, for the parameters $G=10g$.

working medium is envisioned as a system consists of two two-level identical interacting atoms which resonantly interact with a single-mode optical cavity. A cycle sketch of the thermal entangled quantum engine consists of four stages as seen from Fig. 4. The quantum heat-engine cycle consists of four branches labeled by A, B, C, and D; this is schematically illustrated in Fig. 4. The four-stroke entangled QOE includes two isothermal processes (1 and 3) and two quantum adiabatic processes (2 and 4).

(i) Stage 1: $A \rightarrow B$ the working medium of two two-level identical interacting atoms is coupled to a hot reservoir of temperature T_H and its energy structure is kept fixed. In this isothermal process, the population the four discrete levels is changing from the population $P_{i2},(i=1,2,3,4)$ to the population $P_{i1},(i=1,2,3,4)$ and only heat is transferred in this stage to yield a change in the occupation probabilities, and no work done as there is no change in the values of the energy levels,

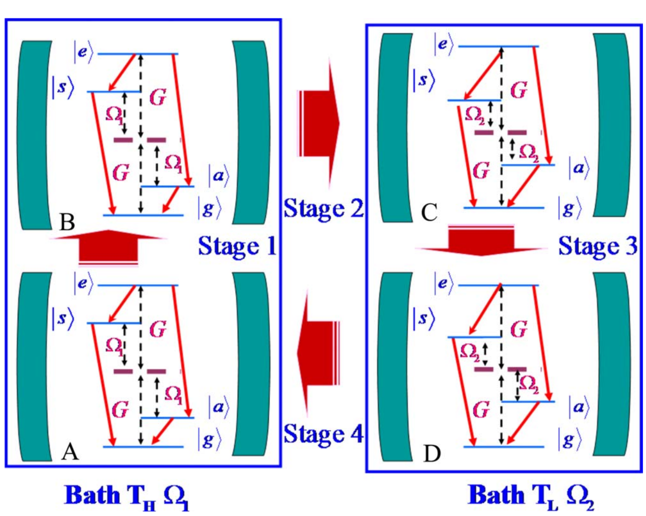

FIG. 4. (Color online) A schematic diagram of a thermal entangled quantum Otto engine based on two two-level atoms of quantum system. The process from A to B (C to D) is the isothermal expansion (compression) process, in which the working substance is put in contact with the high (low)-temperature heat bath. The processes from B to C and from D to A are two adiabatic processes.

$$P_{i1} = \begin{cases} P_{11} = \dfrac{\exp\left(\dfrac{-\hbar G}{kT_H}\right)}{Z_H} \\ P_{21} = \dfrac{\exp\left(\dfrac{\hbar \Omega_1}{kT_H}\right)}{Z_H} \\ P_{31} = \dfrac{\exp\left(\dfrac{-\hbar \Omega_1}{kT_H}\right)}{Z_H} \\ P_{41} = \dfrac{\exp\left(\dfrac{\hbar G}{kT_H}\right)}{Z_H} \end{cases} \tag{15}$$

and

$$P_{i2} = \begin{cases} P_{12} = \dfrac{\exp\left(\dfrac{-\hbar G}{kT_L}\right)}{Z_L} \\ P_{22} = \dfrac{\exp\left(\dfrac{\hbar \Omega_2}{kT_L}\right)}{Z_L} \\ P_{32} = \dfrac{\exp\left(\dfrac{-\hbar \Omega_2}{kT_L}\right)}{Z_L} \\ P_{42} = \dfrac{\exp\left(\dfrac{\hbar G}{kT_L}\right)}{Z_L}, \end{cases} \tag{16}$$

where

$$Z_H = \exp\left(\frac{-\hbar G}{kT_H}\right) + \exp\left(\frac{\hbar \Omega_1}{kT_H}\right) + \exp\left(\frac{-\hbar \Omega_1}{kT_H}\right) + \exp\left(\frac{\hbar G}{kT_H}\right)$$

and

$$Z_L = \exp\left(\frac{-\hbar G}{kT_L}\right) + \exp\left(\frac{\hbar \Omega_2}{kT_L}\right) + \exp\left(\frac{-\hbar \Omega_2}{kT_L}\right) + \exp\left(\frac{\hbar G}{kT_L}\right).$$

(ii) Stage 2: $B \rightarrow C$, This is an adiabatic process in the sense that the total occupation probability of the working medium remains unchanged. In this process, the occupation probability $P_{i1},(i=1,2,3,4)$ is kept fixed. The working medium is decoupled from the hot reservoir, and the energy structure is varied from $E_{i1},(i=1,2,3,4)$ to E_{i2}, $(i=1,2,3,4)$. In this stage, provided the expansion rate is sufficiently slow according to the quantum adiabatic theorem, the occupation probabilities for the two states remain unchanged. The entangled QOE performs an amount of positive work when the energy spacings decrease,

$$E_{i1}=\begin{cases}E_{11}=-\hbar G\\ E_{21}=\hbar\Omega_1\\ E_{31}=-\hbar\Omega_1\\ E_{41}=\hbar G\end{cases}\tag{17}$$

and

$$E_{i2}=\begin{cases}E_{12}=-\hbar G\\ E_{22}=\hbar\Omega_2\\ E_{32}=-\hbar\Omega_2\\ E_{42}=\hbar G.\end{cases}\tag{18}$$

(iii) Stage 3: $C\rightarrow D$, Stage 3 is almost an inverse process of Stage 1. The working medium is coupled to a cold reservoir of temperature T_L and its energy structure is kept fixed. In this isothermal process, the population of the four discrete level is changing from the initial population $P_{i1},(i=1,2,3,4)$ to the population $P_{i2},(i=1,2,3,4)$ and some heat is thus transferred but no work is performed in this stage.

(iv) Stage 4: $D\rightarrow A$, Stage 4 is also an adiabatic process in the sense that the total occupation probability of the working medium remains unchanged. In this process, the occupation probability $P_{i2},(i=1,2,3,4)$ is kept fixed. The working medium is decoupled from the cold reservoir, and the energy structure is varied from $E_{i2},(i=1,2,3,4)$ to $E_{i1},(i=1,2,3,4)$. In this stage an amount of work is done on the system.

The expectation value of the measured energy of the system with coupling four discrete energy levels is

$$U=\langle E\rangle=\sum_{i=1,}^{4}P_{ij}E_{ij}\tag{19}$$

in which $E_{ij},(i=1,2,3,4;j=1,2)$ are the energy levels and $P_{ij},(i=1,2,3,4;j=1,2)$ are the corresponding occupation probabilities. Infinitesimally,

$$dU=\sum_{i=1}^{4}E_{ij}dP_{ij}+P_{ij}dE_{ij}\tag{20}$$

from which we make the following identifications for infinitesimal heat transferred dQ and work done dW:

$$\begin{cases}dQ=\sum_{i=1}^{4}E_{ij}dP_{ij}\\ dW=\sum_{i=1}^{4}P_{ij}dE_{ij}.\end{cases}\tag{21}$$

According to the quantum interpretations of heat transferred and work done in Eqs. (21), the heat transferred in stage 1 Q_H and in stage 3 Q_L is given by

$$\begin{aligned}Q_H=\sum_{i=1}^{4}E_{i1}(P_{i1}-P_{i2})&=\hbar G\left[\frac{\exp\left(\frac{\hbar G}{kT_H}\right)-\exp\left(\frac{-\hbar G}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar G}{kT_L}\right)-\exp\left(\frac{\hbar G}{kT_L}\right)}{Z_L}\right]\\&+\hbar\Omega_1\left[\frac{\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)-\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)-\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}{Z_L}\right]\end{aligned}\tag{22}$$

and

$$\begin{aligned}Q_L&=\sum_{i=1}^{4}E_{i2}(P_{i1}-P_{i2})\\&=\hbar G\left[\frac{\exp\left(\frac{\hbar G}{kT_H}\right)-\exp\left(\frac{-\hbar G}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar G}{kT_L}\right)-\exp\left(\frac{\hbar G}{kT_L}\right)}{Z_L}\right]\\&+\hbar G_2\left[\frac{\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)-\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)-\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}{Z_L}\right].\end{aligned}\tag{23}$$

From the law of conservation of energy, the net work done by the entangled QOE in two quantum adiabatic processes, i.e., stage 2 and stage 4, is

$$\begin{aligned}W&=Q_H-Q_L\\&=\sum_{i=1}^{4}(E_{i1}-E_{i2})(P_{i1}-P_{i2})\\&=\hbar(\Omega_1-\Omega_2)\left[\frac{\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)-\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)-\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}{Z_L}\right].\end{aligned}\tag{24}$$

Then the efficiency of the entangled QOE reads

$$\eta=\frac{W}{Q_H}=1-\frac{Q_L}{Q_H}=1-G\left[\frac{\exp\left(\frac{\hbar G}{kT_H}\right)-\exp\left(\frac{-\hbar G}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar G}{kT_L}\right)-\exp\left(\frac{\hbar G}{kT_L}\right)}{Z_L}\right]+\Omega_2\left[\frac{\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)-\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)}{Z_H}\right.$$
$$\left.+\frac{\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)-\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}{Z_L}\right]\Bigg/G\left[\frac{\exp\left(\frac{\hbar G}{kT_H}\right)-\exp\left(\frac{-\hbar G}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar G}{kT_L}\right)-\exp\left(\frac{\hbar G}{kT_L}\right)}{Z_L}\right]$$
$$+\Omega_1\left[\frac{\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)-\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)}{Z_H}+\frac{\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)-\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}{Z_L}\right]. \tag{25}$$

The entanglement under our consideration is that of two thermal equilibrium states at the end of stage 1 and stage 3, denoted by C_1 and C_2, respectively. They are

$$C_1=\begin{cases}0 & T_H\geq T_{H,C_1}=\dfrac{\hbar\Omega_1}{k\,\mathrm{arcsinh}(1)},\\[2ex] -\dfrac{2}{\exp\left(\frac{-\hbar G}{kT_H}\right)+\exp\left(\frac{\hbar G}{kT_H}\right)+\exp\left(\frac{-\hbar\Omega_1}{kT_H}\right)+\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)}+\dfrac{\exp\left[\frac{(G+2\Omega_1)\hbar}{kT_H}\right]-\exp\left(\frac{\hbar G}{kT_H}\right)}{\exp\left(\frac{\hbar G}{kT_H}\right)+\exp\left(\frac{\hbar\Omega_1}{kT_H}\right)+\exp\left[\frac{(2G+\Omega_1)\hbar}{kT_H}\right]+\exp\left[\frac{(G+2\Omega_1)\hbar}{kT_H}\right]}, & T_H<T_{H,C_1}=\dfrac{\hbar\Omega_1}{k\,\mathrm{arcsinh}(1)},\end{cases} \tag{26a}$$

$$C_2=\begin{cases}0 & T_L\geq T_{L,C_2}=\dfrac{\hbar\Omega_2}{k\,\mathrm{arcsinh}(1)},\\[2ex] -\dfrac{2}{\exp\left(\frac{-\hbar G}{kT_L}\right)+\exp\left(\frac{\hbar G}{kT_L}\right)+\exp\left(\frac{-\hbar\Omega_2}{kT_L}\right)+\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)}+\dfrac{\exp\left[\frac{(G+2\Omega_2)\hbar}{kT_L}\right]-\exp\left(\frac{\hbar G}{kT_L}\right)}{\exp\left(\frac{\hbar G}{kT_L}\right)+\exp\left(\frac{\hbar\Omega_2}{kT_L}\right)+\exp\left[\frac{(2G+\Omega_2)\hbar}{kT_L}\right]+\exp\left[\frac{(G+2\Omega_2)\hbar}{kT_L}\right]}, & T_L<T_{L,C_2}=\dfrac{\hbar\Omega_2}{k\,\mathrm{arcsinh}(1)}.\end{cases} \tag{26b}$$

Now, we explore the relation between entanglement and basic thermodynamics quantities and the efficiency of the entangled QOE. From Eqs. (26a) and (26b) we find

$$\Omega_1 = \frac{kT_H}{\hbar}\ln\left\{ -C_1 - 2\exp\left(\frac{\hbar G}{kT_H}\right) - C_1\exp\left(\frac{2\hbar G}{kT_H}\right) - \sqrt{C_1^2 + 4C_1\exp\left(\frac{\hbar G}{kT_H}\right) + 8\exp\left(\frac{2\hbar G}{kT_H}\right) - 2C_1^2\exp\left(\frac{2\hbar G}{kT_H}\right) + 4C_1\exp\left(\frac{3\hbar G}{kT_H}\right) + C_1^2\exp\left(\frac{4\hbar G}{kT_H}\right)} \Bigg/ 2\left[-\exp\left(\frac{\hbar G}{kT_H}\right) + C_1\exp\left(\frac{\hbar G}{kT_H}\right)\right]\right\}, \tag{27a}$$

$$\Omega_2 = \frac{kT_L}{\hbar}\ln\left\{ -C_2 - 2\exp\left(\frac{\hbar G}{kT_L}\right) - C_2\exp\left(\frac{2\hbar G}{kT_L}\right) - \sqrt{C_2^2 + 4C_2\exp\left(\frac{\hbar G}{kT_L}\right) + 8\exp\left(\frac{2\hbar G}{kT_L}\right) - 2C_2^2\exp\left(\frac{2\hbar G}{kT_L}\right) + 4C_2\exp\left(\frac{3\hbar G}{kT_L}\right) + C_2^2\exp\left(\frac{4\hbar G}{kT_L}\right)} \Bigg/ 2\left[-\exp\left(\frac{\hbar G}{kT_L}\right) + C_2\exp\left(\frac{\hbar G}{kT_L}\right)\right]\right\}. \tag{27b}$$

We first start to explore the case of zero atom-cavity coupling strength $G=0$. Equations (27a) and (27b) become

$$\Omega_1 = \frac{kT_H}{\hbar}\ln\left(\frac{1 + C_1 + \sqrt{2}\sqrt{1 + C_1}}{1 - C_1}\right), \tag{28a}$$

$$\Omega_2 = \frac{kT_L}{\hbar}\ln\left(\frac{1 + C_2 + \sqrt{2}\sqrt{1 + C_2}}{1 - C_2}\right). \tag{28b}$$

By simple deduction, Eqs. (22)–(24) become

$$Q_H = \frac{2k[(2+\sqrt{2}\sqrt{1+C_1})C_2 + \sqrt{2}(-\sqrt{1+C_1} + \sqrt{1+C_2}) - C_1(2+\sqrt{2}\sqrt{1+C_2})]\ln\left[\frac{1+C_1+\sqrt{2}\sqrt{1+C_1}}{1-C_1}\right]T_H}{(2+\sqrt{2}\sqrt{1+C_1})(2+\sqrt{2}\sqrt{1+C_2})}, \tag{29}$$

$$Q_L = \frac{2k[(2+\sqrt{2}\sqrt{1+C_1})C_2 + \sqrt{2}(-\sqrt{1+C_1} + \sqrt{1+C_2}) - C_1(2+\sqrt{2}\sqrt{1+C_2})]\ln\left[\frac{1+C_2+\sqrt{2}\sqrt{1+C_2}}{1-C_2}\right]T_L}{(2+\sqrt{2}\sqrt{1+C_1})(2+\sqrt{2}\sqrt{1+C_2})}, \tag{30}$$

$$W = 2k[(2+\sqrt{2}\sqrt{1+C_1})C_2 + \sqrt{2}(-\sqrt{1+C_1} + \sqrt{1+C_2}) - C_1(2+\sqrt{2}\sqrt{1+C_2})]\left\{\ln\left[\frac{1+C_1+\sqrt{2}\sqrt{1+C_1}}{1-C_1}\right]T_H - \ln\left[\frac{1+C_2+\sqrt{2}\sqrt{1+C_2}}{1-C_2}\right]T_L\right\} \Bigg/ (2+\sqrt{2}\sqrt{1+C_1})(2+\sqrt{2}\sqrt{1+C_2}). \tag{31}$$

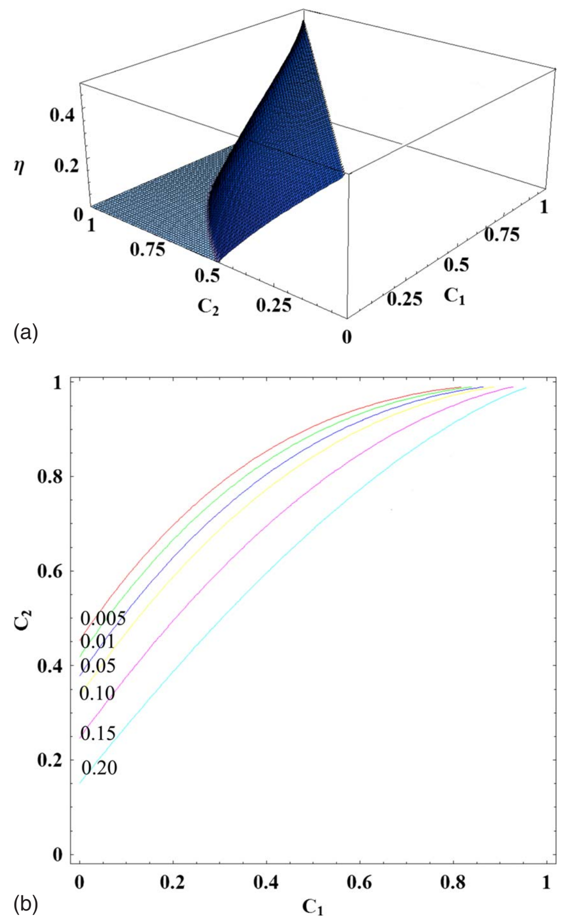

FIG. 5. (Color online) The three-dimensional graph and isoline map of efficiency η varying with the variables C_1 and C_2 for the parameters $G=0$, $T_H=2.0\times10^{-5}$ K, and $T_L=1.0\times10^{-5}$ K.

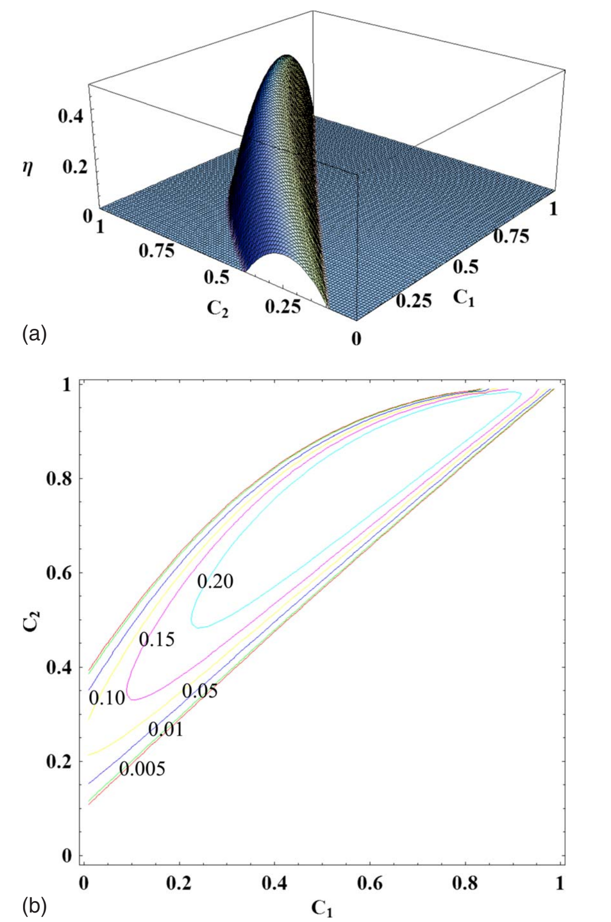

FIG. 6. (Color online) The three-dimensional graph and isoline map of efficiency η varying with the variables C_1 and C_2 for the parameters $G=10g$, $T_H=2.0\times10^{-5}$ K, and $T_L=1.0\times10^{-5}$ K.

The efficiency of the entangled QOE in the case of zero atom-cavity coupling strength is given by

$$\eta = 1 - \frac{\ln\left[\frac{1+C_2+\sqrt{2}\sqrt{1+C_2}}{1-C_2}\right]T_L}{\ln\left[\frac{1+C_1+\sqrt{2}\sqrt{1+C_1}}{1-C_1}\right]T_H} \le \eta_C = 1 - \frac{T_L}{T_H}. \tag{32}$$

It can be verified that the entangled quantum Carnot engine (QCE) is more efficient than the entangled QOE, even for any finite cycle.

Equations (29)–(32) analytically give the expressions for basic thermodynamic quantities and the efficiency in terms of two thermal concurrences C_1 and C_2, respectively. It is found from Fig. 5 that when the case of $G=0$, each isoline of efficiency is an open curve and the efficiency is only a monotonically increasing function of C_1 and a monotonically decreasing function of C_2. That is to say, the larger the thermal concurrence C_1, the larger the efficiency and while the smaller the thermal concurrence C_2, the larger the efficiency.

Combining Eqs. (25) and (27), one can calculate the efficiency of the thermal entangled quantum heat engine in the case of $G\neq0$. But it is too complicate to yield a simple analytical solution. In order to intuitively investigate how entanglement affects η, one can obtain the three-dimensional performance characteristic curves and isoline maps of the efficiency η varying with the thermal concurrences C_1 and C_2 for three representative atom-cavity coupling strength $G=10g$, $G=30g$, and $G=50g$, as shown in Figs. 6–8.

As can be seen from Figs. 6–8 that in the case of nonzero $(G\neq0)$, the efficiency η no longer increases monotonically with C_1 or decreases monotonically with C_2. It is found that in a high enough G $(G\neq0)$, each isoline of efficiency becomes a loop instead of an open curve in zero G $(G=0)$. This indicates that there exists a maximum value efficiency $\eta_{\max}$ when the thermal concurrences C_1 and C_2 attain certain values. That is to say, the efficiency η is doubtlessly affected by nonzero G $(G\neq0)$.

It is clearly seen from Figs. 6–8 that in a high-enough atom-cavity coupling strength, the loops also appear when $C_1>C_2$ whereas in a low atom-cavity coupling strength it seems that only thermal concurrence $C_2>C_1$ is relevant. This can be explained in Fig. 2 that entanglement could increase with the increase of temperature in certain enough G $(G\neq0)$. Thereby $C_1>C_2$ would possibly occur for some G $(G\neq0)$ values. Furthermore, we find that the maximum efficiency $\eta_{\max}$ increase with the increase of G $(G\neq0)$ in the case of thermal concurrence $C_1>C_2$ but the maximum effi-

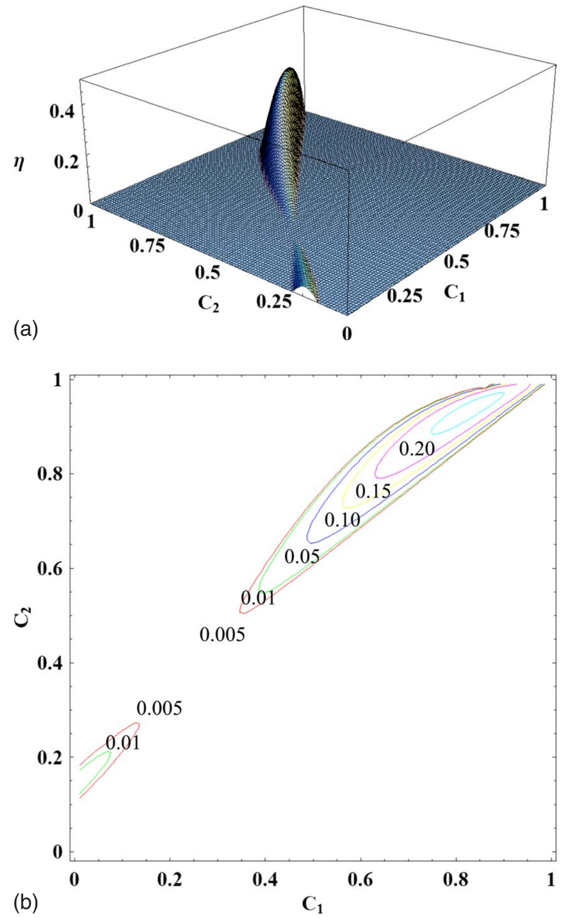

FIG. 7. (Color online) The three-dimensional graph and isoline map of efficiency η varying with the variables C_1 and C_2 for the parameters $G=30g$, $T_H=2.0\times10^{-5}$ K, and $T_L=1.0\times10^{-5}$ K.

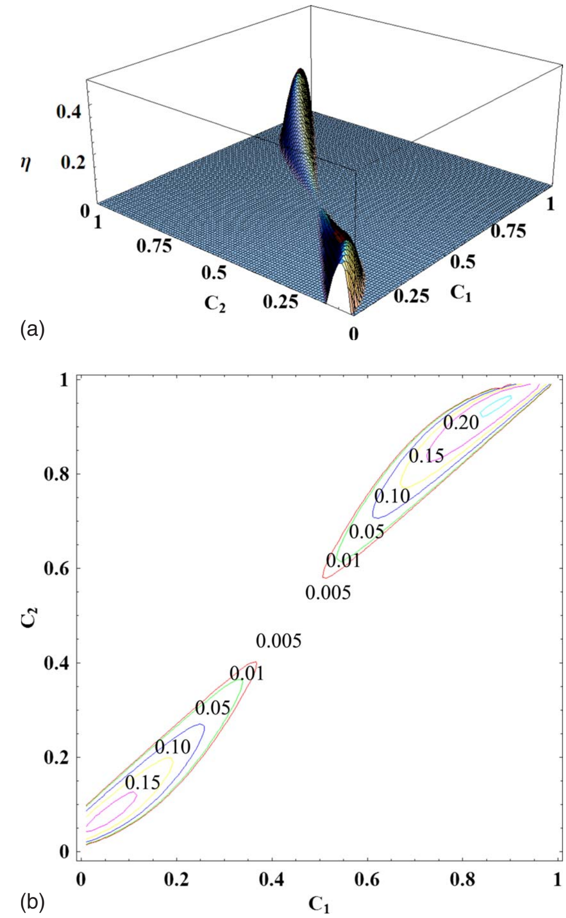

FIG. 8. (Color online) The three-dimensional graph and isoline map of efficiency η varying with the variables C_1 and C_2 for the parameters $G=50g$, $T_H=2.0\times10^{-5}$ K, and $T_L=1.0\times10^{-5}$ K.

ciency $\eta_{\max}$ decrease with the increase of G $(G\neq0)$ in the case of thermal concurrence $C_1<C_2$. It is noteworthy that the maximum efficiency will be different for different G $(G\neq0)$. Moreover, we find that $\eta_C=1-\frac{T_L}{T_H}$ is not achievable in these four figures. Therefore the second law of thermodynamics holds all the while.

V. CONCLUSION

In conclusion, we find that the thermal entanglement exists for the system of two two-level identical atoms couple to single-mode optical cavity. Based on the mode, we have introduced a kind of entangled QOE model. We have presented some interesting results in the entangled QOE. This quantum heat engine can extract work like a two-level quantum heat engine in high and low temperatures, whereas it works in a different way at intermediate temperatures. Expressions for several important performance parameters such as the heat transferred, the work done in a cycle, and the efficiency of the entangled QOE in zero G $(G=0)$ are derived in terms of thermal concurrence. We investigate the influence of entanglement on the efficiency η for zero G $(G=0)$ and nonzero G $(G\neq0)$. For zero G $(G=0)$ and nonzero G $(G\neq0)$, we graphically explore the variation in four thermodynamic quantities and the efficiency with C_1 and C_2, respectively. Some intriguing features and their qualitative explanations are given.

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